SECOND EDITION

THE FRANK J. FABOZZI SERIES

financial management & analysis

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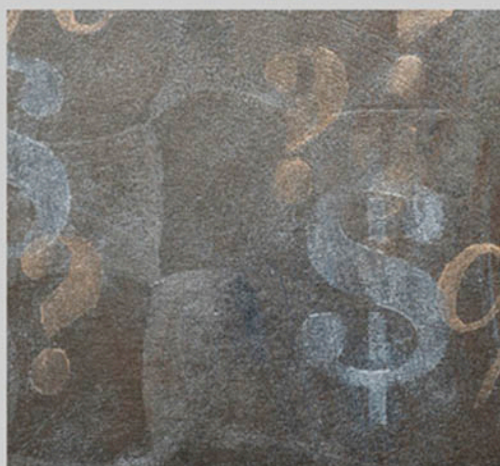

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Financial Management and Analysis

Second Edition

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Financial Management and Analysis

Second Edition

FRANK J. FABOZZI

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To my wife and children, Francesco, Patricia, and Karly

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To my children, Ken and Erica

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Preface

Financial Management and Analysis is an introduction to the concepts, tools, and applications of finance. The purpose of this textbook is to communicate the fundamentals of financial management and financial analysis. This textbook is written in a way that will enable students who are just beginning their study of finance to understand financial decision-making and its role in the decision-making process of the entire firm.

Throughout the textbook, you'll see how we view finance. We see financial decision-making as an integral part of the firm's decision-making, not as a separate function. Financial decision-making involves coordination among personnel specializing in accounting, marketing, and production aspects of the firm.

The principles and tools of finance are applicable to all forms and sizes of business enterprises, not only to large corporations. Just as there are special problems and opportunities for small family-owned businesses (such as where to obtain financing), there are special problems and opportunities for large corporations (such as agency problems that arise when management of the firm is separated from the firm's owners). But the fundamentals of financial management are the same regardless of the size or form of the business. For example, a dollar today is worth more than a dollar one year from today, whether you are making decisions for a sole proprietorship or a large corporation.

We view the principles and tools of finance as applicable to firms around the globe, not just to U.S. business enterprises. While customs and laws may differ among nations, the principles, theories, and tools of financial management do not. For example, in evaluating whether to buy a particular piece of equipment, you must evaluate what happens to the firm's future cash flows (How much will they be? When will they occur? How uncertain are they?), whether the firm is located in the United States, Great Britain, or elsewhere.

In addition, we believe that a strong foundation in finance principles and the related mathematical tools are necessary for you to understand how investing and financing decisions are made. But building that foundation need not be strenuous. One way that we try to help you build that foundation is to present the principles and theories of finance using intuition, instead of with proofs and theorems. For example, we walk you through the intuition of capital structure theory with numerical and real world examples, not equations and proofs. Another way we try to assist you is to approach the tools of finance using careful, step-by-step examples and numerous graphs.

ORGANIZATION

Financial Management and Analysis is presented in seven parts. The first two parts (Parts One and Two) cover the basics, including the objective of financial management, valuation principles, and the relation between risk and return. Financial decision-making is covered in Parts Three, Four, and Five where we present long-term investment management (commonly referred to as capital budgeting), the management of long-term sources of funds, and working capital management. Part Six covers financial statement analysis which includes financial ratio analysis, earnings analysis, and cash flow analysis. The last part (Part Seven) covers several specialized topics: international financial management, borrowing via structured financial transactions (i.e., asset securitization), project financing, equipment leasing, and financial planning and strategy.

DISTINGUISHING FEATURES OF THE TEXTBOOK

Logical structure. The text begins with the basic principles and tools, followed by long-term investment and financing decisions. The first two parts lay out the basics; Part Three then focuses on the “left side” of the balance sheet (the assets) and the Part Four is the “right side” of the balance sheet (the liabilities and equity). Working capital decisions, which are made to support the day-to-day operations of the firm, are discussed in Part Five. Part Six provides the tools for analyzing a firm’s financial statements. In the last chapter of the book, you are brought back full-circle to the objective of financial management: the maximization of owners’ wealth.

Graphical illustrations. Graphs and illustrations have been carefully and deliberately developed to depict and provide visual reinforcement of mathematical concepts. For example, we show the growth of a bank balance through compound interest several ways: mathematically, in a time-line, and with a bar graph.

Applications. As much as possible, we develop concepts and mathematics using examples of actual practice. For example, we first present financial analysis using a simplified set of financial statements for a fictitious company. After you've learned the basics using the fictitious company, we demonstrate financial analysis tools using data from Wal-Mart Stores, Inc. Actual examples help you better grasp and retain major concepts and tools. We integrate over 100 actual company examples throughout the text, so you're not apt to miss them. Considering both the examples throughout the text and the research questions and problems, you are exposed to hundreds of actual companies.

Extensive coverage of financial statement analysis. While most textbooks provide some coverage of financial statement analysis, we have provided you with much more detail in Part Six of the textbook. Chapter 6 and the three chapters in Part Six allow an instructor to focus on financial statement analysis.

Extensive coverage of alternative debt instruments. Because of the innovations in the debt market, alternative forms debt instruments can be issued by a corporation. In Chapter 15, you are introduced to these instruments. We then devote one chapter to the most popular alternative to corporate bond issuance, the creation and issuance of asset-backed securities.

Coverage of leasing and project financing. We provide in-depth coverage of leasing in Chapter 27, demystifying the claims about the advantages and disadvantages of leasing you too often read about in some textbooks and professional articles. Project financing has grown in importance for not only corporations but for countries seeking to develop infrastructure facilities. Chapter 28 provides the basic principles for understanding project financing.

Early introduction to derivative instruments. Derivative instruments (futures, swaps, and options) play an important role in finance. You are introduced to these instruments in Chapter 4. While derivative instruments are viewed as complex instruments, you are provided with an introduction that makes clear their basic investment characteristics. By the early introduction of derivative instruments, you will be able to appreciate the difficulties of evaluating securities that have embedded options (Chapter 9), how there are real options embedded in capital budgeting decisions (Chapter14), and how derivative instruments can be used to reduce or to hedge the cost of borrowing (Chapter 15).

Stand-alone nature of the chapters. Each chapter is written so that chapters may easily be rearranged to fit different course structures. Concepts, terminology, and notation are presented in each chapter so that no chapter is dependent upon another. This means that instructors can tailor the use of this book to fit their particular time frame for the course and their students' preparation (for example, if students enter the course with sufficient background in accounting and taxation, Chapters 5 and 6 can be skipped).

We believe that our approach to the subject matter of financial management and analysis will help you understand the key issues and provide the foundation for developing a skill set necessary to deal with real world financial problems.

Frank J. Fabozzi
Pamela P. Peterson

About the Authors

Frank J. Fabozzi, Ph.D., CFA, CPA is the Frederick Frank Adjunct Professor of Finance in the School of Management at Yale University. Prior to joining the Yale faculty, he was a Visiting Professor of Finance in the Sloan School at MIT. Professor Fabozzi is a Fellow of the International Center for Finance at Yale University and the editor of the *Journal of Portfolio Management*. He earned a doctorate in economics from the City University of New York in 1972. In 1994 he received an honorary doctorate of Humane Letters from Nova Southeastern University and in 2002 was inducted into the Fixed Income Analysts Society's Hall of Fame. He is the honorary advisor to the Chinese Asset Securitization website.

Pamela Parrish Peterson, Ph.D., CFA is a Professor of finance at Florida State University where she teaches undergraduate courses in corporate finance and doctoral courses in valuation theory. She received her Ph.D. from the University of North Carolina and has taught at FSU since receiving her degree in 1981. Professor Peterson is a co-author with Don Chance of *Real Options* (AIMR Research Foundation, 2002), is a co-author with Frank J. Fabozzi of *Capital Budgeting* (John Wiley & Sons, 2002) and *Analysis of Financial Statements* (published by Frank J. Fabozzi Associates, 1999), co-author with David R. Peterson of the AIMR monograph *Company Performance and Measures of Value Added* (1996), and author of *Financial Management and Analysis* (published by McGraw-Hill, 1994). Professor Peterson has published articles in journals including the *Journal of Finance*, the *Journal of Financial Economics*, the *Journal of Banking and Finance*, *Financial Management*, and the *Financial Analysts Journal*.

PART One

Foundations

CHAPTER 1

Introduction to Financial Management and Analysis

Finance is the application of economic principles and concepts to business decision-making and problem solving. The field of finance can be considered to comprise three broad categories: financial management, investments, and financial institutions:

- ***Financial management.*** Sometimes called ***corporate finance*** or ***business finance***, this area of finance is concerned primarily with financial decision-making within a business entity. Financial management decisions include maintaining cash balances, extending credit, acquiring other firms, borrowing from banks, and issuing stocks and bonds.
- ***Investments.*** This area of finance focuses on the behavior of financial markets and the pricing of securities. An investment manager's tasks, for example, may include valuing common stocks, selecting securities for a pension fund, or measuring a portfolio's performance.
- ***Financial institutions.*** This area of finance deals with banks and other firms that specialize in bringing the suppliers of funds together with the users of funds. For example, a manager of a bank may make decisions regarding granting loans, managing cash balances, setting interest rates on loans, and dealing with government regulations.

No matter the particular category of finance, business situations that call for the application of the theories and tools of finance generally involve either investing (using funds) or financing (raising funds).

Managers who work in any of these three areas rely on the same basic knowledge of finance. In this book, we introduce you to this common body of knowledge and show how it is used in financial decision-making. Though the emphasis of this book is financial management, the basic principles and tools also apply to the areas of investments and financial institutions. In this introductory chapter, we'll consider the types of decisions financial managers make, the role of financial analysis, the forms of business ownership, and the objective of managers' decisions. Finally, we will describe the relationship between owners and managers.

FINANCIAL MANAGEMENT

Financial management encompasses many different types of decisions. We can classify these decisions into three groups: investment decisions, financing decisions, and decisions that involve both investing and financing. Investment decisions are concerned with the use of funds—the buying, holding, or selling of all types of assets: Should we buy a new die stamping machine? Should we introduce a new product line? Sell the old production facility? Buy an existing company? Build a warehouse? Keep our cash in the bank?

Financing decisions are concerned with the acquisition of funds to be used for investing and financing day-to-day operations. Should managers use the money raised through the firms' revenues? Should they seek money from outside of the business? A company's operations and investment can be financed from outside the business by incurring debts, such as though bank loans and the sale of bonds, or by selling ownership interests. Because each method of financing obligates the business in different ways, financing decisions are very important.

Many business decisions simultaneously involve both investing and financing. For example, a company may wish to acquire another firm—an investment decision. However, the success of the acquisition may depend on how it is financed: by borrowing cash to meet the purchase price, by selling additional shares of stock, or by exchanging existing shares of stock. If managers decide to borrow money, the borrowed funds must be repaid within a specified period of time. Creditors (those lending the money) generally do not share in the control of profits of the borrowing firm. If, on the other hand, managers decide to raise funds by selling ownership interests, these funds never have to be paid back. However, such a sale dilutes the control of (and profits accruing to) the current owners.

Whether a financial decision involves investing, financing, or both, it also will be concerned with two specific factors: expected return and risk. And throughout your study of finance, you will be concerned with these factors. ***Expected return*** is the difference between potential benefits and potential costs. ***Risk*** is the degree of uncertainty associated with these expected returns.

Financial Analysis

Financial analysis is a tool of financial management. It consists of the evaluation of the financial condition and operating performance of a business firm, an industry, or even the economy, and the forecasting of its future condition and performance. It is, in other words, a means for examining risk and expected return. Data for financial analysis may come from other areas within the firm, such as marketing and production departments, from the firm's own accounting data, or from financial information vendors such as Bloomberg Financial Markets, Moody's Investors Service, Standard & Poor's Corporation, Fitch Ratings, and Value Line, as well as from government publications, such as the *Federal Reserve Bulletin*. Financial publications such as *Business Week*, *Forbes*, *Fortune*, and the *Wall Street Journal* also publish financial data (concerning individual firms) and economic data (concerning industries, markets, and economies), much of which is now also available on the Internet.

Within the firm, financial analysis may be used not only to evaluate the performance of the firm, but also its divisions or departments and its product lines. Analyses may be performed both periodically and as needed, not only to ensure informed investing and financing decisions, but also as an aid in implementing personnel policies and rewards systems.

Outside the firm, financial analysis may be used to determine the creditworthiness of a new customer, to evaluate the ability of a supplier to hold to the conditions of a long-term contract, and to evaluate the market performance of competitors.

Firms and investors that do not have the expertise, the time, or the resources to perform financial analysis on their own may purchase analyses from companies that specialize in providing this service. Such companies can provide reports ranging from detailed written analyses to simple creditworthiness ratings for businesses. As an example, Dun & Bradstreet, a financial services firm, evaluates the creditworthiness of many firms, from small local businesses to major corporations. As another example, three companies—Moody's Investors Service, Standard & Poor's, and Fitch—evaluate the credit quality of debt obligations issued by corporations and express these views in the form of a rating that is published in the reports available from these three organizations.

FORMS OF BUSINESS ENTERPRISE

Financial management is not restricted to large corporations: It is necessary in all forms and sizes of businesses. The three major forms of business organization are the sole proprietorship, the partnership, and the corporation. These three forms differ in a number of factors, of which those most important to financial decision-making are:

- The way the firm is taxed.
- The degree of control owners may exert on decisions.
- The liability of the owners.
- The ease of transferring ownership interests.
- The ability to raise additional funds.
- The longevity of the business.

Sole Proprietorships

The simplest and most common form of business enterprise is the ***sole proprietorship***, a business owned and controlled by one person—the proprietor. Because there are very few legal requirements to establish and run a sole proprietorship, this form of business is chosen by many individuals who are starting up a particular business enterprise. The sole proprietor carries on a business for his or her own benefit, without participation of other persons except employees. The proprietor receives all income from the business and alone decides whether to reinvest the profits in the business or use them for personal expenses.

A proprietor is liable for all the debts of the business; in fact, it is the proprietor who incurs the debts of the business. If there are insufficient business assets to pay a business debt, the proprietor must pay the debt out of his or her personal assets. If more funds are needed to operate or expand the business than are generated by business operations, the owner either contributes his or her personal assets to the business or borrows. For most sole proprietorships, banks are the primary source of borrowed funds. However, there are limits to how much banks will lend a sole proprietorship, most of which are relatively small.

For tax purposes, the sole proprietor reports income from the business on his or her personal income tax return. Business income is treated as the proprietor's personal income.

The assets of a sole proprietorship may also be sold to some other firm, at which time the sole proprietorship ceases to exist. Or the life of a sole proprietorship ends with the life of the proprietor, although the assets of the business may pass to the proprietor's heirs.

Partnerships

A ***partnership*** is an agreement between two or more persons to operate a business. A partnership is similar to a sole proprietorship except instead of one proprietor, there is more than one. The fact that there is more than one proprietor introduces some issues: Who has a say in the day-to-day operations of the business? Who is liable (that is, financially responsible) for the debts of the business? How is the income distributed among the owners? How is the income taxed? Some of these issues are resolved with the partnership agreement; others are resolved by laws. The partnership agreement describes how profits and losses are to be shared among the partners, and it details their responsibilities in the management of the business.

Most partnerships are ***general partnerships***, consisting only of general partners who participate fully in the management of the business, share in its profits and losses, and are responsible for its liabilities. Each general partner is personally and individually liable for the debts of the business, even if those debts were contracted by other partners.

A ***limited partnership*** consists of at least one general partner and one ***limited partner.*** Limited partners invest in the business but do not participate in its management. A limited partner's share in the profits and losses of the business is limited by the partnership agreement. In addition, a limited partner is not liable for the debts incurred by the business beyond his or her initial investment.

A partnership is not taxed as a separate entity. Instead, each partner reports his or her share of the business profit or loss on his or her personal income tax return. Each partner's share is taxed as if it were from a sole proprietorship.

The life of a partnership may be limited by the partnership agreement. For example, the partners may agree that the partnership is to exist only for a specified number of years or only for the duration of a specific business transaction. The partnership must be terminated when any one of the partners dies, no matter what is specified in the partnership agreement. Partnership interests cannot be passed to heirs; at the death of any partner, the partnership is dissolved and perhaps renegotiated.

One of the drawbacks of partnerships is that a partner's interest in the business cannot be sold without the consent of the other partners. So a partner who needs to sell his or her interest because of, say, personal financial needs may not be able to do so.[1]

Another drawback is the partnership's limited access to new funds. Short of selling part of their own ownership interest, the partners can

[1] Still another problem involves ending a partnership and settling up, mainly because it is difficult to determine the value of the partnership and of each partner's share.

raise money only by borrowing from banks—and here too there is a limit to what a bank will lend a (usually small) partnership.

In certain businesses—including accounting, law, architecture, and physician's services—firms are commonly organized as partnerships. The use of this business form may be attributed primarily to state laws, regulations of the industry, and certifying organizations meant to keep practitioners in those fields from limiting their liability.[2]

Corporations

A ***corporation*** is a legal entity created under state laws through the process of incorporation. The corporation is an organization capable of entering into contracts and carrying out business under its own name, separate from it owners. To become a corporation, state laws generally require that a firm must do the following: (1) file articles of incorporation, (2) adopt a set of bylaws, and (3) form a board of directors.

The ***articles of incorporation*** specify the legal name of the corporation, its place of business, and the nature of its business. This certificate gives "life" to a corporation in the sense that it represents a contract between the corporation and its owners. This contract authorizes the corporation to issue units of ownership, called ***shares***, and specifies the rights of the owners, the ***shareholders***.

