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Fuzzy Sets and Systems 114 (2000) 1–9

FUZZY sets and systems

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Extensions of the TOPSIS for group decision-making under fuzzy environment

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Received June 1997; received in revised form October 1997

Abstract

The aim of this paper is to extend the TOPSIS to the fuzzy environment. Owing to vague concepts frequently represented in decision data, the crisp value are inadequate to model real-life situations. In this paper, the rating of each alternative and the weight of each criterion are described by linguistic terms which can be expressed in triangular fuzzy numbers. Then, a vertex method is proposed to calculate the distance between two triangular fuzzy numbers. According to the concept of the TOPSIS, a closeness coefficient is defined to determine the ranking order of all alternatives by calculating the distances to both the fuzzy positive-ideal solution (FPIS) and fuzzy negative-ideal solution (FNIS) simultaneously. Finally, an example is shown to highlight the procedure of the proposed method at the end of this paper. © 2000 Elsevier Science B.V. All rights reserved.

Keywords: TOPSIS; Linguistic variables; Triangular fuzzy number; MCDM

1. Introduction

Decision-making problems is the process of finding the best option from all of the feasible alternatives. In almost all such problems the multiplicity of criteria for judging the alternatives is pervasive. That is, for many such problems, the decision maker wants to solve a multiple criteria decision making (MCDM) problem. A MCDM problem can be concisely expressed in matrix format as

$$\boldsymbol{D} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \end{array},$$

$$W = [w_1\ w_2\ \cdots\ w_n],$$

where $A_1, A_2, \ldots, A_m$ are possible alternatives among which decision makers have to choose, $C_1, C_2, \ldots, C_n$ are criteria with which alternative performance are measured, x_{ij} is the rating of alternative A_i with respect to criterion C_j, w_j is the weight of criterion C_j.

In classical MCMD methods, the ratings and the weights of the criteria are known precisely [5, 10, 13]. A survey of the methods has been presented in Hwang and Yoon [10]. Technique for order

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PII: S0165-0114(97)00377-1

performance by similarity to ideal solution (TOPSIS), one of the known classical MCDM method, was first developed by Hwang and Yoon [10] for solving a MCDM problem. It bases upon the concept that the chosen alternative should have the shortest distance from the positive ideal solution (PIS) and the farthest from the negative ideal solution (NIS). In the process of TOPSIS, the performance ratings and the weights of the criteria are given as crisp values.

Under many conditions, crisp data are inadequate to model real-life situations. Since human judgements including preferences are often vague and cannot estimate his preference with an exact numerical value. A more realistic approach may be to use linguistic assessments instead of numerical values, that is, to suppose that the ratings and weights of the criteria in the problem are assessed by means of linguistic variables [1, 3, 4, 6, 9, 15]. In this paper, we further extended the concept of TOPSIS to develop a methodology for solving multi-person multi-criteria decision-making problems in fuzzy environment. Considering the fuzziness in the decision data and group decision-making process, linguistic variables are used to assess the weights of all criteria and the ratings of each alternative with respect to each criterion. We can convert the decision matrix into a fuzzy decision matrix and construct a weighted normalized fuzzy decision matrix once the decision makers' fuzzy ratings have been pooled. According to the concept of TOPSIS, we define the fuzzy positive ideal solution (FPIS) and the fuzzy negative ideal solution (FNIS). And then, a vertex method is proposed in this paper to calculate the distance between two triangular fuzzy ratings. Using the vertex method, we can calculate the distance of each alternative from FPIS and FNIS, respectively. Finally, a closeness coefficient of each alternative is defined to determine the ranking order of all alternatives. The higher value of closeness coefficient indicates that an alternative is closer to FPIS and farther from FNIS simultaneously.

In order to develop the linguistic TOPSIS method, the paper is organized as follows. Next section introduces the basic definitions and notations of the fuzzy number and linguistic variable. Section 3 presents the linguistic TOPSIS method in group decision making and the choice process. And then, the proposed method is illustrated with an example. Finally, some conclusions are pointed out in the end of this paper.

