A Revised VIKOR Model for Multiple Criteria Decision Making - The Perspective of Regret Theory

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Abstract. VIKOR is one of the multiple criteria decision making (MCDM) models to determine the preference ranking from a set of alternatives in the presence of conflicting criteria. The justification of VIKOR is to use the concept of the compromise programming to determine the preference ranking by the results of the individual and group regrets. However, VIKOR has a critical problem in deriving the preference ranking of alternative and this issue will be discussed later. In this paper, the perspective of regret theory is applied to explain the content of VIKOR and one revised model of VIKOR is proposed base on the concept of regret theory. Then, two examples are given to justify the proposed model and compare it with VIKOR and the regret model.

Keywords: multiple criteria decision making (MCDM); VIKOR; compromise programming; regret model; preference ranking.

1 Introduction

Decision-making process involves a series of identifying the problems, constructing the preferences, evaluating the alternatives, and determining the best alternative [1-3]. Decision making is extremely intuitive while considering the single criterion problems, since we only need to choose the alternative with the highest preference rating. However, when decision makers evaluate the alternatives with the multiple criteria, many problems, such as weights of criteria, preference dependence, and conflicts among criteria, seem to complicate the decision problems and should be overcome by more sophisticate methods.

The field of multiple criteria decision making (MCDM) concerns the problems that how decision makers should ideally do when facing multiple conflicting criteria. There are considerable methods and models have been proposed for various MCDM problems with respect to different perspectives and theories. In this paper, we focus on the VIKOR method which was proposed by [4,5] to determine the ranking and select from a set of alternatives in the presence of conflicting criteria. The major characteristic of VIKOR is it introduces the multiple criteria ranking index based on the

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particular measure of closeness to the ideal solution [4] to derive the preference ranking of alternatives.

Recently, VIKOR has been widely applied for dealing with MCDM problems of various fields, such as location selection [6], environmental policy [7] and data envelopment analysis [8]. However, it may derive the false preference ranking, even if an easy MCDM problem is considered. This problem will be highlighted later.

In this paper, we propose a revised VIKOR model, which is based on the concept of regret theory, to overcome the defeat of VIKOR. The major characteristic of the proposed model is its levels of regret are measured by a fixed reference and other alternatives. In addition, we give two examples to compare the result of the proposed model with VIKOR and the regret model.

2 VIKOR

The procedure of VIKOR can be described as follows. Assuming that each alternative is evaluated according to each criterion function, the compromise ranking could be performed by comparing the measure of regret (i.e., closeness to the ideal alternative). The multi-criteria measure for compromise ranking is developed from the L_p – norm used as an aggregating function in a compromise programming method [9,10]. The various J alternatives are denoted as $a_1, a_2, \ldots, a_J$. For alternative a_j, the rating of the *i*th criterion is denoted by f_{ji}, i.e., f_{ji} is the value of ith criterion function for the alternative a_j and n is the number of criteria. The levels of regret in VIKOR can be defined as:

$$L_{p,j} = \{\sum_{i=1}^{n} [w_i (f_i^* - f_{ji}) / (f_i^* - f_i^-)]^p\}^{1/p}, 1 \leq p \leq \infty; j = 1, 2, \ldots, J \,, \quad (1)$$

where $L_{1,j}$ is defined as the maximum group utility and $L_{\infty,j}$ is defined as the minimum individual regret of the opponent.

The procedure of VIKOR for ranking alternatives can be described as the following steps:

(a) Determine that best f_i^* and the worst f_i^- values of all criterion functions, where $i = 1, 2, \ldots, n$. If the *i*th function represents a benefit then $f_i^* = \max_j f_{ji}$, $f_i^- = \min_j f_{ji}$.

(b) Compute the S_j (the maximum group utility) and R_j (the minimum individual regret of the opponent) values, $j = 1, 2, \ldots, J$, by the relations

$$S_j = L_{1,j} = \sum_{i=1}^{n} w_i (f_i^* - f_{ji}) / (f_i^* - f_i^-), \quad (2)$$

$$R_j = L_{\infty,j} = \max_i [\sum_{i=1}^{n} w_i (f_i^* - f_{ji}) / (f_i^* - f_i^-)], \tag{3}$$

where w_i is the weight of the ith criterion which expresses the relative importance of criteria.