The bylaws are the rules of governance for the corporation. The bylaws define the rights and obligations of officers, members of the board of directors, and shareholders. In most large corporations, it is not possible for each owner to participate in monitoring the management of the business. For example, at the end of 2001, Emerson Electric Co. had approximately 33,700 shareholders. It would not be practical for each of these owners to watch over Emerson's management directly. Therefore, the owners of a corporation elect a board of directors to represent them in the major business decisions and to monitor the activities of the corporation's management. The board of directors, in turn, appoints and oversees the officers of the corporation. Directors who are also employees of the corporation are called ***insider directors***; those who have no other position within the corporation are ***outside directors*** or ***independent directors***. In the case of Emerson Electric Co., for example, there were 18 directors in 2002, six inside directors and 13 outside directors. Generally it is believed that the greater the proportion of outside directors, the greater the board's independence from the management of the company. The proportion of

[2] Many states have allowed some types of business, such as accounting firms, that were previously restricted to the partnership form to become limited liability companies (a form of business discussed later in this chapter).

outside directors on corporate boards varies significantly. For example, in 2002 only 44% of Kraft Foods' board are outsiders, whereas 89% of Texas Instrument's board is comprised of outside directors.

The state recognizes the existence of the corporation in the corporate charter. Corporate laws in many states follow a uniform set of laws referred to as the ***Model Business Corporations Act.***[3] Once created, the corporation can enter into contracts, adopt a legal name, sue or be sued, and continue in existence forever. Though owners may die, the corporation continues to live. The liability of owners is limited to the amounts they have invested in the corporation through the shares of ownership they purchased.

Unlike the sole proprietorship and partnership, the corporation is a taxable entity. It files its own income tax return and pays taxes on its income. That income is determined according to special provisions of the federal and state tax codes and is subject to corporate tax rates different from personal income tax rates.

If the board of directors decides to distribute cash to the owners, that money is paid out of income left over after the corporate income tax has been paid. The amount of that cash payment, or dividend, must also be included in the taxable income of the owners (the shareholders). Therefore, a portion of the corporation's income (the portion paid out to owners) is subject to double taxation: once as corporate income and once as the individual owner's income.

The dividend declared by the directors of a corporation is distributed to owners in proportion to the numbers of shares of ownership they hold. If Owner A has twice as many shares as Owner B, he or she will receive twice as much money.

The ownership of a corporation, also referred to as *stock* or *equity*, is represented as shares of stock. A corporation that has just a few owners who exert complete control over the decisions of the corporation is referred to as a ***close corporation*** or a ***closely-held corporation***. A corporation whose ownership shares are sold outside of a closed group of owners is referred to as a ***public corporation*** or a ***publicly-held corporation***. Mars Inc., producer of M&M candies and other confectionery products, is a closely-held corporation; Hershey Foods, also a producer of candy products among other things, is a publicly-held corporation.

The shares of public corporations are freely traded in securities markets, such as the New York Stock Exchange. Hence, the ownership of a publicly-held corporation is more easily transferred than the ownership of a proprietorship, a partnership, or a closely-held corporation.

[3] A Model act is a statute created and proposed by the National Conference of Commissioners of Uniform State Laws. A Model act is available for adoption—with or without modification—by state legislatures.

Companies whose stock is traded in public markets are required to file an initial registration statement with the ***Securities and Exchange Commission*** (SEC), a federal agency created to oversee the enforcement of U. S. securities laws. The statement provides financial statements, articles of incorporation, and descriptive information regarding the nature of the business, the debt and stock of the corporation, the officers and directors, any individuals who own more than 10% of the stock, among other items.

Other Forms of Business

In addition to the proprietorship, partnership, and corporate forms of business, an enterprise may be conducted using other forms of business, such as the master limited partnership, the professional corporation, the limited liability company, and the joint venture.

A ***master limited partnership*** is a partnership with limited partner ownership interests that are traded on an organized exchange. For example, more than two dozen master limited partnerships are listed on the New York Stock Exchange, including the Boston Celtics, Cedar Fair, and Red Lion Inns partnerships. Ownership interests, which represent a specified ownership percentage, are traded in much the same way as the shares of stock of a corporation. One difference, however, is that a corporation can raise new capital by issuing new ownership interests, whereas a master limited partnership cannot. It is not possible to sell more than a 100% interest in the partnership, yet it is possible to sell additional shares of stock in a corporation. Another difference is that the income of a master limited partnership is taxed only once, as partners' individual income.

Another variant of the corporate form of business is the professional corporation. A ***professional corporation*** is an organization that is formed under state law and treated as a corporation for federal tax law purposes, yet that has unlimited liability for its owners—the owners are personally liable for the debts of the corporation. Businesses that are likely to form such corporations are those that provide services and require state licensing, such as physicians', architects', and attorneys' practices, since it is generally felt that it is in the public interest to hold such professionals responsible for the liabilities of the business.

More recently, companies are using a hybrid form of business, the ***limited liability company*** (LLC), which combines the best features of a partnership and a corporation. In 1988 the Internal revenue Service ruled that the LLC be treated as a partnership for tax purposes, while its owners are not liable for its debts. Since this ruling, every state has passed legislation permitting limited liability companies.

Though state laws vary slightly, in general, the owners of the LLC have limited liability. The IRS considers the LLC to be taxed as a partnership if the company has no more than two of the following characteristics: (1) limited liability, (2) centralized management, (3) free transferability of ownership interests, and (4) continuity of life. If the company has more than two of these, it will be treated as a corporation for tax purposes, subjecting the income to taxation at both the company level and the owners'.

A ***joint venture***, which may be structured as either a partnership or as a corporation, is a business undertaken by a group of persons or entities (such as a partnership or corporation) for a specific business activity and, therefore, does not constitute a continuing relationship among the parties. For tax and other legal purposes, a joint venture partnership is treated as a partnership and a joint venture corporation is treated as a corporation.

U.S. corporations have entered into joint ventures with foreign corporations, enhancing participation and competition in the global marketplace. For example, the Coca-Cola Company entered a joint venture with FEMSA, Mexico's largest beverage company, in 1993, expanding its opportunities within Mexico. Joint ventures are an easy way of entering a foreign market and of gaining an advantage in a domestic market. For example, Burger King, the second largest fast food chain in America, entered the Japanese market through a joint venture with Japan Tobacco Inc., which is two-thirds owned by Japan's Ministry of Finance, to form Burger King Japan. This joint venture gives Burger King (owned by the British firm, Grand Metropolitan PLC) a fighting chance in competing against McDonald's almost 2,000 outlets in Japan.

Joint ventures are becoming increasingly popular as a way of doing business. Participants—whether individuals, partnerships, or corporations—get together to exploit a specific business opportunity. Afterward, the venture can be dissolved. Recent alliances among communication and entertainment firms have sparked thought about what the future form of doing business will be. Some believe that what lies ahead is a virtual enterprise—a temporary alliance without all the bureaucracy of the typical corporation—that can move quickly and decisively to take advantage of profitable business opportunities.

Prevalence

The advantages and disadvantages of the three major forms of business from the point of view of financial decision-making are summarized in Exhibit 1.1. Firms tend to evolve from proprietorship to partnership to corporation as they grow and as their needs for financing increase. Sole proprietorship is the choice for starting a business, whereas the corporation is the choice to accommodate growth. The great majority of business firms in the United States are sole proprietorships, but most business income is generated by corporations.

The Objective of Financial Management

So far we have seen that financial managers are primarily concerned with investment decisions and financing decisions within business organizations. The great majority of these decisions are made within the corporate business structure, which better accommodates growth and is responsible for 89% of U.S. business income. Hence, most of our discussion in the remainder of this book focuses on financial decision-making in corporations, but many of the issues apply generally to all forms of business.

EXHIBIT 1.1 **Characteristics of the Three Basic Forms of Business**

Sole Proprietorship

Advantages

1. The proprietor is the sole business decision-maker.
2. The proprietor receives all income from business.
3. Income from the business is taxed once, at the individual taxpayer level.

Disadvantages

1. The proprietor is liable for all debts of the business (unlimited liability).
2. The proprietorship has a limited life.
3. There is limited access to additional funds.

General Partnership

Advantages

1. Partners receive income according to terms in partnership agreement.
2. Income from business is taxed once as the partners' personal income.
3. Decision-making rests with the general partners only.

Disadvantages

1. Each partner is liable for all the debts of the partnership.
2. The partnership's life is determined by agreement or the life of the partners.
3. There is limited access to additional funds.

Corporation

Advantages

1. The firm has perpetual life.
2. Owners are not liable for the debts of the firm; the most that owners can lose is their initial investment.
3. The firm can raise funds by selling additional ownership interest.
4. Income is distributed in proportion to ownership interest.

Disadvantages

1. Income paid to owners is subjected to double taxation.
2. Ownership and management are separated in larger organizations.

One such issue concerns the objective of financial decision-making. What goal (or goals) do managers have in mind when they choose between financial alternatives—say, between distributing current income among shareholders and investing it to increase future income? There is actually one financial objective: the maximization of the economic well-being, or wealth, of the owners. Whenever a decision is to be made, management should choose the alternative that most increases the wealth of the owners of the business.

The Measure of Owner's Economic Well-Being

The price of a share of stock at any time, or its ***market value***, represents the price that buyers in a free market are willing to pay for it. The ***market value of shareholders' equity*** is the value of all owners' interest in the corporation. It is calculated as the product of the market value of one share of stock and the number of shares of stock outstanding:

Market value of shareholders' equity
= Market value of a share of stock × Number of shares of stock outstanding

The number of shares of stock outstanding is the total number of shares that are owned by shareholders. For example, at the end of June 2002 there were 2,040 million Walt Disney Company shares outstanding. The price of Disney stock at the end of June 2002 was $18.90 per share. Therefore, the market value of Disney's equity at the end of June 2002 was over $38.5 billion.

Investors buy shares of stock in anticipation of future dividends and increases in the market value of the stock. How much are they willing to pay today for this future—and hence uncertain—stream of dividends? They are willing to pay exactly what they believe it is worth today, an amount that is called the ***present value***, an important financial concept explained in Chapter 7. The present value of a share of stock reflects the following factors:

- The uncertainty associated with receiving future payments.
- The timing of these future payments.
- Compensation for tying up funds in this investment.

The market price of a share is a measure of owners' economic well-being. Does this mean that if the share price goes up, management is doing a good job? Not necessarily. Share prices often can be influenced by factors beyond the control of management. These factors include expectations regarding the economy, returns available on alternative investments (such as bonds), and even how investors view the firm and the idea of investing.

These factors influence the price of shares through their effects on expectations regarding future cash flows and investors' evaluation of those cash flows. Nonetheless, managers can still maximize the value of owners' equity, given current economic conditions and expectations. They do so by carefully considering the expected benefits, risk, and timing of the returns on proposed investments.

Economic Profit versus Accounting Profit: Share Price versus Earnings Per Share

When you studied economics, you saw that the objective of the firm is to maximize profit. In finance, however, the objective is to maximize owners' wealth. Is this a contradiction? No. We have simply used different terminology to express the same goal. The difference arises from the distinction between accounting profit and economic profit.

Economic profit is the difference between revenues and costs, where costs include both the actual business costs (the explicit costs) and the implicit costs. The implicit costs are the payments that are necessary to secure the needed resources, the ***cost of capital***. With any business enterprise, someone supplies funds, or capital, that the business then invests. The supplier of these funds may be the business owner, an entrepreneur, or banks, bondholders, and shareholders. The cost of capital depends on both the time value of money—what could have been earned on a risk-free investment—and the uncertainty associated with the investment. The greater the uncertainty associated with an investment, the greater the cost of capital.

Consider the case of the typical corporation. Shareholders invest in the shares of a corporation with the expectation that they will receive future dividends. But shareholders could have invested their funds in any other investment, as well. So what keeps them interested in keeping their money in the particular corporation? Getting a return on their investment that is better than they could get elsewhere, considering the amount of uncertainty of receiving the future dividends. If the corporation cannot generate economic profits, the shareholders will move their funds elsewhere.

Accounting profit, however, is the difference between revenues and costs, recorded according to accounting principles, where costs are primarily the actual costs of doing business. The implicit costs—opportunity cost and normal profit—which reflect the uncertainty and timing of future cash flows, are not taken into consideration in accounting profit. Moreover accounting procedures, and hence the computation of accounting profit, can vary from firm to firm. For both these reasons, accounting profit is not a reasonable gauge of shareholders' return on their investment, and the maximization of accounting profit is not equivalent to the maximization of shareholder wealth.[4]

Many U. S. corporations, including Coca–Cola, Briggs & Stratton, and Boise Cascade, are embracing a relatively new method of evaluating and rewarding management performance that is based on the idea of compensating management for economic profit, rather than for accounting profit. The most prominent of recently developed techniques to evaluate a firm's performance are economic value–added and market value-added.[5]

Economic value-added (*EVA*®) is another name for the firm's economic profit. Key elements of estimating economic profit are:

1. calculating the firm's operating profit from financial statement data, making adjustments to accounting profit to better reflect a firm's operating results for a period,
2. calculating the cost of capital, and
3. comparing operating profit with the cost of capital.

The difference between the operating profit and the cost of capital is the estimate of the firm's economic profit, or economic value–added.

A related measure, ***market value added*** (*MVA*), focuses on the market value of capital, as compared to the cost of capital. The key elements of market value added are:

1. calculating the market value of capital,
2. calculating the amount of capital invested (i.e., debt and equity), and
3. comparing the market value of capital with the capital invested.

The difference between the market value of capital and the amount of capital invested is the market value added. In theory, the market value added is the present value of all expected future economic profits.

The application of economic profit is relatively new in the measurement of performance, yet the concept of economic profit is not new. What this recent emphasis on economic profit has accomplished is to focus attention away from accounting profit and toward clearing the cost of capital hurdle.

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[4] When economic profit is zero, as an example, investors are getting a return that just compensates them for bearing the risk of the investment. When accounting profit is zero, investors would be much better off investing elsewhere and just as well off by keeping their money under their mattresses.

[5] One of the first to advocate using economic profit in compensating management is G. Bennett Stewart III, *The Quest for Value* (New York: HarperCollins Publishers, Inc., 1991).

Share Prices and Efficient Markets

We have seen that the price of a share of stock today is the present value of the dividends and share price the investor expects to receive in the future. What if these expectations change?

Suppose you buy a share of stock of IBM. The price you are willing to pay is the present value of future cash flows you expect from dividends paid on one share of IBM stock and from the eventual sale of that share. This price reflects the amount, the timing, and the uncertainty of these future cash flows. Now what happens if some news—good or bad—is announced that changes the expected IBM dividends? If the market in which these shares are traded is efficient, the price will fall very quickly to reflect that news.

In an efficient market, the price of assets—in this case shares of stock—reflects all publicly available information. As information is received by investors, share prices change rapidly to reflect the new information. How rapidly? In U.S. stock markets, which are efficient markets, information affecting a firm is reflected in share prices of its stock within minutes.

What are the implications for financing decisions? In efficient markets, the current price of a firm's shares reflects all publicly available information. Hence, there is no good time or bad time to issue a security. When a firm issues stock, it will receive what that stock is worth—no more and no less. Also, the price of the shares will change as information about the firm's activities is revealed. If the firm announces a new product, investors will use whatever information they have to figure out how this new product will change the firm's future cash flows and, hence, the value of the firm—and the share price—will change accordingly. Moreover, in time, the price will be such that investors' economic profit approaches zero.

Financial Management and the Maximization of Owners' Wealth

Financial managers are charged with the responsibility of making decisions that maximize owners' wealth. For a corporation, that responsibility translates into maximizing the value of shareholders' equity. If the market for stocks is efficient, the value of a share of stock in a corporation should reflect investors' expectations regarding the future prospects of the corporation. The value of a stock will change as investors' expectations about the future change. For financial managers' decisions to add value, the present value of the benefits resulting from decisions must outweigh the associated costs, where costs include the costs of capital.

If there is a separation of the ownership and management of a firm—that is, the owners are not also the managers of the firm—there are additional issues to confront. What if a decision is in the best interests of the firm, but not in the best interest of the manager? How can owners insure that managers are watching out for the owners' interests? How can owners motivate managers to make decisions that are best for the owners? We will address these issues, and more, in the next section.

THE AGENCY RELATIONSHIP

If you are the sole owner of a business, then you make the decisions that affect your own well-being. But what if you are a financial manager of a business and you are not the sole owner? In this case, you are making decisions for owners other than yourself; you, the financial manager, are an agent. An ***agent*** is a person who acts for—and exerts powers of—another person or group of persons. The person (or group of persons) the agent represents is referred to as the ***principal***. The relationship between the agent and his or her principal is an ***agency relationship***. There is an agency relationship between the managers and the shareholders of corporations.

Problems with the Agency Relationship

In an agency relationship, the agent is charged with the responsibility of acting for the principal. Is it possible the agent may not act in the best interest of the principal, but instead act in his or her own self-interest? Yes—because the agent has his or her own objective of maximizing personal wealth.

In a large corporation, for example, the managers may enjoy many fringe benefits, such as golf club memberships, access to private jets, and company cars. These benefits (also called perquisites, or "perks") may be useful in conducting business and may help attract or retain management personnel, but there is room for abuse. What if the managers start spending more time at the golf course than at their desks? What if they use the company jets for personal travel? What if they buy company cars for their teenagers to drive? The abuse of perquisites imposes costs on the firm—and ultimately on the owners of the firm. There is also a possibility that managers who feel secure in their positions may not bother to expend their best efforts toward the business. This is referred to as shirking, and it too imposes a cost to the firm.

Finally, there is the possibility that managers will act in their own self-interest, rather than in the interest of the shareholders when those interests clash. For example, management may fight the acquisition of their firm by some other firm even if the acquisition would benefit shareholders. Why? In most takeovers, the management personnel of the acquired firm generally lose their jobs. Envision that some company is making an offer to acquire the firm that you manage. Are you happy that the acquiring firm is offering the shareholders of your firm more for their stock than its current market value? If you are looking out for their best interests, you should be. Are you happy about the likely prospect of losing your job? Most likely not.