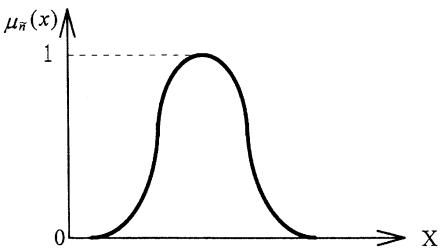

Fig. 1. A fuzzy number $\tilde{n}$.

2. Preliminaries

In the following, we briefly review some basic definitions of fuzzy sets from [2, 11, 12, 14–16]. These basic definitions and notations below will be used throughout the paper until otherwise stated.

Definition 2.1. A fuzzy set $\tilde{A}$ in a universe of discourse X is characterized by a membership function $\mu_{\tilde{A}}(x)$ which associates with each element x in X a real number in the interval $[0, 1]$. The function value $\mu_{\tilde{A}}(x)$ is termed the grade of membership of x in $\tilde{A}$ [14].

Definition 2.2. A fuzzy set $\tilde{A}$ of the universe of discourse X is convex if and only if for all x_1, x_2 in X,

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geqslant \mathrm{Min}(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)), \qquad (1)$$

where $\lambda \in [0, 1]$.

Definition 2.3. A fuzzy set $\tilde{A}$ of the universe of discourse X is called a normal fuzzy set implying that $\exists x_i \in X,\ \mu_{\tilde{A}}(x_i) = 1$.

Definition 2.4. A fuzzy number is a fuzzy subset in the universe of discourse X that is both convex and normal. Fig. 1 shows a fuzzy number $\tilde{n}$ of the universe of discourse X which is both convex and normal.

Definition 2.5. The α-cut of fuzzy number $\tilde{n}$ is defined

$$\tilde{n}^{\alpha} = \{x_i:\ \mu_{\tilde{n}}(x_i) \geqslant \alpha,\ x_i \in X\}, \qquad (2)$$

where $\alpha \in [0, 1]$.

$\tilde{n}^{\alpha}$ is a non-empty bounded closed interval contained in X and it can be denoted by $\tilde{n}^{\alpha} = [n_{\ell}^{\alpha}, n_{\mathrm{u}}^{\alpha}]$, n_{ℓ}^{α} and n_{u}^{α} are the lower and upper bounds of the closed interval, respectively [11, 16]. Fig. 2 shows a fuzzy

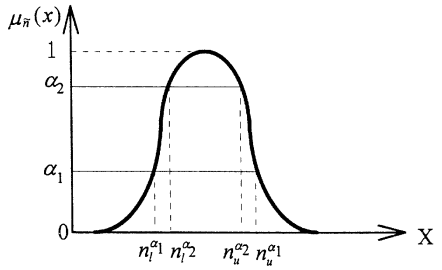

Fig. 2. Fuzzy number $\tilde{n}$ with α-cuts.

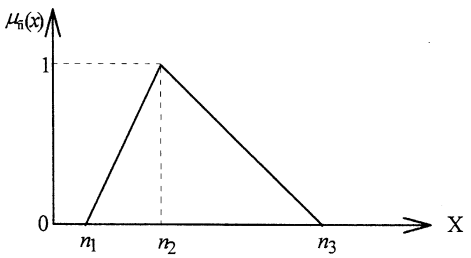

Fig. 3. A triangular fuzzy number $\tilde{n}$.

number $\tilde{n}$ with α-cuts, where

$$\tilde{n}^{\alpha_1} = [n_\ell^{\alpha_1}, n_u^{\alpha_1}], \qquad \tilde{n}^{\alpha_2} = [n_\ell^{\alpha_2}, n_u^{\alpha_2}]. \tag{3}$$

From Fig. 2, we can see that if $\alpha_2 \geqslant \alpha_1$, then $n_\ell^{\alpha_2} \geqslant n_\ell^{\alpha_1}$ and $n_u^{\alpha_1} \geqslant n_u^{\alpha_2}$.