(c) Compute the value $Q_j, j = 1, 2, \ldots, J,$ by the relation

$$Q_j = v(S_j - S^*) / (S^- - S^*) + (1 - v)(R_j - R^*) / (R^- - R^*), \tag{4}$$

where $S^* = \min_j S_j$, $S^- = \max_j S_j$, $R^* = \min_j R_j$, $R^- = \max_j R_j$, and v is introduced as weight of the strategy of S_j and R_j.

(d) Rank the alternatives, sorting by the S, R and Q values, in decreasing order. The results are three ranking lists.

(e) Propose as a compromise solution the alternative (a') which is ranked the best by the minimum Q if the following two conditions are satisfied:
C1. "Acceptable advantage":
$Q(a'') - Q(a') \geq DQ$, where a'' is the alternative with second position in the ranking list by Q, $DQ = 1/(J - 1)$ and J is the number of alternatives.
C2. "Acceptable stability in decision making":
Alternative a' must also be the best ranked by S or/and R. This compromise solution is stable within a decision making process, which could be: "voting by majority rule" (when $v > 0.5$ is needed), or "by consensus" $v \approx 0.5$, or "with vote" ($v < 0.5$). Here, v is the weight of the decision making strategy "the majority of criteria" (or "the maximum group utility"). In this paper, $v = 0.5$ is used in numerical examples.

If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of

(i) Alternative a' and a'' if only condition **C2** is not satisfied, or

(ii) Alternative a', a'', $\ldots, a^{(M)}$ if condition **C1** is not satisfied; and $a^{(M)}$ is determined by the relation $Q(a^{(M)} - Q(a')) < DQ$ for maximum M (the positions of these alternatives are in closeness).

VIKOR is a helpful tool in multi-criteria decision making, particularly in a situation where the decision maker is not able to express his/her preference at the beginning of system design. The obtained solution is compromised by a maximum group utility (represented by $\min S_j$) of the majority, and a minimum of the individual regret (represented by $\min R_j$) of the opponent.

Next, we give an example to emphasize the problem of VIKOR as follows. Consider a house selection problem. Three candidate houses in a choice set are evaluated by two criteria, Size (C_1) and Age (C_2), to determine the best alternative, as shown in Table 1. For simplicity, we transformed the realistic value into the classical utility, where is normalized into [0,1]. By using the procedure of VIKOR, we can calculate the *S*, *R* and *Q* values as shown in Table 1 to derive the preference ranking of the alternatives.

Table 1. The problem of VIKOR

House	Criteria		VIKOR			Preference ranking
	C_1	C_1	*S*	*R*	*Q*	
A_1	0.8	0.5	0.375	0.375	0.000	
A_2	0.6	0.8	0.500	0.500	0.667	$A_1 \succ A_2 \succ A_3$
A_3	0.7	0.4	0.750	0.500	1.000	
Weight	0.5	0.5				

From the result of VIKOR, A_1 should be the best alternative since it has the lowest *S*, *R* and *Q* values with respect to other alternatives. However, it is clearly wrong. The reason is that obviously A_2 should be better than A_1 since the total utility level of A_2 is higher than that of A_1, while two criteria are equal in importance. This induction can be justified if we only consider A_1 and A_2 in the choice set.

The above problem is caused by the inappropriate definitions of calculating *S* and *R* values, i.e., Equations (2) and (3). The original concept of VIKOR is to compromise two different kinds of regret, i.e., *S* and *R* values, to obtain the preference ranking. However, since regret is only matter with the difference between alternatives and the best value of each criterion, it is unnecessary to normalize the levels of regret, i.e., considering the denominator of Equation (2) or (3). In other words, if we normalize the levels of regret, it will be affected by both the best and worst values of each criterion, instead of only the best value of each criterion. To overcome the above problem, we will propose a revised VIKOR model in the next section.