Many managers faced this dilemma in the merger mania of the 1980s. So what did they do? Among the many tactics,

- Some fought acquisition of their firms—which they labeled ***hostile takeovers***—by proposing changes in the corporate charter or even lobbying for changes in state laws to discourage takeovers.
- Some adopted lucrative executive compensation packages—called ***golden parachutes***—that were to go into effect if they lost their jobs.

Such defensiveness by corporate managers in the case of takeovers, whether it is warranted or not, emphasizes the potential for conflict between the interests of the owners and the interests of management.

Costs of the Agency Relationship

There are costs involved with any effort to minimize the potential for conflict between the principal's interest and the agent's interest. Such costs are called ***agency costs***, and they are of three types: monitoring costs, bonding costs, and residual loss.

Monitoring costs are costs incurred by the principal to monitor or limit the actions of the agent. In a corporation, shareholders may require managers to periodically report on their activities via audited accounting statements, which are sent to shareholders. The accountants' fees and the management time lost in preparing such statements are monitoring costs. Another example is the implicit cost incurred when shareholders limit the decision-making power of managers. By doing so, the owners may miss profitable investment opportunities; the foregone profit is a monitoring cost.

The board of directors of corporation has a ***fiduciary duty*** to shareholders; that is the legal responsibility to make decisions (or to see that decisions are made) that are in the best interests of shareholders. Part of that responsibility is to ensure that managerial decisions are also in the best interests of the shareholders. Therefore, at least part of the cost of having directors is a monitoring cost.

Bonding costs are incurred by agents to assure principals that they will act in the principal's best interest. The name comes from the agent's promise or bond to take certain actions. A manager may enter into a contract that requires him or her to stay on with the firm even though another company acquires it; an implicit cost is then incurred by the manager, who foregoes other employment opportunities.

Even when monitoring and bonding devices are used, there may be some divergence between the interests of principals and those of agents. The resulting cost, called the ***residual loss***, is the implicit cost that results because the principal's and the agent's interests cannot be perfectly aligned even when monitoring and bonding costs are incurred.

Motivating Managers: Executive Compensation

One way to encourage management to act in shareholders' best interests, and so minimize agency problems and costs, is through executive compensation—how top management is paid. There are several different ways to compensate executives, including:

Salary. The direct payment of cash of a fixed amount per period.

Bonus. A cash reward based on some performance measure, say earnings of a division or the company.

Stock appreciation right. A cash payment based on the amount by which the value of a specified number of shares has increased over a specified period of time (supposedly due to the efforts of management).

Performance shares. Shares of stock given the employees, in an amount based on some measure of operating performance, such as earnings per share.

Stock option. The right to buy a specified number of shares of stock in the company at a stated price—referred to as an *exercise price* at some time in the future. The exercise price may be above, at, or below the current market price of the stock.

Restricted stock grant. The grant of shares of stock to the employee at low or no cost, conditional on the shares not being sold for a specified time.

The salary portion of the compensation—the minimum cash payment an executive receives—must be enough to attract talented executives. But a bonus should be based on some measure of performance that is in the best interests of shareholders—not just on the past year's accounting earnings. For example, a bonus could be based on gains in market share. Recently, several companies have adopted programs that base compensation, at least in part, on value added by managers as measured by economic profits.

The basic idea behind stock options and restricted stock grants is to make managers owners, since the incentive to consume excessive perks and to shirk are reduced if managers are also owners. As owners, managers not only share the costs of perks and shirks, but they also benefit financially when their decisions maximize the wealth of owners. Hence, the key to motivation through stock is not really the *value* of the stock, but rather *ownership* of the stock. For this reason, stock appreciation rights and performance shares, which do not involve an investment on the part of recipients, are not effective motivators.

Stock options do work to motivate performance if they require owning the shares over a long time period; are exercisable at a price *above* the current market price of the shares, thus encouraging managers to get the share price up, and require managers to tie up their own wealth in the shares.

Currently, there is a great deal of concern in some corporations because executive compensation is not linked to performance. In recent years, many U.S. companies have downsized, restructured, and laid off many employees and allowed the wages of employees who survive the cuts to stagnate. At the same time, corporations have increased the pay of top executives through both salary and lucrative stock options. If these changes lead to better value for shareholders, shouldn't the top executives be rewarded?

There are two issues here. First, such a situation results in anger and disenchantment among both surviving employees and former employees. Second, the downsizing, restructuring, and lay-offs may not result in immediate (or even, eventual) increased profitability. Consider AT&T in 1995: In a year in which the company restructured, barely made a profit, eliminated 40,000 jobs, and its stock had lackluster returns, the chief executive officer (CEO) received salary and bonuses of $5.2 million and options valued at $11 million. If the restructuring pays off in the long-run, the CEO's pay may be justified, but meanwhile, there may be some unhappy AT&T shareholders: The average annual return on AT&T stock over the period 1996–2001 was –23.19%.[6]

Another problem is that compensation packages for top management are designed by the board of directors, which often includes top management. Moreover, reports disclosing these compensation packages to shareholders (the proxy statements) are often confusing. Both problems can be avoided by adequate and understandable disclosure of executive compensation to shareholders, and with compensation packages determined by board members who are not executives of the firm.

[6] Joann S. Lublin, "AT&T Board Faces Protest Over CEO Pay," *Wall Street Journal* (April 16, 1996), pp. A3, A6.

Owners have one more tool with which to motivate management—the threat of firing. As long as owners can fire managers, managers will be encouraged to act in the owners' interest. However, if the owners are divided or apathetic—as they often are in large corporations—or if they fail to monitor management's performance and the reaction of directors to that performance, the threat may not be credible. The removal of a few poor managers can, however, make this threat palpable.

Shareholder Wealth Maximization and Accounting "Irregularities"

Recently, there have been a number of scandals and allegations regarding the financial information that is being reported to shareholders and the market. Financial results reported in the income statements and balance sheets of some companies indicated much better performance than the true performance or much better financial condition than actual. Examples include Xerox, which was forced to restate earnings for several years because it had inflated pre-tax profits by $1.4 billion, Enron, which is accused of inflating earnings and hiding substantial debt, and Worldcom, which failed to properly account for $3.8 billion of expenses. Along with these financial reporting issues, the independence of the auditors and the role of financial analysts have been brought to the forefront.[7]

It is unclear at this time the extent to which these scandals and problems were the result of simply bad decisions or due to corruption. The eagerness of managers to present favorable results to shareholders and the market appears to be a factor in several instances. And personal enrichment at the expense of shareholders seems to explain some cases. Whatever the motivation, chief executive officers (CEOs), chief financial officers (CFOs), and board members are being held directly accountable for financial disclosures. For example, in 2002, the Securities and Exchange Commission ordered sworn statements attesting to the accuracy of financial statements. The first deadline for such statements resulted in several companies restating financial results.

The accounting scandals are creating an awareness of the importance of corporate governance, the importance of the independence of the public accounting auditing function, the role of financial analysts, and the responsibilities of CEOs and CFOs.

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[7] For example, the public accounting firm of Arthur Andersen was found guilty of obstruction of justice in 2002 for their role in the shredding of documents relating to Enron. As an example of the problems associated with financial analysts, the securities firm of Merrill Lynch paid a $100 million fine for their role in hyping stocks to help win investment-banking business.

Shareholder Wealth Maximization and Social Responsibility

When financial managers assess a potential investment in a new product, they examine the risks and the potential benefits and costs. If the risk-adjusted benefits do not outweigh the costs, they will not invest. Similarly, managers assess current investments for the same purpose; if benefits do not continue to outweigh costs, they will not continue to invest in the product but will shift their investment elsewhere. This is consistent with the goal of shareholder wealth maximization and with the allocative efficiency of the market economy.

Discontinuing investment in an unprofitable business may mean closing down plants, laying off workers, and, perhaps destroying an entire town that depends on the business for income. So decisions to invest or disinvest may affect great numbers of people.

All but the smallest business firms are linked in some way to groups of persons who are dependent to a degree on the business. These groups may include suppliers, customers, the community itself, and nearby businesses, as well as employees and shareholders. The various groups of persons that depend on a firm are referred to as its ***stakeholders***; they all have some *stake* in the outcome of the firm. For example, if the Boeing Company lays off workers or increases production, the effects are felt by Seattle and the surrounding communities.

Can a firm maximize the wealth of shareholders and stakeholders at the same time? Probably. If a firm invests in the production of goods and services that meet the demand of consumers in such a way that benefits exceed costs, the firm will be allocating the resources of the community efficiently, employing assets in their most productive use. If later the firm must disinvest—perhaps close a plant—it has a responsibility to assist employees and other stakeholders who are affected. Failure to do so could tarnish its reputation, erode its ability to attract new stakeholder groups to new investments, and ultimately act to the detriment of shareholders.

The effects of a firm's actions on others are referred to as ***externalities***. Pollution is an externality that keeps increasing in importance. Suppose the manufacture of a product creates air pollution. If the polluting firm acts to reduce this pollution, it incurs a cost that either increases the price of its product or decreases profit and the market value of its stock. If competitors do not likewise incur costs to reduce their pollution, the firm is at a disadvantage and may be driven out of business through competitive pressure.

The firm may try to use its efforts at pollution control to enhance its reputation in the hope that this will lead to a sales increase large enough to make up for the cost of reducing pollution. This is called a ***market***

solution: The market places a value on the pollution control and rewards the firm (or an industry) for it. If society really believes that pollution is bad and that pollution control is good, the interests of owners and society can be aligned.

It is more likely, however, that pollution control costs will be viewed as reducing owners' wealth. Then firms must be forced to reduce pollution through laws or government regulations. But such laws and regulations also come with a cost—the cost of enforcement. Again, if the benefits of mandatory pollution control outweigh the cost of government action, society is better off. In such a case, if the government requires all firms to reduce pollution, then pollution control costs simply become one of the conditions under which owner wealth-maximizing decisions are to be made.

SUMMARY

- Finance comprises three areas: financial management, investments, and financial institutions. These three areas are linked together through a common body of knowledge that includes the theories and tools of finance.
- The decision-making of financial managers can be broken down into two broad classes: investment decisions and financing decisions. Investment decisions are those decisions that involve the use of the firm's funds. Financing decisions are those decisions that involve the acquisition of the firm's funds.
- Financial managers assess the potential risks and rewards associated with investment and financing decisions through the application of financial analysis.
- The information necessary for financial decisions and analysis includes the accounting information that describes the company and its industry as well as economic information relating to the company, the industry, and the economy in general.
- A business enterprise may be formed as a sole proprietorship, a partnership, corporation, or a hybrid of one or more of these forms. The hybrid forms include the master limited partnership, the professional corporation, the limited liability company, and the joint venture. The choice of the form of business is influenced by concerns about the life of the enterprise, the liability of its owners, the taxation of income, and access to funds. In turn, the form of business influences financial decision-making through its effect on taxes, governance, and the liability of owners.

- Corporations are entities created by law that limit the liability of owners and subject income to an additional layer of taxation. The corporation's owners—the shareholders—are represented by the board of directors, which oversees the management of the firm.
- The objective of financial decision-making in a business is the maximization of the wealth of owners. For a corporation, this is equivalent to the maximization of the market value of the stock.
- If markets for securities are price efficient, share prices will reflect all available information. When information is revealed to investors, it is rapidly figured into share prices.
- Since managers' self-interest may not be consistent with owners' best interests, owners must devise ways to align mangers' and owners' interests. One means of doing this is through executive compensation. By designing managers' compensation packages to encourage long-term investment in the stock of a corporation, the interests of managers and shareholders can be aligned.
- Recent scandals have created an awareness of the responsibility of CEOs, CFOs, and board members to shareholders and the market.
- Shareholder wealth maximization is consistent with the best interests of stakeholders and society if market forces reward firms for taking actions that are in society's interest or if the government steps in to force actions that are in society's interest.

QUESTIONS

1. Which of following actions are the result of a financing decision? Which of the following actions are the result of an investment decision?
   a. A firm introduces a new product.
   b. A firm issues new bonds.
   c. A corporation issues new shares of stock.
   d. A firm expands its existing manufacturing facilities.
   e. A firm leases a new building to be used in its manufacturing.
2. Suppose you are the financial manager of a large national food processing firm. In your travels, you run across a small regional food processor that you believe will provide your firm with annual returns of over 30%. Returns on your firm's typical investments are around 20%. Should you propose that your firm acquire this regional food processor? What factors need to be considered in this decision?
3. McDonald's Corporation, licensor and operator of a chain of fast-food restaurants, was founded in 1953 as a partnership and within

six months was incorporated. Why would this operator of fast-food restaurants incorporate so soon after being established? What factors influence the decision to incorporate?

4. Briefly describe each of the following forms of business: (a) master limited partnership, (b) professional corporation, (c) joint venture.
5. Corporations contribute the greatest share of business income in the United States, yet there are fewer corporations than sole proprietorships. Explain why these facts seem reasonable, considering the evolution of a firm.
6. If the share price of a corporation's stock declines, does this mean that the management of the company is not maximizing shareholder wealth? If the share price of a corporation's stock increases, does this mean that the management of the company is maximizing shareholder wealth? Explain.
7. Why is the maximizing of shareholder wealth not necessarily equivalent to the maximizing of earnings per share?
8. Through 1997, the Burlington Coat Factory Warehouse Corporation had not paid any dividends. Why were investors willing to pay over $10 for a share of Burlington stock in 1997?
9. The Rising Corporation has had 20 consecutive quarters of increasing earnings per share, but its share price has remained at about the same price over this same time period. Is this consistent? Explain.
10. Which forms of business have limited liability for all owners? Which forms of business have unlimited liability for all owners?
11. Why may a firm's share price increase when the firm announces lower earnings?
12. The Clockwork Corporation would like to issue $2 million in new shares of stock. The President of Clockwork believes that if the company waits two weeks, they could get a better price for their shares. The Chair of the board of directors disputes this. She says that because markets are price efficient, there is no "timing" possible on the stock issue and Clockwork should issue the shares when they need the funds, and not worry about "timing." Who is right?
13. What is an agency cost? Give three examples of agency costs.
14. The Sununu Corporation is having a bit of a problem: The executives are using the corporation's jets for personal reasons, such as traveling on vacation and visiting doctors in other cities. The board of directors wants management to cut down on this type of activity.
    a. In terms of the different types of agency costs, how would we classify the misuse of corporate jets?
    b. What measures can the board take to reduce or eliminate the misuse of the corporate jets?

15. Suppose that you start your own small retail business. As business increases, you expand the hours and hire someone to manage the business during the evening hours.
    a. Describe the agency relationship involved in your business.
    b. What possible problems can arise in this relationship?
    c. How could you reduce the costs associated with this agency relationship?
16. List four kinds of compensation for a firm's management. Identify the arrangements that would be most effective in aligning the interests of shareholders and management.
17. Can shareholder wealth maximization be consistent with a firm's social responsibility? Explain. Consider International Business Machines (IBM), whose headquarters are located in Armonk, New York, but whose manufacturing and sales operations span the globe. Who are IBM's stakeholders? If IBM trims is work force, what obligations does it have to its stakeholders?
18. On Tuesday, February 16, 2000, the L Corp. announced that its fourth quarter 1999 earnings per share rose to 67 cents, up from 55 cents for the fourth quarter of 1991. On the same day, M Corp. announced fourth quarter earnings of 63 cents per share, compared to the previous year's fourth quarter earnings of 66 cents. On February 16, 2000, L Corp.'s share price fell from $27.375 to $25.375 and M Corp.'s share price fell from $40.125 to $37.375. Why would the share prices of both companies fall when these earnings figures are announced?
19. Why would a firm choose to be a closely-held corporation instead of a publicly-held corporation? Why would a firm choose to be a publicly-held corporation instead of a closely-held corporation?
20. Compare performance shares with a restricted stock grant as a means of motivating management to act in shareholders' best interests. Which do you believe is more effective? Explain your reasoning.
21. Mary, Martin, and Michael invested $20,000, $30,000, and $50,000, respectively, in a business enterprise. After operating the business unsuccessfully for five years, they decided to terminate it. At the time they ceased business operations, the assets of the business were worth only $40,000 and the debts of the business were $10,000.
    a. If this business were formed as a partnership, with the sharing of profits and losses based on the proportion of each partner's original investment, what would be the financial consequences of the dissolution of the business to Mary, Martin, and Michael?
    b. If this business were formed as a corporation, with the proportion of ownership based on the proportion of each shareholder's original investment, what would be the financial consequences of the dissolution of the business to Mary, Martin, and Michael?

CHAPTER 2

Securities and Markets

The objective of any financial decision, whether it is a financing or investment decision, should be to maximize owners' wealth. For a corporation this translates into maximizing the market value of the ownership interest—the value of the stock. So a financial manager's decisions must be made with an eye on the value of the firm's stock and the markets in which the stock is traded.

If a firm needs funds, should it issue stock or borrow? If it issues new stock, will present investors lose? If it borrows, what interest rate will its lenders—the investors in its bonds—require? How soon could the loan be paid off? How soon should it be paid off?

If a firm has funds to invest, should financial managers invest it until it is needed? In what kind of financial instrument? What characteristics must the investment vehicle have? What types of risk must they take on with their investment?

Financial managers must understand the wide range of securities available and the markets in which they are bought and sold. This chapter provides an overview of both. Its purpose is twofold. First, we acquaint you with the terms and definitions we use in this book. Then, we give you an idea how markets for securities function so that you will know how security prices are determined.

SECURITIES

A ***security*** is a document that gives the owner a claim on future cash flows. A security may represent an ownership claim on an asset (such as a share of stock) or a claim on the repayment of borrowed funds, with interest (such as a bond). The document may be a piece of paper (such as a stock certificate or a bond) or an entry in a register (which may, in turn, be a computer record). A ***securities market*** is an arrangement for buying and selling securities. It may be a physical location or simply a computer or telephone network.