Definition 2.6. A triangular fuzzy number $\tilde{n}$ can be defined by a triplet (n_1, n_2, n_3) shown in Fig. 3. The membership function $\mu_{\tilde{n}}(x)$ is defined as [11]:

$$\mu_{\tilde{n}}(x) = \begin{cases} 0, & x < n_1, \\ \dfrac{x - n_1}{n_2 - n_1}, & n_1 \leqslant x \leqslant n_2, \\ \dfrac{x - n_3}{n_2 - n_3}, & n_2 \leqslant x \leqslant n_3, \\ 0, & x > n_3. \end{cases} \tag{4}$$

Definition 2.7. If $\tilde{n}$ is a fuzzy number and $n_\ell^\alpha > 0$ for $\alpha \in [0, 1]$, then $\tilde{n}$ is called a positive fuzzy number [2, 9].

Given any two positive fuzzy numbers $\tilde{m}, \tilde{n}$ and a positive real number r, the α-cut of two fuzzy numbers are $\tilde{m}^\alpha = [m_\ell^\alpha, m_u^\alpha]$ and $\tilde{n}^\alpha = [n_\ell^\alpha, n_u^\alpha]$ $(\alpha \in [0, 1])$, respectively. According to the interval of confidence [11], some main operations of positive fuzzy numbers $\tilde{m}$ and $\tilde{n}$ can be expressed as follows:

$$(\tilde{m}(+)\tilde{n})^\alpha = [m_\ell^\alpha + n_\ell^\alpha, m_u^\alpha + n_u^\alpha], \tag{5}$$

$$(\tilde{m}(-)\tilde{n})^\alpha = [m_\ell^\alpha - n_u^\alpha, m_u^\alpha - n_\ell^\alpha], \tag{6}$$

$$(\tilde{m}(\cdot)\tilde{n})^\alpha = [m_\ell^\alpha \cdot n_\ell^\alpha, m_u^\alpha \cdot n_u^\alpha], \tag{7}$$

$$(\tilde{m}(:)\tilde{n})^\alpha = \left[\frac{m_\ell^\alpha}{n_u^\alpha}, \frac{m_u^\alpha}{n_\ell^\alpha}\right], \tag{8}$$

$$(\tilde{m}^\alpha)^{-1} = \left[\frac{1}{m_u^\alpha}, \frac{1}{m_\ell^\alpha}\right], \tag{9}$$

$$(\tilde{m}(\cdot)r)^\alpha = [m_\ell^\alpha \cdot r, m_u^\alpha \cdot r], \tag{10}$$

$$(\tilde{m}(:)r)^\alpha = \left[\frac{m_\ell^\alpha}{r}, \frac{m_u^\alpha}{r}\right]. \tag{11}$$

Definition 2.8. Let $\tilde{m} = (m_1, m_2, m_3)$ and $\tilde{n} = (n_1, n_2, n_3)$ be two triangular fuzzy numbers. If $\tilde{m} = \tilde{n}$, then $m_1 = n_1$, $m_2 = n_2$ and $m_3 = n_3$.

Definition 2.9. If $\tilde{n}$ is a triangular fuzzy number and $n_\ell^\alpha > 0$, $n_u^\alpha \leqslant 1$ for $\alpha \in [0, 1]$, then $\tilde{n}$ is called a normalized positive triangular fuzzy number [12].

Definition 2.10. $\tilde{\boldsymbol{D}}$ is called a fuzzy matrix, if at least an entry in $\tilde{\boldsymbol{D}}$ is a fuzzy number [2].

Definition 2.11. A linguistic variable is a variable whose values are linguistic terms [15].

The concept of linguistic variable is very useful in dealing with situations which are too complex or too ill-defined to be reasonably described in conventional quantitative expressions [15]. For example, "weight" is a linguistic variable, its values are very low, low, medium, high, very high, etc. These linguistic values can also be represented by fuzzy numbers.