3 A Revised VIKOR Model

Before revising VIKOR, we first review the concept of regret theory. Regret theory was first proposed by [11,12], and the idea of regret theory is humans' decision making are affected by emotions. Emotions can be considered as mental and psychological states and used to be thought as the one of the main reasons for triggering decision making. Therefore, [11,12] introduced the feelings of regret and rejoicing (later, [13] added the feelings of disappointment and elation) and proposed a regret model for decision making under uncertainty. The main concept of regret theory is that the classical utility function is modified by incorporating the anticipated feelings of a decision-maker [14]. The contents of regret theory can be described as follows.

Assume the degree of regret or rejoicing that an individual experiences depends only on the difference between the choiceless utility of what is and the choiceless utility of what might have been. Note that the choiceless utility is the utility that an individual would derive from the consequence if he/she experienced it without having chosen it and is defined independently of choices [12]. Then, we can define a regret-rejoicing function to modify the classical utility [15] such that

$$u_{ik}^{i} = u_{ji} + R(u_{ji} - u_{ki}), \tag{5}$$

where u_{ij}^{k} denotes the modified utility of the *j*th alternative with respect to the *k*th alternative in the *i*th criterion, u_{ji} is the classic utility of the *j*th alternative in the *i*th criterion defined over outcomes, and $R(\cdot)$ denotes a regret-rejoicing function such that $R'(\cdot) > 0$ and $R''(\cdot) < 0$.

Therefore, for a MCDM problem, the modified utility for the *j*th alternative which calculated by regret theory can be presented as

$$U_j = \sum_{i=1}^{n} w_i [u_{ji} - \frac{1}{J-1} \sum_{j=1}^{J} R(f_{ji} - f_{ki})], \tag{6}$$

where w_i denotes the weight of the *i*th criterion and J is the number of criteria.

From Equation (5), it can be seen that the regret function proposed by Loomes and Sugden [12] is a regret-rejoicing function, since $R(f_{ji} - f_{ki})$ may be large than zero (i.e., rejoicing) or less than zero (i.e., regret). Thus, the degrees of regret and rejoicing are reflected by a singular function. However, this assumption seems to be against our intuition since post studies (e.g., Inman, et al. [16]) suggested that the influences of regret are much more important than that of rejoicing.

Here, VIKOR is revised by using the concept of regret theory as follows. The distinction between VIKOR and regret theory is that VIKOR defines the regret as the difference between alternatives and the best value of each criterion, namely, the discontent utility in this paper, and regret theory defines the regret as the choiceless utility.

In our revised VIKOR model, values S_j and R_j denotes the choiceless and discontent utilities, respectively, and can be defined as:

$$S_j = \begin{cases} \| \sum_{i=1}^{n} w_i \frac{1}{J-1} \sum_{k=1}^{J} (f_{ki} - f_{ji}) \|_p, if\ f_{ji} < f_{ki}, \\ 0 \qquad\qquad , \text{otherwise} \end{cases} \tag{7}$$

and

$$R_j = \| \sum_{i=1}^{n} w_i (f_i^* - f_{ji}) \|_p, \tag{8}$$

where f_i^* is the best value of the ith criterion and $\|\cdot\|_p$ denotes the L_p - norm. Note that in this paper, we use L_2 - norm to calculate S_j and R_j values.

Next, we can propose the synthesized index Q_j as

$$Q_j = v(S_j - S^*)/(S^- - S^*) + (1-v)(R_j - R^*)/(R^- - R^*) \tag{9}$$

where $S^* = \min_j S_j$, $S^- = \max_j S_j$, $R^* = \min_j R_j$, $R^- = \max_j R_j$, and v is introduced as weight of the strategy of S_j and R_j. The procedure of preference ranking of alternative in the proposed model is similar to VIKOR. Next, we will give two examples to illustrate the proposed method and compare the preference ranking with the VIKOR and regret models.

4 Examples

In this section, we give two examples to demonstrate the proposed method and uncover the irrational results of the VKIOR and regret models.

Example 1. In this example, we review the previous example as shown in the section of VIKOR. Besides VIKOR, we also employ the regret model and the proposed model to derive the preference ranking of the alternatives, as shown in Table 2.