Securities are classified into three groups: money market securities, capital market securities, and derivative securities—based on their maturity and the source of their value. The word "maturity" is often used loosely to refer to the length of time before repayment of a debt. Other terms using the word "maturity" are more specific. The ***maturity date*** of a security is the pre-set date on which the amount borrowed (called the ***face value***, the ***par value***, the ***principal***, or the ***maturity value***) is repaid. The security is said to mature on its maturity date. The ***original maturity*** is the time between the date a security is issued and its maturity date.

Money Market Securities

Money market securities are short-term indebtedness. By "short term" we usually imply an original maturity of one year or less. The most common money market securities are Treasury bills, commercial paper, negotiable certificates of deposit, and bankers acceptances.

Treasury bills (*T-bills*) are short-term securities issued by the U.S. government; they have original maturities of either four weeks, three months, or six months. Unlike other money market securities, T-bills carry no stated interest rate. Instead, they are sold on a ***discounted basis***: Investors obtain a return on their investment by buying these securities for less than their face value and then receiving the face value at maturity. T-Bills are sold in $10,000 denominations; that is, the T-Bill has a face value of $10,000.

Commercial paper is a promissory note—a written promise to pay—issued by a large, creditworthy corporation. These securities have original maturities ranging from one day to 270 days and usually trade in units of $100,000. Most commercial paper is backed by bank lines of credit, which means that a bank is standing by ready to pay the obligation if the issuer is unable to. Commercial paper may be either interest-bearing or sold on a discounted basis.

Certificates of deposit (CDs) are written promises by a bank to pay a depositor. Nowadays they have original maturities from six months to three years. ***Negotiable certificates of deposit*** are CDs issued by large commercial banks that can be bought and sold among investors. Negotiable CDs typically have original maturities between one month and one year and are sold in denominations of $100,000 or more. Negotiable certificates of deposit are sold to investors at their face value and carry a fixed interest rate. On the maturity date, the investor is repaid the amount borrowed, plus interest.

Eurodollar certificates of deposit are CDs issued by foreign branches of U.S. banks, and ***Yankee certificates of deposit*** are CDs issued by foreign banks located in the United States. Both Eurodollar CDs and Yankee CDs are denominated in U.S. dollars. In other words, interest payments and the repayment of principal are both in U.S. dollars.

Bankers' acceptances are short-term loans, usually to importers and exporters, made by banks to finance specific transactions. An acceptance is created when a draft (a promise to pay) is written by a bank's customer and the bank "accepts" it, promising to pay. The bank's acceptance of the draft is a promise to pay the face amount of the draft to whomever presents it for payment. The bank's customer then uses the draft to finance a transaction, giving this draft to her supplier in exchange for goods. Since acceptances arise from specific transactions, they are available in a wide variety of principal amounts. Typically, bankers' acceptances have maturities of less than 180 days. Bankers' acceptances are sold at a discount from their face value, and the face value is paid at maturity. Since acceptances are backed by both the issuing bank *and* the purchaser of goods, the likelihood of default is very small.

Money market securities are backed solely by the issuer's ability to pay. With money market securities, there is no ***collateral***; that is, no item of value (such as real estate) is designated by the issuer to ensure repayment. The investor relies primarily on the reputation and repayment history of the issuer in expecting that he or she will be repaid.

CAPITAL MARKET SECURITIES

Capital market securities are long-term securities issued by corporations and governments. Here "long-term securities" refers to securities with original maturities greater than 1 year and perpetual securities (those with no maturity). There are two types of capital market securities: those that represent shares of ownership interest, also called ***equity***, issued by corporations, and those that represent indebtedness, issued by corporations and by the U.S. and state and local governments.

Equity

The equity of a corporation is referred to as "stock"; ownership of stock is represented by shares. Investors who own stock are referred to as ***shareholders***. Every corporation has common stock, and some corporations have another type of stock, preferred stock, as well.

Common stock is the most basic ownership interest in a corporation. Common shareholders are the residual owners of the firm. If the business is liquidated, the common shareholders can claim the business' assets, but only those assets that remain after all other claimants have been satisfied.

Since common stock represents ownership of the corporation, and since the corporation has a perpetual life, common stock is a perpetual security; it has no maturity. Common shareholders may receive cash payments—dividends—from the corporation. They may also receive a return on their investment in the form of increased value of their stock as the corporation prospers and grows.

Preferred stock also represents ownership interest in a corporation and, like common stock, is a perpetual security. However, preferred stock differs from common stock in several important ways. First, preferred shareholders are usually promised a fixed annual dividend, whereas common shareholders receive what the board of directors decides to distribute. And although the corporation is not legally bound to pay the preferred stock's dividend, preferred shareholders must be paid their dividends before any common dividends are paid. Second, preferred shareholders are not residual owners; their claim on a liquidated corporation takes precedence over that of common shareholders. And finally, preferred shareholders generally do not have a say in corporate matters, whereas common stockholders have the right to vote for members of the board of directors and on major issues.

Indebtedness

A capital market debt obligation is a financial instrument whereby the borrower promises to repay the face amount of the obligation by the maturity date and, in most cases, to make periodic interest payments to the holder of the debt obligation, referred to as the ***lender***. These debt obligations can be broken into two categories: bank loans and debt securities.

While at one time, bank loans were not considered capital market instruments, in recent years a market for the buying and selling of these debt obligations has developed. One form of bank loan that is bought and sold in the market is a ***syndicated bank loan***. This is a loan in which a group (or syndicate) of banks provides funds to the borrower. The need for a group of banks arises because the amount sought by a borrower may be too large for any one bank to be exposed to the credit risk of that borrower.

Debt securities include (1) bonds, (2) notes, (3) medium-term notes, and (4) asset-backed securities. The distinction between a bond and a note has to do with the number of years until the obligation matures when the security is originally issued. Historically, a note is a debt security with a maturity at issuance of 10 years or less; a bond is a debt security with a maturity greater than10 years.[1] The distinction between a note and a medium-term note has nothing to do with the maturity but rather the way the security is issued and we will explain this in Chapter 15. Throughout most of this book we will simply refer to a bond, a note, or a medium-term note as simply a bond. We will refer to the investors in any debt obligation as either the ***debtholder***, ***bondholder***, or ***note holder***.

A debt security may provide a promise to pay the investor periodic interest (referred to as a ***coupon***); a debt security that does not include a promise to pay interest is referred to as a ***zero-coupon debt***. In the case of debt that pays interest, interest is generally paid at regular intervals (say, semi-annually) and may be a fixed or floating (or variable) rate. The interest rate for a floating rate security is usually tied to the interest rate on a market interest rate, the price of a commodity, or the return on some financial instrument.

Bonds, notes, and medium-term notes are issued by corporations, the U.S. government, U.S. government agencies, and municipal governments. Corporate debt securities backed by specific assets as collateral are referred to as ***secured notes*** or ***secured bonds***. If they are not backed by specific assets, they are referred to as ***debentures***. If a debt obligation is secured and the borrower is unable to make interest or principal payments when promised, in theory the creditors may be able to force the sale of the collateral for the purpose of collecting what is due them. Collateral therefore reduces the security's riskiness and the level of return, or ***yield***, the issuer (the borrower) must pay. As we will see in later chapters in this book, riskiness is an important determinant of the return on as investment. The claims of debtholders take precedence over those of shareholders, but debtholders are unlikely to be paid the full face value for their securities if a corporation must be liquidated.

U.S. government notes and bonds are interest-bearing securities backed by the "full faith and credit" of the United States; there is little uncertainty regarding whether the interest and principal will be paid as promised. The bonds and notes of U.S. government agencies, such as the Tennessee Valley Authority, are also backed by the government. The securities of government sponsored enterprises, such as the United

[1] This distinction between notes and bonds is not precisely true, but is consistent with common usage of the terms "note" and "bond." In fact, notes and bonds are distinguished by whether or not there is an indenture agreement, a legal contract specifying the terms of the borrowing and any restrictions, and identifying a trustee to watch out for the debtholders' interests. A bond has an indenture agreement, whereas a note does not. For our purposes in this chapter, we will use the terms notes and bonds in their common usage, distinguished on the term to maturity.

States Postal Service and the Federal Home Loan Bank are not explicitly backed by the government, yet there is little uncertainty whether the interest and principal on these securities will be paid as promised.

Bonds issued by state and local governments are called municipal bonds. They are either ***general obligation bonds***, which are backed by the general taxing power of the issuing government, or ***revenue bonds***, which are bonds issued to finance a specific project and are repaid with the revenues from that project.

Interest on federal government bonds is taxed as income by the federal government, but in most cases not by the states. The interest on municipal bonds is generally taxed as income by the states, but not by the federal government. The exclusion of interest on municipal bonds from federal income tax makes these bonds attractive to investors. It also allows local governments to pay lower-than-average interest on their bonds.

The major financing instrument for corporations that developed in the 1990s was the ***asset-backed security***. This is a debt security that is backed by loans or receivables. For example, Ford Credit, a subsidiary of Ford Motor Company, has issued securities backed by a pool of automobile loans. The process of issuing securities backed by a pool of loans or receivables is referred to as ***securitization***. We'll see the advantages of a corporation issuing an asset-backed security relative to a corporate bond in Chapter 26.

Derivative Instruments

A ***derivative instrument*** is any contract that gets its value directly from another security, a market interest rate, the price of a commodity, or a financial index. Derivative instruments include: (1) options, (2) futures/forwards, (3) swaps, and (4) caps and floors. In Chapter 4 we will discuss these derivative instruments.

What is important to understand is that derivative instruments can be used to control the wide range of risk faced by corporations and investors. This is one reason why derivatives are often referred to as ***risk control instruments***. We must postpone a detailed discussion of the risk reducing role of derivative instruments at this juncture since we have not discussed the various risks faced by corporations and investors. This key role played by derivative instruments in global financial markets was stated in a 1994 report published by the U.S. General Accounting Office:

> Derivatives serve an important function of the global financial marketplace, providing end-users with opportunities to better manage financial risks associated with their business transactions. The rapid growth and increasing

> complexity of derivatives reflect both the increased demand from end-users for better ways to manage their financial risks and the innovative capacity of the financial services industry to respond to market demands.[2]

Unfortunately, derivative markets are too often viewed by the general public—and sometimes regulators and legislative bodies—as vehicles for pure speculation (that is, legalized gambling). Without derivative instruments and the markets in which they trade, the financial systems throughout the world would not be as integrated as they are today and it would be difficult for corporations and investors to protect themselves against unwanted risks.

SECURITIES MARKETS

The primary function of a securities market—whether or not it has a physical location—is to bring together buyers and sellers of securities. Securities markets can be classified by whether they are involved in original sales or resales of securities, and by whether or not they involve a physical trading location.

Primary and Secondary Markets

When a security is first issued, it is sold in the primary market. This is the market in which new issues are sold and new capital is raised. So it is the market whose sales directly benefit the issuer of the securities.

There are three ways to raise capital in the primary market. The first is the direct sale, in which the investor purchases, say, stock directly from the issuer. Many venture capital firms invest in small, growing businesses in this way. Also, many corporations sell securities directly to large investors, such as pension funds. By doing so, the issuer can tailor the features of the security (such as maturity) to suit the desires of the investor. This type of selling is referred to as ***private placement***.

A second method is through ***financial institutions***, which are firms that obtain money from investors in return for the institution's securities and then invest that money. For example, a bank issues bank accounts in return for depositors' money and then loans that money to a firm. Besides banks, firm such as mutual funds and pension funds operate as financial institutions.

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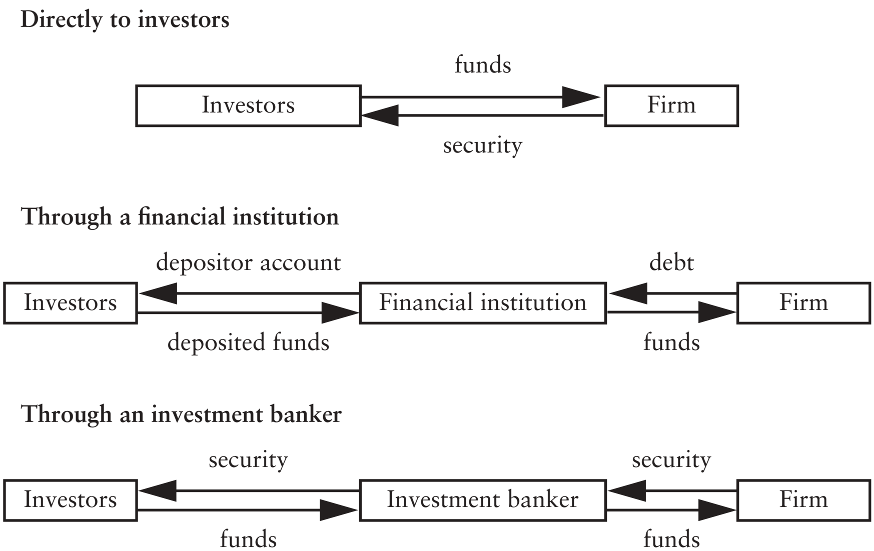
[2] U.S. General Accounting Office (GAO), *Financial Derivatives: Actions Needed to Protect the Financial System*, May 1994, p. 6.

EXHIBIT 2.1 **The Three Methods of Raising Capital in the Primary Markets**



The third method for primary market transactions operates through investment bankers, who buy the securities issued by corporations and then sell those securities to investors for a higher price. This process of buying shares from the issuer and reselling them to investors is called ***underwriting***. For example, Kraft Foods' 2001 offering of newly issued common shares was underwritten by a syndicate of 15 underwriters, including Credit Suisse First Boston, Salomon Smith Barney, Deutsche Banc Alexander Brown, and J. P. Morgan. The offering raised \$8.7 billion, with Kraft Foods receiving over \$8.4 billion:

	Per share	Total proceeds
Initial public offering price	\$31.0000	\$8,680,000,000
Underwriting discount	\$0.8471	\$237,181,000
Proceeds, before expenses, to Kraft	\$30.1529	\$8,442,819,000

The three methods of raising capital in the primary market are illustrated in Exhibit 2.1. We discuss the underwriting of securities and the role of investment bankers in the next chapter.

A ***secondary market*** is one in which securities are resold among investors. No new capital is raised and the issuer of the security does not benefit directly from the sale. Trading takes place among investors. Investors who buy and sell securities on the secondary markets may obtain the services of *stock brokers*, individuals who buy or sell securities for their clients.

We can use the market for college textbooks to illustrate the difference between primary and secondary markets. Suppose one of your instructors decides to use this book, *Financial Management and Analysis*, as the class text. The instructor notifies the school bookstore, which buys copies of the text from the publisher, John Wiley & Sons, and then puts them up for sale at a somewhat higher price than was paid. You then buy your new copy of this book from the bookstore. The market for new books, in which you, the publisher, and the bookstore have operated as buyer, seller, and intermediary, respectively, is a ***primary market***. The bookstore has acted as a sort of textbook "investment banker," but most of the money invested in the book has gone to the issuer (the publisher). The bookstore received a profit for performing as an intermediary, a facilitator of the transaction between you and the publisher. The publisher would have been hard put to sell to each member of the class individually.

At the end of the term you may wish to sell your used copy of *Financial Management and Analysis*. You can sell it directly to a friend who is about to take the course, or you can sell it back to the bookstore for resale to another student. Both these transactions take place in the secondary textbook market, because the publisher (the issuer) is not a party to them.

If a firm can raise new funds only through the primary market, why should financial managers be concerned about the secondary market on which the firm's securities trade? Because investors may not be interested in buying securities that are not liquid—that they could not sell at a fair price at any time. And the secondary markets provide the liquidity. For example, suppose IBM wants to issue new common shares to pay for its expansion program; investors would not be willing to buy such shares if they could not expect to sell them on the secondary market should the need arise. IBM counts on the existence of a healthy secondary market to entice investors to buy its new stock issue.

Exchanges and Over-the-Counter Markets

There are two types of secondary securities markets: exchanges and over-the-counter markets. ***Exchanges*** are actual places where buyers and sellers (or their representatives) meet to trade securities. Examples are the New York Stock Exchange and the Tokyo Stock Exchange. ***Over-the-counter*** (*OTC*) ***markets*** are arrangements in which investors or their representatives trade securities without sharing a physical location. For the most part, computer and telephone networks are used for this purpose. These networks are owned and managed by the market's members. An example is the Nasdaq system, which is operated by the National Association of Securities Dealers (NASD).

Exchanges may be privately owned, as are those in the United States and the United Kingdom. Privately owned exchanges are managed by their owners, or members (typically brokerage firms), who may pay hundreds of thousands of dollars for the privilege of owning a seat (a membership) on the exchange. Private exchanges are self-regulated; that is, they determine the rules and regulations that must be followed by their members, by traders, and by companies whose securities are ***listed***, or accepted for trading, on the exchange.

Exchanges may be owned and operated by banks or banking organizations, as are many European exchanges—those in Luxembourg and Germany, for example. If the exchanges are owned by the banking institutions, these institutions then control both the primary and secondary markets for securities. Both bank-owned and privately owned exchanges are, of course, subject to regulation by the countries in which they are located.

Finally, there are state-controlled exchanges, such as those in France, Belgium, and several Latin American countries. These are generally the most restrictive exchanges and are characterized by stringent listing standards, especially for foreign companies.