Definition 2.12. Let $\tilde{m} = (m_1, m_2, m_3)$ and $\tilde{n} = (n_1, n_2, n_3)$ be two triangular fuzzy numbers, then the vertex method is defined to calculate the distance between them as

$$d(\tilde{m}, \tilde{n}) = \sqrt{\tfrac{1}{3}[(m_1 - n_1)^2 + (m_2 - n_2)^2 + (m_3 - n_3)^2]}. \tag{12}$$

Definition 2.13. Let $\tilde{A}$ and $\tilde{B}$ be two triangular fuzzy numbers. The fuzzy number $\tilde{A}$ is closer to fuzzy number $\tilde{B}$ as $d(\tilde{A}, \tilde{B})$ approaches 0.

Many distance measurement functions are proposed in [17], but here the vertex method is an effective and simple method to calculate the distance between two triangular fuzzy numbers. Some important properties of the vertex method are described as follows:

Property 1. *If both $\tilde{m}$ and $\tilde{n}$ are real numbers, then the distance measurement $d(\tilde{m},\tilde{n})$ is identical to the Euclidean distance.*

Proof. Suppose that both $\tilde{m}=(m_1,m_2,m_3)$ and $\tilde{n}=(n_1,n_2,n_3)$ are two real numbers, then let $m_1=m_2=m_3=m$ and $n_1=n_2=n_3=n$. The distance measurement ($d(\tilde{m},\tilde{n})$) can be calculated as

$$\begin{aligned} &d(\tilde{m},\tilde{n})\\ &=\sqrt{\tfrac{1}{3}[(m_1-n_1)^2+(m_2-n_2)^2+(m_3-n_3)^2]}\\ &=\sqrt{\tfrac{1}{3}[(m-n)^2+(m-n)^2+(m-n)^2]}\\ &=\sqrt{(m-n)^2}\\ &=|m-n|. \end{aligned}$$

Property 2. *Two triangular fuzzy numbers $\tilde{m}$ and $\tilde{n}$ are identical if and only if $d(\tilde{m},\tilde{n})=0$.*

Proof. Let $\tilde{m}=(m_1,m_2,m_3)$ and $\tilde{n}=(n_1,n_2,n_3)$ be two triangular fuzzy numbers.

(I) If $\tilde{m}$ and $\tilde{n}$ are identical, then $m_1=n_1$, $m_2=n_2$ and $m_3=n_3$. The distance between $\tilde{m}$ and $\tilde{n}$ is

$$\begin{aligned} &d(\tilde{m},\tilde{n})\\ &=\sqrt{\tfrac{1}{3}[(m_1-n_1)^2+(m_2-n_2)^2+(m_3-n_3)^2]}\\ &=\sqrt{\tfrac{1}{3}[(0)^2+(0)^2+(0)^2]}\\ &=0. \end{aligned}$$

(II) If $d(\tilde{m},\tilde{n})=0$, then

$$\begin{aligned} &d(\tilde{m},\tilde{n})\\ &=\sqrt{\tfrac{1}{3}[(m_1-n_1)^2+(m_2-n_2)^2+(m_3-n_3)^2]}\\ &=0. \end{aligned}$$

Implies that $m_1=n_1$, $m_2=n_2$ and $m_3=n_3$. Therefore, two triangular fuzzy numbers $\tilde{m}$ and $\tilde{n}$ are identical and the property has been proved.

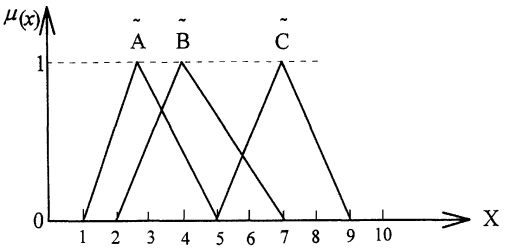

Fig. 4. Three triangular fuzzy numbers.