Table 2. The problem of house selection for Example 1

House	Criteria		VIKOR			Regret model	Revised VIKOR		
	C_1	C_2	S	R	Q		S	R	Q
A_1	0.8	0.5	0.375	0.375	0.000	0.628	0.150	0.212	0.462
A_2	0.6	0.8	0.500	0.500	0.667	0.688	0.100	0.141	0.000
A_3	0.7	0.4	0.750	0.500	1.000	0.505	0.260	0.292	1.000
Weight	0.5	0.5							
Preference ranking			$A_1 \succ A_2 \succ A_3$			$A_2 \succ A_1 \succ A_3$	$A_2 \succ A_1 \succ A_3$		

From Table 2, it can be seen that except VIKOR, regret theory and the proposed model can derive the correct preference ranking of the alternatives.

Example 2. Here, we employ the example from [17] to describe the problem of VIKOR and regret theory by considering the following two cases.

Case 1: Car A is rated "very high reliability" with classical utility of 1.0 out of a possible 1.0. Car B is rated "high reliability" with a classical utility of 0.8. For the other criteria the preference is Car A□ Car B.

Case 2: Car C is rated "average reliability" with a classical utility of 0.6 out of a possible 1.0. Car D is rated "below average reliability" with a classical utility of 0.4. For the other criteria the preference is Car C ⊔ Car D.

According to the above statement with respect to two cases, a decision-maker should feel worse in Case 2 than in Case 1 for having chosen the car with the lower reliability. However, the above induction is inconsistent with the regret model since it only considers the choiceless utility of alternatives and ignores the discontent utility. That is, the level of regret to choose Car D and forgo Car C is identical to that of choosing Car B and forgoing Car A. In contrast, since the proposed model considers both the choiceless and discontent utilities, the different regret of Case 1 and Case 2 can be reflected by the discontent utility. Therefore, the proposed model should be more accurately reflect the realistic MCDM problems. Next, we will give the discussion in detail according to the results of the examples.

5 Discussions

The major problem of VIKOR is that it may derive the incorrect preference ranking of alternatives because of the problematic Equations (2) and (3). More specifically, the normalization of the S and R values will result in the levels of regrets are influenced by the worst values of criteria. However, according to the perspective of regret theory, the levels of regret are only affected by the best values of criteria.

On the other hand, regret theory suggests that the level of regret comes from the choiceless utility which is the utility that an individual would derive from the consequence if he/she experienced it without having chosen it. However, regret theory ignores the problem of the discontent utility. From the description of Example 2, we can conclude that regret can also be influenced by the best values of criteria.

In this paper, we revised the VIKOR model to consider both the choiceless and discontent utilities so that the S and R values are represented the choiceless and discontent utilities, respectively. In addition, the levels of regrets are not normalized in order to avoid the possible problem of the preference ranking of alternatives. The comparison of the above three models can be summarized as shown in Table 3.

Table 3. The comparison of the VIKOR, regret and the proposed models

Dimension	**VIKOR**	**Regret model**	**Revised VIKOR**
Feeling	Regret	Regret and rejoicing	Regret
Utility	Discontent utility	Choiceless utility	Both
Index	Multiple indices	Single index	Multiple indices
Ranking	The smaller the better	The larger the better	The smaller the better
Core concept	Compromise programming	Regret theory	Regret theory

It should be highlighted that although the S and R values are both used in VIKOR and the proposed model, their meanings are absolutely different. The S value in our model represents the choiceless utility. However, the choiceless utility is not considered in VIKOR. On the other hand, the R value in the proposed model includes the S or R values in VIKOR while we use L_1-norm or L_∞-norm to measure the discontent utility.

6 Conclusions

In this paper, we highlight the problem of VIKOR for dealing with MCDM problems in order to avoid possible applications. The main error of VIKOR is caused by the problematic equations to calculate regrets from the best values of criteria. In order to revise the VIKOR model, the perspective of regret theory is employed. In the proposed model, two different kinds of regret, namely the discontent and choiceless utilities, are included to reflect the choice behavior of decision-makers. From the results of examples, we can conclude that the proposed model is more suitable for dealing with realistic MCDM problems than VIKOR and regret models.

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