There are two types of pricing systems for securities: the pure auction and the dealer market. In the ***pure auction process***, investors wanting to buy or sell shares of stock submit their bids through their brokers, who relay these bids to a centralized location, where bids are matched and the transaction is executed. The party that does the matching is referred to as the ***specialist***. For each stock in the market, there is only one matchmaker, one specialist. In a ***dealer market***, individual dealers buy and sell shares of stock, trading with individuals and other dealers. We refer to these dealers as market makers since they "make" a market in the stock, providing liquidity to the market. In a dealer market, there may be many dealers for a given stock. Though a market can use either or some combination of the two systems, exchanges tend to use the auction process and over-the-counter markets use a dealer market.

Markets in the United States

Governments provide no guarantees regarding securities. However, through legislation and regulation of markets, transactions, and transactors, the U.S. government has attempted to guard against fraudulent practices and manipulative behavior on the part of market participants. The federal organization charged with the regulation of U.S. financial markets is the Securities and Exchange Commission (SEC). The SEC is a federal agency that administers federal securities laws and was established by the Securities and Exchange Act of 1934. The SEC consists of five members, each appointed by the President of the United States for a term of five years. The SEC carries out the following activities:

- Issues rules that clarify securities laws or trading procedure issues.
- Requires disclosure of specific information.
- Makes public statements on current issues.
- Oversees self-regulation of the securities industry by the stock exchanges and professional groups such as the National Association of Securities Dealers.

Major federal legislation is listed in Exhibit 2.2; in addition, the states have all passed laws that reinforce or extend federal legislation.

EXHIBIT 2.2 **Federal Regulation of Securities Markets in the United States**

Law	Description
Securities Act of 1933	Regulates new offerings of securities to the public. It requires the filing of a registration statement containing specific information about the issuing corporation and prohibits fraudulent and deceptive practices related to security offers.
Securities and Exchange Act of 1934	Establishes the Securities and Exchange Commission (SEC) to enforce securities regulations and extends regulation to the secondary markets.
Investment Company Act of 1940	Gives the SEC regulatory authority over publicly-held companies that are in the business of investing and trading in securities.
Investment Advisers Act of 1940	Requires registration of investment advisors and regulates their activities.
Federal Securities Act of 1964	Extends the regulatory authority of the SEC to include the over-the-counter securities markets.
Securities Investor Protection Act of 1970	Creates the Securities Investor Protection Corporation, which is charged with the liquidation of securities firms that are in financial trouble and which insures investors' accounts with brokerage firms.
Insider Trading Sanctions Act of 1984	Provides for treble damages to be assessed against violators of securities laws.
Insider Trading and Securities Fraud Enforcement Act of 1988	Provides preventative measures against insider trading and establishes enforcement procedures and penalties for the violation of securities laws.

Money Markets

Money market securities are not traded in a physical location; rather these securities are traded over-the-counter through banks and dealers that are networked together by telephone and computer lines. These intermediaries bring together buyers and sellers from around the world. In the United States, most trading is centered around large banks (called ***money center banks***) located in the major financial centers of the country. Many banks and dealers specialize in specific instruments, such as commercial paper or bankers' acceptances.

Equity Markets

In the United States, there are two national stock exchanges: (1) the New York Stock Exchange (NYSE), commonly called the "Big Board," and (2) the American Stock Exchange (AMEX or ASE), also called the "Curb." National stock exchanges trade stocks of not only U.S. corporations but also non-U.S. corporations. In addition to the national exchanges, there are regional stock exchanges in Boston, Chicago (called the Midwest Exchange), Cincinnati, San Francisco (called the Pacific Coast Exchange), and Philadelphia. Regional exchanges primarily trade stocks from corporations based within their region.

The major OTC market in the United States is Nasdaq (the National Association of Securities Dealers Automated Quotation System), which is owned and operated by the NASD (the National Association of Securities Dealers). The NASD is a securities industry self-regulatory organization (SRO) that operates subject to the oversight of the SEC. Nasdaq is a national market. During 1998, Nasdaq and AMEX merged to form the Nasdaq–AMEX Market Group, Inc., each maintaining their respective markets and forming a large market that takes advantage of both the floor-based market structure and the OTC market structure.

The NYSE is the largest exchange in the United States, with approximately 2,800 companies' shares listed and dominates other markets in terms of the value and volume of shares traded. The AMEX is the second largest national stock exchange in the United States, with more than 750 issues listed for trading. Nasdaq has a greater number of listed stocks (4,200) but with much less market capitalization than the NYSE.

According to the Securities Act of 1934 (see Exhibit 2.2), there are two categories of traded stocks. The first is exchange traded stocks, which are also called ***listed stocks***. The second is OTC stocks, which are also non-exchange traded stocks and are, thereby, by inference, nonlisted. However, as we will describe later in this chapter, certain Nasdaq stocks have listing requirements (the Nasdaq National Market and the

Nasdaq Small Capitalization Market). Thus, a more useful and practical categorization of these categories is as follows:

- Exchange listed stocks (national and regional exchanges).
- Nasdaq listed OTC stocks.
- Non-Nasdaq OTC stocks.

Stock Exchanges

Stock exchanges are formal organizations, approved and regulated by the Securities and Exchange Commission (SEC). They are made up of members who use the exchange facilities and systems to exchange or trade listed stocks. These exchanges are physical locations where members assemble to trade. Stocks that are traded on an exchange are said to be listed stocks. That is, these stocks are individually approved for trading on the exchange by the exchange. To be listed, a company must apply and satisfy requirements established by the exchange for minimum capitalization, shareholder equity, average closing share price, and other criteria. Even after being listed, exchanges may delist a company's stock if it no longer meets the exchange requirements.

To have the right to trade securities or make markets on an exchange floor, firms or individuals must become a member of the exchange, which is accomplished by buying a seat on the exchange. The number of seats is fixed by the exchange and the cost of a seat is determined by supply and demand of those who want to sell or buy seats. In early 2001, there were 1,366 seats on the NYSE.

Two kinds of stocks are listed on the five regional stock exchanges: (1) stocks of companies that either could not qualify for listing on one of the major national exchanges or could qualify for listing but chose not to list; and (2) stocks that are also listed on one of the major national exchanges. The latter are called ***dually listed stocks***. The motivation of a company for dual listing is that a local brokerage firm that purchases a membership on a regional exchange can trade their listed stocks without having to purchase a considerably more expensive membership on the national stock exchange where the stock is also listed. Alternatively, a local brokerage firm could use the services of a member of a major national stock exchange to execute an order, but in this case it would have to give up part of its commission.

The regional stock exchanges compete with the NYSE for the execution of smaller trades. Major national brokerage firms have in recent years routed such orders to regional exchanges because of the lower fee they charge for executing orders or better prices, as we will discuss later.

OTC Market

The OTC market is called the market for unlisted stocks. As explained previously, technically while there are listing requirements for exchanges, there are also listing requirements for the Nasdaq National and Small Capitalization OTC markets. Nevertheless, exchange traded stocks are called listed, and stocks traded on the OTC markets are called unlisted. There are three parts to the OTC market: two under the aegis of NASD (the Nasdaq markets) and a third market for truly unlisted stocks, the non-Nasdaq OTC markets.

The Nasdaq stock market is the flagship market of the NASD. Nasdaq is essentially a telecommunication network that links thousands of geographically dispersed, market-making participants. Nasdaq is an electronic quotation system that provides price quotations to market participants on Nasdaq listed stocks. Although there is no central trading floor, Nasdaq has become an electronic "virtual trading floor." Some 535 dealers, known as market-makers, representing some of the world's largest securities firms, provide competing bids to buy and offers to sell Nasdaq stocks to investors.

The Nasdaq stock market has two broad tiers of securities: (1) the Nasdaq National Market and the Small Capitalization Market. Newspapers contain separate sections for these two tiers of stocks (sections labeled the "Nasdaq National Market" and the "Nasdaq Small Capitalization Market"). The Nasdaq National Market is the dominant OTC market in the United States.

Whereas the Nasdaq stock markets are the major parts of the U.S. OTC markets, the vast majority of the OTC issues (about 8,000) do not trade on either of the two Nasdaq systems. There are two types of markets for these stocks. The securities traded on these markets are not listed; that is, they have no listing requirements. The first of these two non-Nasdaq OTC markets is the OTC Bulletin Board (OTCBB), sometimes called simply the Bulletin Board. It includes stocks not traded on NYSE, AMEX, or Nasdaq. The second non-Nasdaq OTC market is the *Pink Sheets* that are published weekly. In addition, an electronic version of the Pink Sheets is updated daily and disseminated over market data vendor terminals. Pink Sheet securities are often pejoratively called ***penny stocks***.

Alternative Trading Systems

It is not necessary for two parties to a transaction to use an intermediary. That is, the services of a broker or a dealer are not required to execute a trade. The direct trading of stocks may take place between two customers without the use of a broker. A number of proprietary alternative trading systems (ATSs) are operated by the NASD members or member affiliates. These ATSs are for-profit "broker's brokers" that match investor orders and report trading activity to the marketplace via Nasdaq or the third market. In a sense, ATSs are similar to exchanges because they are designed to allow two participants to meet directly on the system and are maintained by a third party who also serves a limited regulatory function by imposing requirements on each subscriber.

Broadly, there are two types of ATSs: electronic communications networks and crossing networks. ***Electronic communications networks*** (ECNs) are privately owned broker-dealers that operate as market participants within the Nasdaq system. They display quotes that reflect actual orders and provide institutions and Nasdaq market-makers with an anonymous way to enter orders. Instinet was the first ECN. ***Crossing networks*** are systems developed to allow institutional investors to cross trade—that is, match buyers and sellers directly—typically via computer. These networks are batch processes that aggregate orders for execution at prespecified times.

Stock Market Indicators

Stock market indicators have come to perform a variety of functions, from serving as benchmarks for evaluating the performance of professional investors to answering the question "How did the market do today?" Thus, stock market indicators (indexes or averages) have become a part of everyday life.

The most commonly quoted stock market indicator is the Dow Jones Industrial Average (DJIA). Other stock market indicators cited in the financial press are the Standard & Poor's 500 Composite (S&P 500), the New York Stock Exchange Composite Index (NYSE Composite), the Nasdaq Composite Index, and the Value Line Composite Average (VLCA). Other stock market indicators include the Wilshire stock indexes and the Russell stock indexes, which are followed primarily by institutional money managers.

In general, market indexes rise and fall in fairly similar patterns. Although the correlation is high, the indexes do not move in exactly the same ways at all times. The differences in movement reflect the different ways in which the indexes are constructed. Three factors enter into that construction: the universe of stocks represented by the sample underlying the index, the relative weights assigned to the stocks included in the index, and the method of averaging across all the stocks.

Some indexes represent only stocks listed on an exchange. Examples are DJIA and the NYSE Composite, which represent only stocks listed on the Big Board. By contrast, the Nasdaq Composite Index includes only stocks traded over the counter. A favorite of professionals is the S&P 500 because it contains both NYSE-listed and OTC-traded shares.

Each index relies on a sample of stocks from its universe, and that sample may be small or quite large. The DJIA uses only 30 of the largest corporations, while the NYSE Composite includes every one of the NYSE listed shares. The Nasdaq Composite Index also includes all shares in its universe, while the S&P 500 has a sample that contains only 500 of the more than 8,000 shares in the universe it represents.

The stocks included in a stock market indicator must be combined in certain proportions, and each stock must be given a weight. The three main approaches to weighting are these: (1) weighting by the market capitalization of the stock's company, which is the value of the number of shares times price per share; (2) weighting by the price of the stock; and (3) equal weighting for each stock, regardless of its price or its firm's market value. With the exception of the Dow Jones averages (such as the DJIA) and the VLCA, all of the most widely used indices are market-value weighted. The DJIA is a price-weighted average, and the VLCA is an equally weighted index.

Stock market indicators can be classified into three groups: (1) those produced by stock exchanges based on all stocks traded on the exchanges; (2) those produced by organizations that subjectively select the stocks to be included in indices; and (3) those where stock selection is based on an objective measure, such as the market capitalization of the company. The first group includes the New York Stock Exchange Composite Index, which reflects the market value of all stocks traded on the exchange. Although it is not an exchange, the Nasdaq Composite Index falls into this category because the index represents all stocks tracked by the Nasdaq system.

The three most popular stock market indicators in the second group are the Dow Jones Industrial Average, the Standard & Poor's 500, and the Value Line Composite Average. The DJIA is constructed from 30 of the largest blue-chip industrial companies. The companies included in the average are those selected by Dow Jones & Company, publisher of the *Wall Street Journal*. The S&P 500 represents stocks chosen from the two major national stock exchanges and the over-the-counter market. The stocks in the index at any given time are determined by a committee of Standard & Poor's Corporation, which may occasionally add or delete individual stocks or the stocks of entire industry groups. The aim of the committee is to capture present overall stock market conditions as reflected in a very broad range of economic indicators. The VLCA, produced by Value Line, Inc., covers a broad range of widely held and actively traded NYSE, AMEX, and OTC issues selected by Value Line.

In the third group, we have the Wilshire indexes produced by Wilshire Associates (Santa Monica, California) and Russell indexes produced by the Frank Russell Company (Tacoma, Washington), a consultant to pension funds and other institutional investors. The criterion for inclusion in each of these indexes is solely a firm's market capitalization. The most comprehensive index is the Wilshire 5000, which currently includes more than 6,500 stocks, up from 5,000 at its inception. The Wilshire 4500 includes all stocks in the Wilshire 5000 except for those in the S&P 500. Thus, the shares in the Wilshire 4500 have a smaller capitalization than those in the Wilshire 5000. The Russell 3000 encompasses the 3,000 largest companies in terms of their market capitalization. The Russell 1000 is limited to the largest 1,000 of those, and the Russell 2000 has the remaining smaller firms.

Does it matter in which market a corporation's securities are traded? Yes and no. It is desirable to have your securities traded in a market where there is sufficient activity so that an investor who wants to buy or sell the security can do so readily. Therefore, the marketability that the market provides to the security is important. The more easily a security can be bought and sold, the less its ***marketability risk***, which is the risk than an owner will not be able to sell the security when he or she wants to sell it. Investors are willing to take a lower return when the marketability risk is lower, allowing the corporation to raise additional funds at a lower cost. Therefore, firms want to list their stocks in a market that provides marketability for the stock.

Bond Markets

Almost all bond trading takes place in OTC markets, with the remainder (around 1%) occurring mainly on the New York Stock Exchange Fixed Income Market and the American Stock Exchange. The bond trading that does take place on exchanges consists primarily of small orders, whereas bond trading in the OTC market is for larger—sometimes huge—blocks of bonds, purchased by institutional investors.

Within the OTC market, large banks and large trading firms "make a market" in bonds; that is, they connect buyers with sellers. They negotiate directly with large bond investors such as pension funds, insurance companies, and corporations, and are connected through a computerized network.

As with the stock market, there are bond market indexes that are followed by investors. The wide range of bond market indexes available can be classified as broad-based bond market indexes and specialized bond market indexes. The three broad-based bond market indexes most commonly used by institutional investors are the Lehman Brothers U.S. Aggregate Index, the Salomon Smith Barney (SSB) Broad Investment-Grade Bond Index (BIG), and the Merrill Lynch Domestic Market Index. There are more than 5,500 issues in each index. The specialized bond market indexes focus on one sector of the bond market or a subsector of the bond market. Indexes on sectors of the market are published by the three firms that produce the broad-based bond market indexes. Nonbrokerage firms have created specialized indexes for sectors.

Options and Futures Markets

The first formal options market was the ***Chicago Board Options Exchange*** (CBOE), begun in 1973. Soon after, several exchanges introduced options contracts to their "product lines." Now options are traded on such exchanges as the CBOE, the Chicago Board of Trade (CBOT), the Pacific Stock Exchange, the Philadelphia Stock Exchange, and the American Stock Exchange. As an indicator of the growing interest in options, we note that the dollar value of options traded annually on the CBOE now exceeds the value of the stocks traded annually on the AMEX. Options are traded on both exchanges and in the over-the-counter market, with most of the recent growth in the over-the-counter market.

Futures contracts are traded on (among others) the CBOT, the Chicago Mercantile Exchange, the Mid-America Commodity Exchange, and the New York Futures Exchange. Some futures markets specialize in certain contracts, either by preference or by state law. For example, the International Petroleum exchange specializes in petroleum products such as crude oil and gas oil. However, most commodities exchanges deal with a variety of futures contracts.

Like the equity markets, options and futures markets are subject to state and federal regulations (to different degrees), as well as to self-regulation by the markets themselves.[3]

Efficient Markets

Investors do not like risk and they must be compensated for taking on risk—the larger the risk, the more the compensation. But can investors earn a return on securities beyond that necessary to compensate them for the risk? In other words, can investors earn an ***abnormal profit*** on the secondary markets? Can they beat the market? The answer is "maybe."

An ***efficient market*** is a market in which asset prices rapidly reflect all available information, and the securities markets in the United States are typically thought of as being highly efficient. This means that all available information is already impounded in a security's price, so investors should expect to earn a return necessary to compensate them for their opportunity cost, anticipated inflation, and risk. That would seem to preclude abnormal profits. But according to at least one theory, there are several levels of efficiency: weak form efficient, semi-strong form efficient, and strong form efficient.[4]

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[3] For example, the *Commodity Futures Trading Commission* (*CFTC*) is a regulatory body established by Congress to approve new types of futures contracts and to establish trading rules for futures exchanges.

[4] Eugene F. Fama, "Efficient Capital Markets: A Review of Theory and Empirical Work," *Journal of Finance*, Volume 25, Number 2 (May 1970), pp. 383–417.

In the ***weak form of market efficiency***, current securities prices reflect all past prices and price movements. In other words, all worthwhile information about previous prices of the stock has been used to determine today's price; the investor cannot use that same information to predict tomorrow's price and still earn abnormal profits.[5]

Empirical evidence shows that the securities markets are at least weak-form efficient. In other words, you cannot beat the market by using information on past securities prices.