Property 3. *Let $\tilde{A},\tilde{B}$ and $\tilde{C}$ be three triangular fuzzy numbers. The fuzzy number $\tilde{B}$ is closer to fuzzy number $\tilde{A}$ than the other fuzzy number $\tilde{C}$ if and only if $d(\tilde{A},\tilde{B})<d(\tilde{A},\tilde{C})$.*

This property is trivial. For example, Fig. 4 shows three fuzzy numbers $\tilde{A}=(1,3,5)$, $\tilde{B}=(2,4,7)$ and $\tilde{C}=(5,7,9)$. From Fig. 4, we can easily see that the fuzzy number $\tilde{B}$ is closer to fuzzy number $\tilde{A}$ than the other fuzzy number $\tilde{C}$. According to the vertex method, the distance measurement is calculated as

$$\begin{aligned} d(\tilde{A},\tilde{B})&=\sqrt{\tfrac{1}{3}[(1-2)^2+(3-4)^2+(5-7)^2]}\\ &=\sqrt{2},\\ d(\tilde{A},\tilde{C})&=\sqrt{\tfrac{1}{3}[(1-5)^2+(3-7)^2+(5-9)^2]}=4. \end{aligned}$$

According to the distance measurement and Definition 2.13, we conclude that the fuzzy number $\tilde{B}$ is closer to fuzzy number $\tilde{A}$ than the other fuzzy number $\tilde{C}$.

Property 4. *Let $\tilde{O}=(0,0,0)$ be original. If $d(\tilde{A},\tilde{O})<d(\tilde{B},\tilde{O})$, then fuzzy number $\tilde{A}$ is closer to original than the other fuzzy number $\tilde{B}$.*

According to the Property 3, this property is trivial and easily proved.

3. The proposed method

A systematic approach to extend the TOPSIS to the fuzzy environment is proposed in this section. This method is very suitable for solving the group decision-making problem under fuzzy environment. In

Table 1
Linguistic variables for the importance weight of each criterion

Very low (VL)	(0, 0, 0.1)
Low (L)	(0, 0.1, 0.3)
Medium low (ML)	(0.1, 0.3, 0.5)
Medium (M)	(0.3, 0.5, 0.7)
Medium high (MH)	(0.5, 0.7, 0.9)
High (H)	(0.7, 0.9, 1.0)
Very high (VH)	(0.9, 1.0, 1.0)

Table 2
Linguistic variables for the ratings

Very poor (VP)	(0, 0, 1)
Poor (P)	(0, 1, 3)
Medium poor (MP)	(1, 3, 5)
Fair (F)	(3, 5, 7)
Medium good (MG)	(5, 7, 9)
Good (G)	(7, 9, 10)
Very good (VG)	(9, 10, 10)

this paper, the importance weights of various criteria and the ratings of qualitative criteria are considered as linguistic variables. These linguistic variables can be expressed in positive triangular fuzzy numbers as Tables 1 and 2.

The importance weight of each criterion can be obtained by either directly assign or indirectly using pairwise comparisons [7]. In here, it is suggested that the decision makers use the linguistic variables (shown as Tables 1 and 2) to evaluate the importance of the criteria and the ratings of alternatives with respect to various criteria.

Assume that a decision group has K persons, then the importance of the criteria and the rating of alternatives with respect to each criterion can be calculated as

$$\tilde{x}_{ij} = \frac{1}{K}[\tilde{x}_{ij}^1(+)\tilde{x}_{ij}^2(+)\cdots(+)\tilde{x}_{ij}^K] \tag{13}$$

$$\tilde{w}_j = \frac{1}{K}[\tilde{w}_j^1(+)\tilde{w}_j^2(+)\cdots(+)\tilde{w}_j^K] \tag{14}$$

where $\tilde{x}_{ij}^K$ and $\tilde{w}_j^K$ are the rating and the importance weight of the Kth decision maker.

As stated above, a fuzzy multicriteria group decision-making problem which can be concisely expressed in matrix format as

$$\tilde{\boldsymbol{D}} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{11} & \dots & \tilde{x}_{11} \\ \tilde{x}_{21} & \tilde{x}_{22} & \dots & \tilde{x}_{2n} \\ \vdots & \vdots & \dots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \end{bmatrix},$$

$$\tilde{W} = [\tilde{w}_1,\ \tilde{w}_2,\ \dots,\ \tilde{w}_n]$$

where $\tilde{x}_{ij},\ \forall i,j$ and $\tilde{w}_j,\ j=1,2,\dots,n$ are linguistic variables. These linguistic variables can be described by triangular fuzzy numbers, $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij})$ and $\tilde{w}_j = (w_{j1}, w_{j2}, w_{j3})$.

To avoid the complicated normalization formula used in classical TOPSIS, the linear scale transformation is used here to transform the various criteria scales into a comparable scale. Therefore, we can obtain the normalized fuzzy decision matrix denoted by $\tilde{\boldsymbol{R}}$.

$$\tilde{\boldsymbol{R}} = [\tilde{r}_{ij}]_{m\times n}, \tag{15}$$

where B and C are the set of benefit criteria and cost criteria, respectively, and

$$\tilde{r}_{ij} = \left(\frac{a_{ij}}{c_j^*}, \frac{b_{ij}}{c_j^*}, \frac{c_{ij}}{c_j^*}\right), \quad j \in B;$$

$$\tilde{r}_{ij} = \left(\frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}}\right), \quad j \in C;$$

$$c_j^* = \max_i c_{ij} \quad \text{if } j \in B;$$

$$a_j^- = \min_i a_{ij} \quad \text{if } j \in C.$$

The normalization method mentioned above is to preserve the property that the ranges of normalized triangular fuzzy numbers belong to [0, 1].

Considering the different importance of each criterion, we can construct the weighted normalized fuzzy decision matrix as

$$\tilde{V} = [\tilde{v}_{ij}]_{m\times n}, \quad i=1,2,\dots,m,\ j=1,2,\dots,n, \tag{16}$$

where $\tilde{v}_{ij} = \tilde{r}_{ij}(\cdot)\tilde{w}_j$.

According to the weighted normalized fuzzy decision matrix, we know that the elements $\tilde{v}_{ij},\ \forall i,j$ are normalized positive triangular fuzzy numbers and their ranges belong to the closed interval [0, 1]. Then, we can define the fuzzy positive-ideal solution (FPIS, A^*) and fuzzy negative-ideal solution

(FNIS, A^-) as

$$A^* = (\tilde{v}_1^*, \tilde{v}_2^*, \ldots, \tilde{v}_n^*),$$
$$A^- = (\tilde{v}_1^-, \tilde{v}_2^-, \ldots, \tilde{v}_n^-),$$

where $\tilde{v}_j^* = (1,1,1)$ and $\tilde{v}_j^- = (0,0,0)$, $j = 1,2,\ldots,n$.

The distance of each alternative from A^* and A^- can be currently calculated as

$$d_i^* = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^*), \quad i = 1,2,\ldots,m, \tag{17}$$

$$d_i^- = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^-), \quad i = 1,2,\ldots,m, \tag{18}$$

where $d(\cdot,\cdot)$ is the distance measurement between two fuzzy numbers.

A closeness coefficient is defined to determine the ranking order of all alternatives once the d_i^* and d_i^- of each alternative A_i $(i = 1,2,\ldots,m)$ has been calculated. The closeness coefficient of each alternative is calculated as

$$CC_i = \frac{d_i^-}{d_i^* + d_i^-}, \quad i = 1,2,\ldots,m. \tag{19}$$

Obviously, an alternative A_i is closer to the FPIS (A^*) and farther from FNIS(A^-) as CC_i approaches to 1. Therefore, according to the closeness coefficient, we can determine the ranking order of all alternatives and select the best one from among a set of feasible alternatives.

In sum, an algorithm of the multi-person multi-criteria decision making with fuzzy set approach is given in the following.

Step 1: Form a committee of decision-makers, then identify the evaluation criteria.

Step 2: Choose the appropriate linguistic variables for the importance weight of the criteria and the linguistic ratings for alternatives with respect to criteria.