In the ***semi-strong form of market efficiency***, the current market prices of securities reflect all publicly-available information. So if you trade on the basis of publicly-available information, you cannot earn abnormal profits. This does not mean that prices change instantaneously to reflect new information, but rather that information is impounded *rapidly* into the prices of securities.

Empirical evidence supports the idea that U.S. securities markets are semi-strong form efficient. This, in turn, implies that careful analysis of securities and issuing firms cannot produce abnormal profits.[6]

In the ***strong form of market efficiency***, stock prices reflect all public and private information. In other words, the market (which includes all investors) knows everything about all securities, including information that has not been released to the public.

The strong form implies that you cannot make abnormal profits from trading on inside information, where inside information is information that is not yet public.[7] This form of market efficiency is not supported by the evidence. In fact, we know from recent events that the opposite is true; gains are available from inside information.

As pointed out above, U.S. securities markets are essentially semi-strong efficient. This means that investors can, for the most part, expect securities to be fairly priced. So when a firm issues new securities, it should expect investors to pay a price for those shares that reflects their value. This also means that if new information about the firm is revealed

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[5] This doesn't mean that trying it once may not prove fruitful. What it does mean is that, over the long run, you cannot earn abnormal profits from reading charts of past prices and predicting future prices from these charts. Do investors actually try this? Well, there are financial services in business today that perform analysis of stock prices (called technical analysis), so someone out there is doing it.

[6] Does this mean that financial analysis is worthless? No. We still need financial analysis to help us sort out risk and expected return so that we can properly manage our investments.

[7] There is no exact definition of "inside information" in law. Laws pertaining to insider trading remain a gray area, subject to clarification mainly through judicial interpretation.

to the public (for example, concerning a new product), the price of the stock should change to reflect that new information.

But a semi-strong efficient market also means that an investor can make abnormal profits through trading using information not known to the public. Such trading tends to distort the prices of affected securities and thus to harm at least some investors. For that reason, and because investigators found evidence of such trading during the corporate merger mania of the 1980s, existing anti-insider trading legislation has recently been strengthened and reinforced. Strengthening such legislation tends to ensure the fairness of securities prices.

In essence, it is illegal for any person with an agency relationship to a firm to benefit financially through non-public information obtained as a result of that relationship. This does *not* mean that executives of a corporation cannot buy and sell shares of the firm. Trading by insiders (members of the board of directors and the employees of the firm) is legal *if* it is not motivated by the use of non-public information. What it does mean is that insiders cannot use inside information to make their personal investment decisions; doing so would be ***illegal insider trading***. As another example, an investment banker who is negotiating the merger of two corporations cannot legally purchase the stock of those corporations knowing that the market prices will rise when news of the merger is made public.

SUMMARY

- A security is an instrument that represents ownership in an asset or debt obligation. Securities are classified as either money market securities, capital market securities, or derivative securities.
- Money market securities are marketable securities with original maturities of less than a year and include U.S. T-bills, commercial paper, certificates of deposit, and bankers acceptances. Capital market securities have maturities beyond one year and include common stocks, corporate bonds, and government bonds.
- Derivative instruments are contracts that derive their value from some security or asset, interest rate, exchange rate, or financial index. Derivative instruments include options, futures/forwards, swaps, and caps/floors.
- A securities market is any arrangement in which securities can be bought and sold and can be a formalized market, such as a stock exchange, or an informal market, such as banks acting as dealers in the over-the-counter market for bonds. Securities are bought and sold in primary markets, which provide the issuer with new capital, or in sec-

ondary markets, which involves trading among investors and no new capital for the issuer.

- Stocks, bonds, options, and futures are traded in securities markets. These financial markets may be specialized for one type of security, or may trade in more than one type of security. For example, bonds, futures, and options are all traded on markets organized under the New York Stock Exchange.
- More and more, securities are being bought and sold in countries other than their country of origin. The actual security may not trade outside its domestic market, yet there are means of trading securities that represent ownership of a foreign security, such as ADRs and unit trusts.
- Market indicators provide us with a gauge of the securities markets, giving us an idea of the general movements of securities prices.
- An efficient market is one in which information is quickly reflected in the prices of securities. We can further classify efficient markets according to the kind of information that is reflected: In weak form markets all past price information is contained in securities prices; in semi-strong form markets all publicly available information is reflected in securities prices; and in semi-strong form markets, all public and private information is reflected in securities prices. Evidence supports the idea that U.S. securities markets are semi-strong efficient markets. Trading on inside information, which disrupts market operations and efficiency, is illegal in the United States.

QUESTIONS

1. Ahsin, Inc., is a publicly traded company, but it does not intend to raise any new capital in the next few years. Why should Ahsin's financial managers concern themselves with securities markets?
2. What is the primary distinction between a money market security and a capital market security? From an investor perspective, which security would tend to be riskier? Why?
3. How risky is buying the commercial paper of a corporation relative to, say, buying its common stock? What factors affect the riskiness of a corporation's commercial paper? What factors affect the riskiness of a corporation's common stock?
4. How does collateral affect a security's riskiness? How does collateral affect the return required by investors?
5. Suppose individual income tax rates increase. Ignoring any other changes that may be made in the tax law, how should this affect the demand for municipal bonds?

6. Consider a convertible security that gives the owner the right to exchange it for another security within a specified period of time. Is this right to exchange a call or a put option? Explain.
7. What are derivative instruments and why are they used?
8. Describe the maturity and cash flow characteristics of common stock, preferred stock, and corporate debt securities. Rank these securities in terms of the uncertainty of their future cash flow.
9. What are the main differences between common and preferred stock? From the perspective of an investor, which security is riskier? Why?
10. Suppose International Business Machines (IBM) needs to raise new capital. List and briefly describe the three methods of raising capital.
11. Blockbuster Entertainment initially listed their stock on the Nasdaq system in 1983 and then changed its listing to the NYSE in 1989. Why would they initially list on the Nasdaq system? Why would they want to change their listing to the NYSE?
12. Determine whether each statement is consistent with the semi-strong form of market efficiency.
    a. *Statement X:* A local brokerage firm claims that following their strategy of investing in securities whose company name begins with the letter M, investors can earn a return that more than makes up for the risk associated with these securities.
    b. *Statement Y:* Company Big invested in stocks during 1992 and earned a return of 10%. Company Little earned 15% during the same year.
    c. *Statement Z:* Larry's investment strategy requires him to buy stocks of those companies that announced earnings higher than last year's. He claims that he can earn returns that are more than necessary to compensate him for the securities' risks.
13. What is insider trading? What is illegal insider trading?
14. Suppose an executive exercises her stock options just prior to the year, buying the shares and then selling them immediately, in order to avoid an anticipated increase in tax rates with the new administration. Is this illegal insider trading?
15. Suppose a member of the board of directors is involved in negotiating a merger of the firm with another firm. But suppose the negotiations are not complete and will not be for several months. If the board member buys stock of the other company while the negotiations are going on (but not completed), is this illegal insider trading?
16. Suppose you are manager of a corporation and you feel that it will do better in the future than most analysts believe. Can you buy stock of this firm? Is this illegal insider trading?

CHAPTER 3

Financial Institutions and the Cost of Money

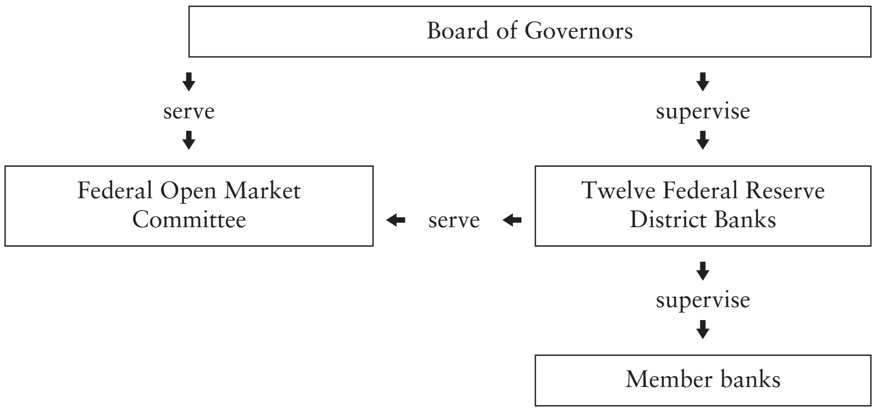
Businesses make their investment and financing decisions in a dynamic financial environment. Financial managers must understand the economy, the role of government in the economy, and the markets in which financial institutions operate. We have already taken a look at the financial markets in Chapter 2. Now we focus on other aspects of the financial environment. In particular, we examine the role of the following:

- the U.S. Federal Reserve System in determining the money supply
- the key role of financial intermediaries in the financial market with a focus on two of them—commercial banks and investment banks
- interest rates, the factors that influence them, and the cost of borrowing.

THE FEDERAL RESERVE SYSTEM

The United States has a central monetary authority known as the Federal Reserve System. The ***Federal Reserve System*** (often referred to as the "Fed") acts as the U.S. central bank, much like the Bank of England and the Bank of France are central banks in their respective countries. The role of a central bank is to carry out monetary policy that serves the best interests of the country's economic well-being. ***Monetary policy*** is the set of tools that a central bank can use to control the availability of loanable funds. These tools can be used to achieve goals for the nation's economy. Along with the U.S. Treasury, the Fed determines policies that affect employment and prices.

EXHIBIT 3.1 The Federal Reserve System



The Federal Reserve System is comprised of 12 district banks, with the Federal Reserve Board of Governors overseeing the activities of the district banks. The members of the Board are appointed by the President of the United States and confirmed by the U.S. Senate, and each serves a term of 14 years, with terms staggered through time. The president also appoints the chairman of the board from among the members on the board. The chairman serves in this capacity for a term of four years. What's the role of the Board? The Board creates rules and regulations that govern all depository institutions, as shown in Exhibit 3.1.

The Federal Reserve District Banks are not-for-profit institutions. Their responsibilities include (1) handling the vast majority of check-clearing in the United States, (2) issuing money, and (3) acting as the bankers' bank, accepting deposits from other financial institutions

The Federal Reserve System consists of the Federal Reserve Banks and member commercial banks. All nationally chartered banks must join the system, but state-chartered banks may also join. A nationally chartered bank is a bank that receives its charter of incorporation (its right to do business) from the federal government, granted by the Comptroller of the Currency; a state-chartered bank receives its charter from the state. The ***Comptroller of the Currency*** is a division of the U.S. Treasury and was established in 1863. The role of the Comptroller is to monitor periodically banks' financial condition and compliance with regulations; states have similar agencies that monitor state-chartered banks. Because national banks represent the largest banking institutions in the United States, more than two-thirds of all bank assets are held by national banks.

The ***Federal Open Market Committee*** (*FOMC*) is a policy making group within the Federal Reserve System. The committee is comprised of the seven members of the Federal Reserve Board, plus presidents or vice-presidents of five Reserve banks. The FOMC is charged with making decisions regarding the Federal Reserve's open market operations, which consist of buying and selling of U.S. government securities. The open market operations of the Fed affect the cost and availability of credit in the economy.

The Fed and the Money Supply

Financial managers and investors are interested in the supply and demand for money because it is the interaction of supply and demand that ultimately affects the interest rates paid to borrow funds and the amount of interest earned on investing funds. The demand for money is determined by the availability of investment opportunities. The supply of money is determined, in large part, by the actions of a nation's central bank.

The decisions of the Fed affect the money supply of the United States. The ***money supply*** consists of cash and cash-like items. In fact, there are different definitions of the money supply, depending on the cash-like items you include. For example, the most basic definition of money supply, *M1*, consists of:

- cash (currency and bills) in circulation,
- demand deposits (non-interest earning deposits at banking institutions that can be withdrawn on demand),
- other deposits that can be readily withdrawn using checks, and
- travelers' checks.

A broader definition of money supply is *M2*, which consists of everything in M1, plus

- savings deposits,
- small denomination time deposits,
- money market mutual funds, and
- money market deposit accounts.

A still broader definition of money supply is *M3*, which consists of everything in M2, plus:

- large denomination time deposits,
- term repurchase agreements issued by commercial banks and thrift institutions, term Eurodollars held by U.S. residents, and
- institution-owned balances in money market funds.

A ***savings deposit*** is an amount held in an account with a financial institution for the purpose of accumulating money. A ***time deposit*** is a type of savings account at a financial institution. A ***certificate of deposit*** (CD) is an example of a time deposit. The term "time" is used to describe the account because originally these accounts required that the saver notify the institution in advance (e.g., 90 days) of making a withdrawal. Though this practice of advance notification is no longer around, the term "time deposit" remains. ***Money market mutual funds*** are funds invested in an account that invests in short-term securities. ***Money market deposit accounts*** are funds deposited at financial institutions such as a bank or a thrift, that can be readily withdrawn. ***Eurodollars*** are deposits of U.S. dollars in foreign banks or in foreign branches of U.S. banks.

The money supply, whether defined as M1, M2, or M3, is managed by the government, and is one of the many tools that the government has to affect the economy. The role of non-M2 elements of the money supply has gained in importance in the money supply over this period, due primarily to the increasing popularity of money market funds.

The Fed affects the country's money supply and, ultimately, its economy through three devices. One device is a change in the ***reserve requirement***, the fraction of deposits that a bank accepts that must be held either on deposit with the district Federal Reserve Bank or in cash in the bank's own vault. The money not held in reserve can be used to make loans and purchase securities. Changing the reserve requirement affects monetary expansion; the lower the reserve requirement, the more funds that can be put into the economy through loans and investments and, hence, the greater the supply of money in the economy. Raising the required ratio reduces the effects of money expansion, and hence the money supply.

Another device is the use of open market operations. The FOMC affects the money supply through its decisions regarding ***open market operations***, which are purchases and sales of government securities by the Fed. Buying securities injects money into the economy; selling securities reduces the amount of money available.

Still another device is the change in the ***discount rate***, the interest rate charged by the Fed for loans to banks for reserves. These loans are secured—that is, backed—by U.S. government securities or other, suitable collateral. These loans are made by the district Federal Reserve Banks through a source referred to as the ***discount window***. Banks can borrow from the discount window to shore up their reserves for short periods of time (generally less than fifteen days). Increasing the discount rate discourages borrowing by banks, which in turn discourages banks from lending funds. Lowering the discount rate has the opposite effect—encouraging bank lending.

These three devices are used by the Fed to control the money supply. Since these devices have an effect on interest rates and the availability of loanable funds, many businesses watch the actions of the Fed with great interest.

The Future of Money: Electronic Cash

Nowadays, what we think of as "money" is quickly changing: electronic cash has entered the picture. ***Electronic cash***, sometimes referred to as ***e-cash***, ***cybercash***, or ***digicash***, is money that is created electronically and that exists outside the world of banks, checks, coin, and currency overseen by the Federal Reserve, and is created electronically. Though still in its infancy, electronic cash exists in the computer world of the Internet and on plastic cards. Who creates this money? Just about anyone. Electronic cash is replacing traditional cash, credit cards, and checks as a medium of exchange. It is more convenient than other forms of money and can result in lower costs of transacting business.

While seen as a universal medium of exchange that can span countries' borders, the existence of electronic cash raises some important issues:

- Who should be able to issue electronic cash?
- Who will regulate the issuers?
- How will transactions be monitored for compliance with tax laws?
- How secure are these transactions? (That is, can someone else wind up using your electronic cash?)
- How will consumers be protected from fraudulent issuers and users?
- How will regulators deal with money laundering and counterfeiting?

Currently, the Federal Reserve and banks are the money creators through the fractional reserve system. The emergence of electronic cash opens up money creation to almost anyone. The Fed may therefore lose its ability to control the economy by manipulating the reserve requirement, open market operations, and the discount rate. Though there are few players in electronic cash creation today on the Internet, several joint ventures between major corporations such as Microsoft, Xerox, and AT&T will bring electronic cash to consumers in the near future.

FINANCIAL INSTITUTIONS

Business entities include nonfinancial and financial enterprises. Nonfinancial enterprises manufacture products (e.g., cars, steel, computers) and/or provide nonfinancial services (e.g., transportation, utilities, computer programming). Financial enterprises, more popularly referred to as ***financial institutions***, provide services related to one or more of the following:

1. Transforming financial assets acquired through the market and constituting them into a different, and more widely preferable, type of asset—which becomes their liability. This is the function performed by ***financial intermediaries***, the most important type of financial institution.
2. Exchanging of financial assets on behalf of customers.
3. Exchanging of financial assets for their own accounts.
4. Assisting in the creation of financial assets for their customers, and then selling those financial assets to other market participants.
5. Providing investment advice to other market participants.
6. Managing the portfolios of other market participants.

Financial intermediaries include ***depository institutions*** (commercial banks, savings and loan associations, savings banks, and credit unions), which acquire the bulk of their funds by offering their liabilities to the public mostly in the form of deposits; insurance companies (life and property and casualty companies); pension funds; and finance companies.

The second and third services in the list above are the ***broker and dealer functions***. The fourth service is referred to as ***underwriting***. As we explain later, typically a financial intermediary that provides an underwriting service also provides a brokerage and/or dealer service. Some nonfinancial enterprises have subsidiaries that provide financial services. For example, many large manufacturing firms have subsidiaries that provide financing for their parent company's customer. These financial institutions are called ***captive finance companies***. Examples include General Motors Acceptance Corporation (a subsidiary of General Motors) and General Electric Credit Corporation (a subsidiary of General Electric).

Role of Financial Intermediaries

As we have seen, financial intermediaries obtain funds by issuing financial claims against themselves to market participants, and then investing those funds. The investments made by financial intermediaries—their assets—can be in loans and/or securities. These investments are referred to as ***direct investments***. Market participants who hold the financial claims issued by financial intermediaries are said to have made ***indirect investments***.