Step 3: Aggregate the weight of criteria to get the aggregated fuzzy weight $\tilde{w}_j$ of criterion C_j, and pool the decision makers' opinions to get the aggregated fuzzy rating $\tilde{x}_{ij}$ of alternative A_i under criterion C_j.

Step 4: Construct the fuzzy decision matrix and the normalized fuzzy decision matrix.

Step 5: Construct the weighted normalized fuzzy decision matrix.

Step 6: Determine FPIS and FNIS.

Step 7: Calculate the distance of each alternative from FPIS and FNIS, respectively.

Step 8: Calculate the closeness coefficient of each alternative.

Step 9: According to the closeness coefficient, the ranking order of all alternatives can be determined.

4. Numerical example

Suppose that a software company desires to hire a system analysis engineer. After preliminary screening, three candidates A_1, A_2 and A_3 remain for further evaluation. A committee of three decision-makers, D_1, D_2 and D_3 has been formed to conduct the interview and to select the most suitable candidate. Five benefit criteria are considered:

(1) emotional steadiness (C_1),
(2) oral communication skill (C_2),
(3) personality (C_3),
(4) past experience (C_4),
(5) self-confidence (C_5).

The hierarchical structure of this decision problem is shown as Fig. 5. The proposed method is currently applied to solve this problem and the computational procedure is summarized as follows:

Step 1: The decision-makers use the linguistic weighting variables (shown in Table 1) to assess the importance of the criteria and present it in Table 3.

Step 2: The decision-makers use the linguistic rating variables (shown in Table 2) to evaluate the rating of alternatives with respect to each criterion and present it in Table 4.

Step 3: Converting the linguistic evaluation (shown in Tables 3 and 4) into triangular fuzzy numbers to construct the fuzzy decision matrix and determine the fuzzy weight of each criterion as Table 5.

Table 3
The importance weight of the criteria

	D_1	D_2	D_3
C_1	H	VH	MH
C_2	VH	VH	VH
C_3	VH	H	H
C_4	VH	VH	VH
C_5	M	MH	MH

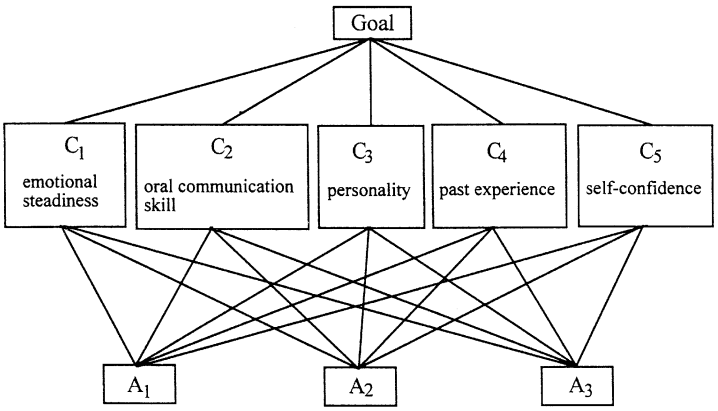

Fig. 5. The hierarchical structure.

Table 4
The ratings of the three candidates by decision makers under all criteria

Criteria	Candidates	Decision-makers		
		D_1	D_2	D_3
C_1	A_1	MG	G	MG
	A_2	G	G	MG
	A_3	VG	G	F
C_2	A_1	G	MG	F
	A_2	VG	VG	VG
	A_3	MG	G	VG
C_3	A_1	F	G	G
	A_2	VG	VG	G
	A_3	G	MG	VG
C_4	A_1	VG	G	VG
	A_2	VG	VG	VG
	A_3	G	VG	MG
C_5	A_1	F	F	F
	A_2	VG	MG	G
	A_3	G	G	MG

Step 4: Constructing the normalized fuzzy decision matrix as Table 6.

Step 5: Constructing the weighted normalized fuzzy decision matrix as Table 7.

Step 6: Determine FPIS and FNIS as

$$A^* = [(1,1,1),(1,1,1),(1,1,1),(1,1,1),(1,1,1)],$$
$$A^- = [(0,0,0),(0,0,0),(0,0,0),(0,0,0),(0,0,0)].$$

Step 7: Calculate the distance of each candidate from FPIS and FNIS, respectively, as Table 8.

Step 8: Calculate the closeness coefficient of each candidate as

$$CC_1 = 0.62, \quad CC_2 = 0.77, \quad CC_3 = 0.71.$$

Step 9: According to the closeness coefficient, the ranking order of the three candidates is A_2, A_3, and A_1. Obviously, the best selection is candidate A_2.

5. Conclusion

In general, multicriteria problems adhere to uncertain and imprecise data, and fuzzy set theory is adequate to deal with it. In this paper, a linguistic decision process is proposed to solve the multiple criteria decision-making problem under fuzzy environment.

In decision-making process, very often, the assessment of alternatives with respect to criteria and the importance weight are suitable to use the linguistic variables instead of numerical values. Here, we propose a vertex method which is an effective and simple method to measure the distance between two triangular fuzzy numbers, and extend the TOPSIS procedure to the fuzzy environment. In fact, the vertex method can be easily applied to calculate the distance between any two fuzzy numbers which their membership function are linear. Under group decision-making process, it is not difficult to use other aggregation functions

Table 5
The fuzzy decision matrix and fuzzy weights of three alternatives

	C_1	C_2	C_3	C_4	C_5
A_1	(5.7, 7.7, 9.3)	(5, 7, 9)	(5.7, 7.7, 9)	(8.33, 9.67, 10)	(3, 5, 7)
A_2	(6.3, 8.3, 9.7)	(9, 10, 10)	(8.3, 9.7, 10)	(9, 10, 10)	(7, 9, 10)
A_3	(6.3, 8, 9)	(7, 9, 10)	(7, 9, 10)	(7, 9, 10)	(6.3, 8.3, 9.7)
Weight	(0.7, 0.9, 1)	(0.9, 1, 1)	(0.77, 0.93, 1)	(0.9, 1, 1)	(0.43, 0.63, 0.83)

Table 6
The fuzzy normalized decision matrix

	C_1	C_2	C_3	C_4	C_5
A_1	(0.59, 0.79, 0.96)	(0.5, 0.7, 0.9)	(0.57, 0.77, 0.9)	(0.83, 0.97, 1)	(0.3, 0.5, 0.7)
A_2	(0.65, 0.86, 1)	(0.9, 1, 1)	(0.83, 0.97, 1)	(0.9, 1, 1)	(0.7, 0.9, 1)
A_3	(0.65, 0.82, 0.93)	(0.7, 0.9, 1)	(0.7, 0.9, 1)	(0.7, 0.9, 1)	(0.63, 0.83, 0.97)

Table 7
The fuzzy weighted normalized decision matrix

	C_1	C_2	C_3	C_4	C_5
A_1	(0.41, 0.71, 0.96)	(0.45, 0.7, 0.9)	(0.44, 0.72, 0.9)	(0.75, 0.97, 1)	(0.13, 0.32, 0.58)
A_2	(0.46, 0.77, 1)	(0.81, 1, 1)	(0.64, 0.9, 1)	(0.81, 1, 1)	(0.3, 0.57, 0.83)
A_3	(0.46, 0.74, 0.93)	(0.63, 0.9, 1)	(0.54, 0.84, 1)	(0.63, 0.9, 1)	(0.27, 0.52, 0.81)

Table 8
The distance measurement

	A^*	A^-
A_1	2.10	3.45
A_2	1.24	4.13
A_3	1.59	3.85

[8] to pool the fuzzy ratings of decision-makers in the proposed method.

Although the proposed method presented in this paper is illustrated by a personal selection problem, however, it can also be applied to problems such as information project selection, material selection and many other areas of management decision problems.

Acknowledgements

The author gratefully acknowledges financial support from the National Science Council, Taiwan, Republic of China, under project NSC 87-2213-E-240-001.

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