Two examples will illustrate this. Most readers of this book are familiar with what a commercial bank does. Commercial banks accept deposits and may use the proceeds to lend funds to consumers and businesses. The deposits represent a liability of the commercial bank and a financial asset owned by the depositor. The loan represents a liability of the borrowing entity and a financial asset of the commercial bank. The commercial bank has made a direct investment in the borrowing entity; the depositor effectively has made an indirect investment in that borrowing entity.

As a second example, consider an investment company or mutual fund, a financial intermediary we focus on later, which pools the funds of market participants and uses those funds to buy a portfolio of securities such as stocks and bonds. Investors providing funds to the investment company receive an equity claim that entitles the investor to a pro rata share of the outcome of the portfolio. The equity claim is issued by the investment company. The portfolio of financial assets acquired by the investment company represents a direct investment that it has made. By owning an equity claim against the investment company, those who invest in the investment company have made an indirect investment.

We have stressed that financial intermediaries play the basic role of transforming financial assets that are less desirable for a large part of the public into other financial assets—their own liabilities—which are more widely preferred by the public. This transformation involves at least one of four economic functions: (1) providing maturity intermediation; (2) reducing risk via diversification; (3) reducing the costs of contracting and information processing; and (4) providing a payments mechanism. Each function is described below.

Maturity Intermediation

In our example of the commercial bank, two things should be noted. First, the maturity of at least a portion of the deposits accepted is typically short term. For example, certain types of deposits are payable upon demand. Others have a specific maturity date, but most are less than two years. Second, the maturity of the loans made by a commercial bank may be considerably longer than two years. In the absence of a commercial bank, the borrower would have to borrow for a shorter term, or find an entity that is willing to invest for the length of the loan sought, and/or investors who make deposits in the bank would have to commit funds for a longer length of time than they want. The commercial bank, by issuing its own financial claims, in essence transforms a longer-term asset into a shorter-term one by giving the borrower a loan for the length of time sought and the investor/depositor a financial asset for the desired investment horizon. This function of a financial intermediary is called ***maturity intermediation***.

Maturity intermediation has two implications for financial markets. First, it provides investors with more choices concerning maturity for their investments; borrowers have more choices for the length of their debt obligations. Second, because investors are naturally reluctant to commit funds for a long period of time, they will require that long-term borrowers pay a higher interest rate than short-term borrowers. A financial intermediary is willing to make longer-term loans, and at a lower cost to the borrower than an individual investor would, by counting on successive deposits providing the funds until maturity (although at some risk—see below). Thus, the second implication is that the cost of longer-term borrowing is likely to be reduced.

Reducing Risk via Diversification

Consider the example of the investor who places funds in an investment company. Suppose that the investment company invests the funds received in the stock of a large number of companies. By doing so, the investment company has diversified and reduced its risk. Investors who have a small sum to invest would find it difficult to achieve the same degree of diversification because they do not have sufficient funds to buy shares of a large number of companies. Yet by investing in the investment company for the same sum of money, investors can accomplish this diversification, thereby reducing risk.

This economic function of financial intermediaries—transforming more risky assets into less risky ones—is called ***diversification***. Although individual investors can do it on their own, they may not be able to do it as cost-effectively as a financial intermediary, depending on the amount of funds they have to invest. Attaining cost-effective diversification in order to reduce risk by purchasing the financial assets of a financial intermediary is an important economic benefit for financial markets.

Reducing the Costs of Contracting and Information Processing

Investors purchasing financial assets should take the time to develop skills necessary to understand how to evaluate an investment. Once those skills are developed, investors should apply them to the analysis of specific financial assets that are candidates for purchase (or subsequent sale). Investors who want to make a loan to a consumer or business will need to write the loan contract (or hire an attorney to do so).

Although there are some people who enjoy devoting leisure time to this task, most prefer to use that time for just that—leisure. Most of us find that leisure time is in short supply, so to sacrifice it, we have to be compensated. The form of compensation could be a higher return that we obtain from an investment.

In addition to the opportunity cost of the time to process the information about the financial asset and its issuer, there is the cost of acquiring that information. All these costs are called ***information processing costs***. The costs of writing loan contracts are referred to as ***contracting costs***. There is also another dimension to contracting costs, the cost of enforcing the terms of the loan agreement.

With this in mind, consider our two examples of financial intermediaries—the commercial bank and the investment company. People who work for these intermediaries include investment professionals who are trained to analyze financial assets and manage them. In the case of loan agreements, either standardized contracts can be prepared, or legal counsel can be part of the professional staff that writes contracts involving more complex transactions. The investment professionals can monitor compliance with the terms of the loan agreement and take any necessary action to protect the interests of the financial intermediary. The employment of such professionals is cost-effective for financial intermediaries because investing funds is their normal business.

In other words, there are economies of scale in contracting and processing information about financial assets because of the amount of funds managed by financial intermediaries. The lower costs accrue to the benefit of the investor who purchases a financial claim of the financial intermediary and to the issuers of financial assets, who benefit from a lower borrowing cost.

Providing a Payments Mechanism

Although the previous three economic functions may not have been immediately obvious, this last function should be. Most transactions made today are not done with cash. Instead, payments are made using checks, credit cards, debit cards, and electronic transfers of funds. These methods for making payments, called ***payment mechanisms***, are provided by certain financial intermediaries.

The ability to make payments without the use of cash is critical for the functioning of a financial market. In short, depository institutions transform assets that cannot be used to make payments into other assets that offer that property.

Below we review each of the financial institutions and their role as intermediaries. The majority of our discussion will focus on the role of commercial banks (a form of depository institution) and investment banks. You will see why these entities are of particular interest to us because of the role that they play in either providing funds directly to entities needing to raise funds, assisting entities in raising funds, and/or facilitating the trading of securities. You will also see that while we made a distinction between commercial banks and investment bank, today a financial institution can provide all of the services provided by both of them.

Deposit Institutions

Traditionally, the United States has had several types of deposit institutions: commercial banks, savings and loan associations (referred to as thrift institutions or simply "thrifts"), mutual savings banks, and credit unions. In addition to accepting deposits, these institutions make loans and provide other financial services. These types of institutions are distinguished by their type of ownership (investor or depositor owned) and the type of loans (business or personal).

Commercial banks are corporations that are owned by investors. These banks lend primarily to businesses. Commercial banks may be independent corporations or may be subsidiaries of bank holding companies. ***Bank holding companies*** are organizations that own one or more other companies in addition to a bank. A common use of a bank holding company is as a device to circumvent regulations regarding bank branching or merging with banks across state lines. The Federal Reserve Board permits bank holding companies to own subsidiaries that are in lines of business related to banking.

Savings and loan associations are owned by their depositors and specialize in making home mortgage loans. The mission of savings and loan associations is to serve the thrift (that is, savings) and home ownership needs of consumers. Federally chartered savings and loans are overseen by the Office of Thrift Supervision (formerly the Federal Home Loan Bank Board, which was created in 1933). ***Mutual savings banks*** are also owned by their depositors and focus primarily on loans to the local community. ***Credit unions*** are non-profit associations that are owned by the members, the depositors, and their primary focus is making personal loans to their members. Exhibit 3.2 is a summary of the features of several of the deposit institutions.

EXHIBIT 3.2 **Summary of Types of Financial Institutions**

Type	Ownership	Primary Mission
Commercial bank	Corporations; owned by investors	Lend to businesses
Savings and loan (S&L)	Either corporations or owned by depositors	Offer savings accounts for individuals and make loans for home ownership
Mutual savings bank	Owned by depositors	Lend to the local community
Credit union	Non-profit; owned by depositors	Lend and provide other financial services to members

Commercial banks traditionally have the widest range of services, including checking accounts, savings accounts, credit cards, business loans, and personal loans. The ***Depository Institutions Deregulation and Monetary Control Act of 1980*** (*DIDMC*) reduced some of the distinctions between commercial banks and other institutions by eliminating restrictions on the type of loans, the interest rates on accounts, and the types of investments these other institutions could make. The effect of this act was to allow savings and loans, mutual savings banks, and credit unions to do business much like commercial banks. Adding to these new freedoms, the ***Garn-St. Germain Depository Act of 1982*** permitted both commercial banks and thrifts to provide money market accounts, enabling these institutions to compete with non-bank companies, such as brokerage firms, that offered money market accounts to individuals.

Deposits of commercial banks and savings institutions are insured by the ***Federal Deposit Insurance Corporation*** (*FDIC*), which is an agency created in 1934. Deposits with FDIC-insured institutions are insured up to $100,000 for each depositor and $100,000 for each depositor's retirement account. The role of the FDIC is to monitor these institutions' earnings and capital. Deposit insurance is intended to make the financial system more stable, preventing bank runs or panics—sudden and massive withdrawals of funds by customers.

Commercial Bank Services

Commercial banks play an important role in the country's money supply. Our purpose in this chapter is not to discuss this role; this topic is typically covered in a course on money and banking or financial markets. Rather, we will discuss the services commercial banks provide to entities seeking to raise funds. These services can be broadly classified as follows: (1) individual banking; (2) institutional banking; and (3) global banking. Of course, different banks are more active in certain of these activities than others.

Individual banking encompasses consumer lending, residential mortgage lending, consumer installment loans, credit card financing, automobile and boat financing, brokerage services, student loans, and individual-oriented financial investment services such as personal trust and investment services. Interest income and fee income are generated from mortgage lending and credit card financing.

Loans to nonfinancial corporations, financial corporations (such as life insurance companies), and government entities (state and local governments in the United States and foreign governments) fall into the category of ***institutional banking***. Also included in this category are commercial real estate financing and other activities that will be discussed elsewhere in this book, leasing and factoring.

It is in the area of global banking that banks began to compete head-to-head with investment banking (or securities) firms. ***Global banking*** covers a broad range of activities involving corporate financing and capital market and foreign-exchange products and services.

Corporate financing involves two components. First is the procuring of funds for a bank's customers. This can go beyond traditional bank loans to involve the underwriting of securities. As we shall explain later, legislation in the form of the Glass-Steagall Act at one time limited bank activities in this area. In assisting customers in obtaining funds, banks also provide bankers acceptances, letters of credit, and other types of guarantees for their customers. That is, if a customer has borrowed funds backed by a letter of credit or other guarantee, its lenders can look to the customer's bank to fulfill the obligation. The second area of corporate financing involves advice on such matters as strategies for obtaining funds, corporate restructuring, divestitures, and acquisitions.

Capital market and foreign exchange products and services involve transactions where the bank may act as a dealer or broker in a service. Some banks, for example, are dealers in U.S. government or other securities. Customers who wish to transact in these securities can do so through the government desk of the bank. Similarly, some banks maintain a foreign-exchange operation, where foreign currency is bought and sold. Bank customers in need of foreign exchange can use the services of the bank.

Regulation of Commercial Bank Activities

Because of the special role that commercial banks play in the financial system, banks are regulated and supervised by several federal and state government entities. At the federal level, supervision is undertaken by the Federal Reserve Board, the Office of the Comptroller of the Currency, and the Federal Deposit Insurance Corporation. While much of the legislation defining these activities dates back to the late 1930s, the nature of financial markets and commercial banking has changed since the 1970s.

The most sweeping legislation in bank regulation in recent years has produced changes in the permissible activities for banks and bank holding companies. The key legislation is the Financial Services Modernization (FSM) Act of 1999, more commonly referred to as the Gramm-Leach-Bliley (GLB) Act, which has changed the shape of the financial services sector by lowering the firewalls that existed between different financial service businesses. To appreciate the major impact of this legislation, we must first review the regulations on permissible bank activities prior to the passage of this legislation.

Early legislation governing bank activities developed because transactions between commercial banks and their securities affiliates that were permitted led to abuses. Against this background, Congress passed the Banking Act of 1933, which, among other provisions, contained four sections that are popularly referred to as the ***Glass-Steagall Act***. Specifically, banks could neither (1) underwrite securities and stock, nor (2) act as dealers in the secondary market for securities and stock (although there were some exceptions).

The Gramm-Leach-Bliley Act of 1999 created a new financial holding company authorized to engage in underwriting and selling securities. Consequently, the underwriting activities described later and the secondary securities market that were primarily the domain of financial entities referred to as investment banking firms were now opened to banks. As a result, subsequent to the act there have been several mergers of large bank holding companies and investment banking firms.

Investment Banking

The primary market involves the distribution to investors of newly issued securities by corporations and other entities seeking to raise funds. The entity issuing a security is referred to as the ***issuer***. The participants in the marketplace that work with issuers to distribute newly issued securities are called ***investment bankers***. The activity of investment banking is undertaken by basically two types of firms: securities houses and commercial banks.

Traditional Process for Underwriting New Issues

The traditional process in the United States for issuing new securities involves investment bankers performing one or more of the following three functions:

1. advising the issuer on the terms and the timing of the offering,
2. buying the securities from the issuer, and
3. distributing the issue to the public.[1]

In the sale of new securities, investment bankers need not undertake the second function—buying the securities from the issuer. An investment banker may merely act as an advisor and/or distributor of the new security. The function of buying the securities from the issuer is what we

[1] When an investment banking firm commits its own funds on a long-term basis by either taking an equity interest or creditor position in companies, this activity is referred to as ***merchant banking***.

referred to earlier as "underwriting." When an investment banking firm buys the securities from the issuer and accepts the risk of selling the securities to investors at a lower price, it is referred to as an "underwriter." When the investment banking firm agrees to buy the securities from the issuer at a set price, the underwriting arrangement is referred to as a ***firm commitment***. In contrast, in a ***best efforts arrangement***, the investment banking firm agrees only to use its expertise to sell the securities—it does not buy the entire issue from the issuer.

The fee earned from underwriting a security is the difference between the price paid to the issuer and the price at which the investment bank reoffers the security to the public. This difference is called the ***gross spread***, or the ***underwriter discount***. There are numerous factors that affect the size of the gross spread.

The typical underwritten transaction involves so much risk of capital loss that a single investment banking firm undertaking it alone would be exposed to the danger of losing a significant portion of its capital. To share this risk, an investment banking firm forms a syndicate of firms to underwrite the issue. The gross spread is then divided among the lead underwriter(s) and the other firms in the underwriting syndicate. The lead underwriter manages the deal (or "runs the books" for the deal). In many cases, there may be more than one lead underwriter.

A successful underwriting of a security requires that the underwriter have a strong sales force. The sales force provides feedback on advance interest in the security, and the traders (also called market makers) provide input in pricing the security as well. It would be a mistake to think that once the securities are all sold the investment banking firm's ties with the deal are ended. In the case of bonds, those who bought the securities will look to the investment banking firm to make a market in the issue.

Regulation of the Primary Market

Underwriting activities are regulated by the Securities and Exchange Commission (SEC). The Securities Act of 1933 governs the issuance of securities. The act requires that a registration statement be filed with the SEC by the issuer of a security. The type of information contained in the registration statement is the nature of the business of the issuer, key provisions or features of the security, the nature of the investment risks associated with the security, and the background of management. Financial statements must be included in the registration statement, and they must be certified by an independent public accountant.

The registration is actually divided into two parts. Part I is the ***prospectus***. It is this part that is typically distributed to the public as an offering of the securities. Part II contains supplemental information, which is not distributed to the public as part of the offering but is available from the SEC upon request. The act provides for penalties in the form of fines and/or imprisonment if the information provided is inaccurate or material information is omitted. One of the most important duties of an underwriter is to perform due diligence.

The filing of a registration statement with the SEC does not mean that the security can be offered to the public. The registration statement must be reviewed and approved by the SEC's Division of Corporate Finance before a public offering can be made. If the staff is satisfied, the SEC will issue an order declaring that the registration statement is "effective," and the underwriter can solicit sales. The approval of the SEC, however, does not mean that the securities have investment merit or are properly priced or that the information is accurate. It merely means that the appropriate information appears to have been disclosed.

The time interval between the initial filing of the registration statement and the time the registration statement becomes effective is referred to as the waiting period: (also called the "cooling-off period"). During the waiting period, the SEC does allow the underwriters to distribute a preliminary prospectus. Because the prospectus has not become effective, its cover page states this in red ink and, as a result, the preliminary prospectus is commonly called a ***red herring***. During the waiting period, the underwriter cannot sell the security, nor may it accept written offers from investors to buy the security.

In 1982 the SEC approved Rule 415, which permits certain issuers to file a single registration document indicating that they intend to sell a certain amount of a certain class of securities at one or more times within the next two years.[2] Rule 415 is popularly referred to as the shelf registration rule because the securities can be viewed as sitting on a "shelf," and can be taken off that shelf and sold to the public without obtaining additional SEC approval. In essence, the filing of a single registration document allows the issuer to come to market quickly because the sale of the security has been preapproved by the SEC. Prior to establishment of Rule 415, there was a lengthy period required before a security could be sold to the public. As a result, in a fast-moving market, issuers could not come to market quickly with an offering to take advantage of what they perceived to be attractive financing opportunities. For example, if a corporation felt that interest rates were low and wanted to issue a bond, it had to file a registration statement and could

[2] The issuer qualifes for Rule 415 registration if the securities are investment-grade securities and/or are the securities of companies that have historically filed registration statements and whose securities comply with minimum flotation requirements.

not issue the bond until the registration statement became effective. The corporation was then taking the chance that during the waiting period interest rates would rise, making the bond offering more costly.

Variations in the Underwriting Process

Not all deals are underwritten using the traditional syndicate process we have described. Variations in the United States and foreign markets include the bought deal for the underwriting of bonds, the auction process for both stocks and bonds, and a rights offering for underwriting common stock.

The mechanics of a ***bought deal*** are as follows. The lead manager or a group of managers offers a potential issuer of debt securities a firm bid to purchase a specified amount of the securities. The issuer is given a day or so (maybe even only a few hours) to accept or reject the bid. If the bid is accepted, the underwriting firm has bought the deal. It can, in turn, sell the securities to other investment banking firms for distribution to their clients and/or distribute the securities to its clients.

Another variation for underwriting securities is the ***auction process***. In this method, the issuer announces the terms of the issue, and interested parties submit bids for the entire issue. The auction form is mandated for certain securities of regulated public utilities and many municipal debt obligations. It is more commonly referred to as a ***competitive bidding underwriting***. For example, suppose that a public utility wishes to issue $300 million of bonds. Various underwriters will form syndicates and bid on the issue. The syndicate that bids the lowest cost to the issuer wins the entire $300 million bond issue and then reoffers it to the public.

A ***preemptive rights offering*** is a method for issuing new common stock directly to existing shareholders. A preemptive right grants existing shareholders the right to buy some proportion of the new shares issued at a price below market value. The price at which the new shares can be purchased is called the ***subscription price***. A rights offering ensures that current shareholders may maintain their proportionate equity interest in the corporation. For the shares sold via a preemptive rights offering, the underwriting services of an investment banker are not needed. However, the issuing corporation may use the services of an investment banker for the distribution of common stock that is not subscribed to. A ***standby underwriting arrangement*** will be used in such instances. This arrangement calls for the underwriter to buy the unsubscribed shares. The issuing corporation pays a standby fee to the investment banking firm. In the United States, the practice of issuing common stock via a preemptive rights offering is uncommon. In other countries it is much more common; in some countries, it is the only means by which a new offering of common stock may be sold.

Private Placement of Securities

In addition to underwriting securities for distribution to the public, securities may be placed with a limited number of financial institutions. ***Private placement***, as this process is known, differs from the public offering of securities that we have described so far. Life insurance companies are the major investors in private placements.

Public and private offerings of securities differ in terms of the regulatory requirements that the issuer must satisfy. The Securities Act of 1933 and the Securities Exchange Act of 1934 require that all securities offered to the general public must be registered with the SEC, unless there is a specific exemption. One exemption from registration under the 1933 act is for "transactions by an issuer not involving any public offering." Regulation D, adopted by the SEC in 1982, sets forth the guidelines that determine if an issue is qualified for exemption from registration. The guidelines require that, in general, the securities cannot be offered through any form of general advertising or general solicitation that would prevail for public offerings. Most importantly, the guidelines restrict the sale of securities to "sophisticated" investors. Such "accredited" investors are defined as those who (1) have the capability to evaluate (or who can afford to employ an advisor to evaluate) the risk and return characteristics of the securities, and (2) have the resources to bear the economic risks.

The exemption of an offering does not mean that the issuer need not disclose information to potential investors. In fact, the issuer must still furnish the same information deemed material by the SEC. The issuer supplies this information in a private placement memorandum, as opposed to a prospectus for a public offering. The distinction between the private placement memorandum and the prospectus is that the former does not include information deemed "nonmaterial" by the SEC, if such information is required in a prospectus. Moreover, unlike a prospectus, the private placement memorandum is not subject to SEC review.

Investment banking firms assist in the private placement of securities in several ways. They work with the issuer and potential investors on the design and pricing of the security. Often it has been in the private placement market that investment bankers first design new security structures. The investment bankers may be involved with lining up the investors as well as designing the issue. Or, if the issuer has already identified the investors, the investment banker may serve only in an advisory capacity. An investment banker can also participate in the transaction on a best efforts underwriting arrangement.

In the United States, one restriction imposed on buyers of privately placed securities is that they may not be resold for two years after acquisition. Thus, there is no liquidity in the market for that time period. Buyers of privately placed securities must be compensated for the lack of liquidity, which raises the cost to the issuer of the securities. In April 1990, however, SEC Rule 144A became effective. This rule eliminates the two-year holding period by permitting large financial institutions to trade securities acquired in a private placement among themselves without having to register these securities with the SEC. Private placements are now classified as ***Rule 144A offerings*** or ***non-Rule 144A offerings***. The latter are more commonly referred to as traditional private placements. Rule 144A offerings are underwritten by investment bankers.

Other Financial Institutions

There are a number of non-deposit financial institutions that hold financial assets. In this section, we briefly describe the role of trust companies, investment companies, pension funds, and insurers.

Trust Companies

A ***trust company*** is corporation formed to act as a trustee according to the terms of a contract (referred to as a trust agreement). A ***trustee*** is a person or a business that has the responsibility of overseeing the management of funds, making sure that they are managed in a way that is in the best interests of the beneficiaries (the persons for whose benefit the trust is established). Though many banks have their own trust departments to serve this function, independent trust companies exist to accept and manage funds according to a trust agreement.

Investment Companies

Investment companies sell shares to individuals and use these funds to invest in a pool of assets. These assets may be stocks, bonds, or some other investment. Investment companies that buy and sell shares in the pool at any time the customer wishes are referred to as ***open-end investment companies***, which are also referred to as ***mutual funds***. Investment companies that sell only a specific number of shares in the pool are referred to as ***closed-end investment companies***. Pools invested in short-term assets are referred to as ***money market funds***. Pools invested in real estate investments are referred to as ***real estate investment trusts***.

Pension Funds

Workers set aside a portion of their income in ***pension funds*** to protect against a loss of income in retirement years. The pension fund then invests the money in stocks, bonds, or other assets, building up the value of the funds to provide for the workers' future retirement.

Life Insurance Companies

Individuals and companies purchase insurance policies that protect policyholders and their families or employers against the risks of premature death or disability. The life insurance company invests and manages the funds, building up these funds for the eventual payout of insurance policy benefits.

Property-Casualty Insurers

Individuals and businesses purchase insurance policies that protect them against risks of loss from weather, crime, personal negligence, or some other type of event. Like the life insurers, property-casualty insurers invest and manage these funds to accommodate future payments to insured individuals and businesses.

THE COST OF MONEY

Money is not a free good. Those who need money are willing to pay for it and those who lend money expect to be compensated. The ***interest rate*** is the cost of money. If you put $1,000 in an account in a savings and loan that pays interest of 5% per year, you will earn $50 interest in one year. The savings and loan is paying you $50 for the use of your $1,000. Similarly, if you buy a $1,000 face value bond with a coupon rate of 5%, you earn $50 interest each year. The issuer is paying $50 interest each year for the use of your $1,000.

Interest Rates and Yields

Because bonds are traded in the secondary market, the price of the bond may change as the supply and demand for funds changes. The interest paid on your bond does not change (you get $50 per year), but the bond's price does. Suppose instead you buy the $1,000 face value bond for $900. The bond still pays $50 interest per year, but you only paid $900 for it. Therefore, you are earning more than the 5% interest rate ($50/$900 = 5.56%). The 5.56% is the ***yield*** on the bond, and 5% is the ***interest rate*** on the bond.

Most bonds are issued at their face or par value, so when they are issued the yield is often equal to the coupon rate. And if you buy a bond when it is issued and hold it until it matures, you will earn the bond's interest rate (i.e., pay $1,000 and get $50 per year). As time marches on, a bond's value change and its yield (that is, what investors can earn if they buy the bond at the time) will often deviate from its interest rate. If you hold the bond to maturity, you don't care about its changing value. But if you buy the bond sometime after it is issued or sell the bond before it matures, you do care about its changing value.

We often use the terms "interest rate" and "yield" interchangeably because they tell us how much we get for the amount we invest. When we are talking about what investors are getting in terms of a return, we generally talk about the "yield"—the return investors get if they buy a security at its current price. And, to make returns comparable across securities with different maturities, we quote these yields in terms of a common time frame—a year. This allows us to compare the yield on, say, a 3-month Treasury Bill with a 1-year Treasury Bill.

Determinants of Interest Rates

Interest rates are determined by the supply and demand for money. The supply of money depends in large part on the actions of the Fed, as we discussed previously. Therefore, let's focus on the demand for money.

The demand for money arises from two sources: transactions demand and asset demand. The ***transactions demand*** arises from individuals' and businesses' need to use money as a medium of exchange in transactions. The more goods and services exchanges take place in the economy, the greater the transactions demand. The ***asset demand*** is individuals' and businesses' need to use money as a store of value—they keep some of their wealth in the form of money (instead of in, say, stocks or bonds), which is risk-free and liquid.

Firms raise funds to invest in capital projects, which are investments that have long-term future cash flow consequences. If a firm has many possible ways to invest—to build a new plant, to start an advertising campaign—it will rank these projects based on profitability and invest in those whose profit exceeds the cost of the funds. Meanwhile, other firms are doing the same thing. As a result, firms compete for funds for their investment projects. Firms with the most profitable investment opportunities get the necessary funds, and firms with the least profitable investment opportunities do not. In other words, money is distributed to the capital projects that are most profitable.

Since money earns little or nothing, how much wealth individuals or firms are willing to keep in the form of money depends not only on how they feel about liquidity and risk, but also on what they could earn on the funds if they invested them elsewhere (say, in bonds). Therefore, the demand for money is affected by interest rates: the higher the interest rate, the lower the demand for money.

The Structure of Interest Rates

There is not one interest rate in any economy. Rather, there is a structure of interest rates. The interest rate that a borrower will have to pay depends on a myriad of factors. We discuss these factors next.

The Base Interest Rate

The securities issued by the U.S. Department of the Treasury, popularly referred to as Treasury securities or simply Treasuries, are backed by the full faith and credit of the U.S. government. Consequently, market participants throughout the world view them as having no credit risk. As a result, historically the interest rates on Treasury securities have served as the benchmark interest rates throughout the U.S. economy as well as in international capital markets.

The U.S. Treasury is the largest single issuer of debt in the world and the large size of any single issue has contributed to making the Treasury market the most active and, hence, the most liquid market in the world. However, in recent years, the U.S. Department of the Treasury has reduced its issuance of Treasury securities, particularly long-term securities, as well as buying back long-term Treasury securities in the market. This has decreased the supply of these securities and, as a result, there are market participants who feel that the yields on Treasury securities are no longer a suitable benchmark for interest rates throughout the world. As a result, as of this writing, there is a search for other possible benchmarks.

The Risk Premium

Market participants talk of interest rates on non-Treasury securities as "trading at a spread" to a particular on-the-run Treasury security (or a spread to any particular benchmark interest rate selected). For example, if the yield on a 10-year non-Treasury security is 7% and the yield on a 10-year Treasury security is 6%, the spread is 100 basis points. This spread reflects the additional risks the investor faces by acquiring a security that is not issued by the U.S. government and, therefore, can be called a risk premium. Thus, we can express the interest rate offered on a non-Treasury security as:

$$\text{Interest rate} = \text{Base interest rate} + \text{Spread}$$

or equivalently,

$$\text{Interest rate} = \text{Base interest rate} + \text{Risk premium}$$

We have discussed the factors that affect the base interest rate. One of the factors is the expected rate of inflation. That is, the base interest rate can be expressed as:

$$\text{Base interest rate} = \text{Real rate of interest} + \text{Expected rate of inflation}$$

Turning to the spread, the factors that affect it are (1) the issuer's perceived creditworthiness; (2) the term or maturity of the instrument; (3) provisions that grant either the issuer or the investor the option to do something; (4) the taxability of the interest received by investors; and (5) the expected liquidity of the issue.

It is important to note that yield spreads must be interpreted relative to the benchmark interest rate used. This is particularly important to keep in mind for the second and last factors that affect the spread when the benchmark interest rate is other than the yield on U.S. Treasury securities.

Perceived Creditworthiness of Issuer Credit risk refers to the risk that the issuer of a debt obligation may be unable to make timely payment of interest and/or the principal amount when it is due. Most market participants rely primarily on commercial rating companies to assess the default risk of an issuer. These companies perform credit analyses and express their conclusions by a system of ratings. The three commercial rating companies in the United States are (1) Moody's Investors Service, (2) Standard & Poor's Corporation, and (3) Fitch Ratings.

In all systems the term ***high grade*** means low credit risk, or conversely, high probability of future payments. The highest-grade bonds are designated by Moody's by the symbol Aaa, and by S&P and Fitch by the symbol AAA. The next highest grade is denoted by the symbol Aa (Moody's) or AA (S&P and Fitch); for the third grade all rating systems use A. The next three grades are Baa or BBB, Ba or BB, and B, respectively. There are also C grades. Moody's uses 1, 2, or 3 to provide a narrower credit quality breakdown within each class, and S&P and Fitch use plus and minus signs for the same purpose.

Bonds rated triple A (AAA or Aaa) are said to be ***prime***; double A (AA or Aa) are of ***high quality***; single A issues are called ***upper medium grade***, and triple B are ***medium grade***. Lower-rated bonds are said to have speculative elements or be distinctly speculative. Bond issues that are assigned a rating in the top four categories are referred to as ***investment-grade bonds***. Issues that carry a rating below the top four categories are referred to as ***noninvestment-grade bonds***, or more popularly as high-yield bonds or junk bonds. Thus, the bond market can be divided into two sectors: the investment-grade and noninvestment-grade markets. The spread between Treasury securities and non-Treasury securities that are identical in all respects except for quality is referred to as a ***quality spread*** or ***credit spread***.

Term to Maturity The price of a financial asset will fluctuate over its life as yields in the market change. It can be demonstrated that the price volatility of a bond is dependent on its maturity. More specifically, with all other factors being constant, the longer the maturity of a bond, the greater the price volatility resulting from a change in market yields. The spread between any two maturity sectors of the market is called a ***maturity spread*** or ***yield curve spread***. The relationship between the yields on comparable securities but different maturities is called the ***term structure of interest rates***. The term-to-maturity topic is of such importance that we discuss in more detail later in this chapter.

Inclusion of Options It is not uncommon for a bond issue to include a provision that gives either the bondholder and/or the issuer an option to take some action against the other party. An option that is included in a bond issue is referred to as an ***embedded option***. The most common type of option in a bond issue is a ***call provision***. This provision grants the issuer the right to retire the debt, fully or partially, before the scheduled maturity date. The inclusion of a call feature benefits issuers by allowing them to replace an old bond issue with a lower interest cost issue should interest rates in the market decline. Effectively, a call provision allows the issuer to alter the maturity of a bond. A call provision is detrimental to the bondholder because the bondholder will be uncertain about maturity and might have to reinvest the proceeds received at a lower interest rate if the bond is called and the bondholder wants to keep his or her funds in issues of similar risk of default.

An issue also may include a provision that allows the bondholder to change the maturity of a bond. An issue with a ***put provision*** grants the bondholder the right to sell the issue back to the issuer at par value on designated dates. Here, the advantage to the investor is that, if interest rates rise after the issue date and result in a price that is less than the par value, the investor can force the issuer to redeem the bond at par value.

A ***convertible bond*** is an issue giving the bondholder the right to exchange the bond for a specified number of shares of common stock. This feature allows the bondholder to take advantage of favorable movements in the price of the issuer's common stock.

The presence of these embedded options has an effect on the spread of an issue relative to a Treasury security and the spread relative to otherwise comparable issues that do not have an embedded option. In general, market participants require a larger spread over a comparable Treasury security for an issue with an embedded option that is favorable to the issuer (e.g., a call option) than for an issue without such an option. In contrast, market participants require a smaller spread over a comparable Treasury security for an issue with an embedded option that is favorable to the investor (for example, put option and conversion option). In fact, for a bond with an option that is favorable to an investor, the interest rate on an issue may be less than that on a comparable Treasury security!

Taxability of Interest Unless exempted under the federal income tax code, interest income is taxable at the federal level. In addition to federal income taxes, there may be state and local taxes on interest income. The federal tax code specifically exempts the interest income from qualified municipal bond issues from taxation at the federal level. Municipal bonds are securities issued by state and local governments and by their creations, such as "authorities" and special districts. The large majority of outstanding municipal bonds are tax-exempt securities. Because of the tax-exempt feature of municipal bonds, the yield on municipal bonds is less than that on Treasuries with the same maturity.

Expected Liquidity of an Issue Bonds trade with different degrees of liquidity. The greater the expected liquidity with which an issue trades, the lower the yield that investors require. As noted earlier, Treasury securities are the most liquid securities in the world. The lower yield offered on Treasury securities relative to non-Treasury securities reflects, to a significant extent, the difference in liquidity.

Term Structure of Interest Rates

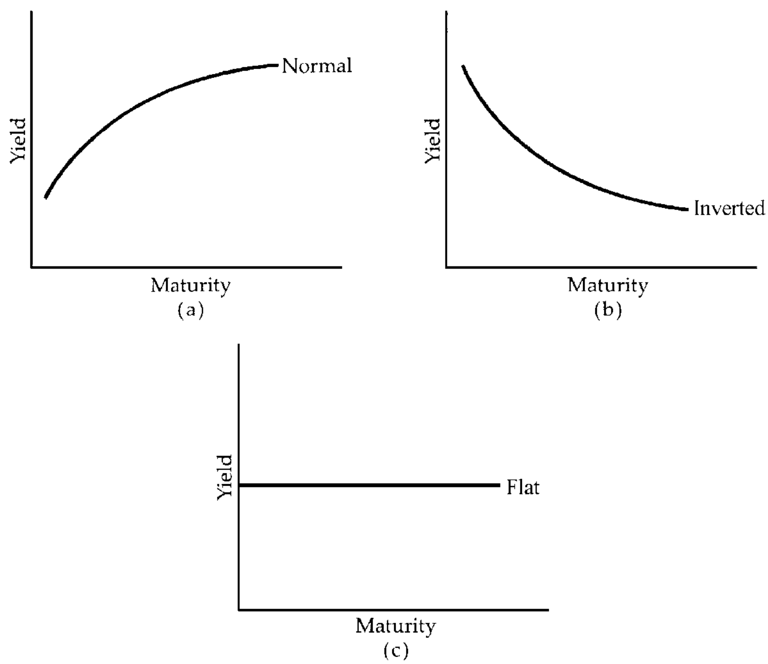
One of the factors that we stated affects the risk premium is the maturity of a debt obligation. The relationship between the yield on a bond and its maturity is the ***term structure of interest rates***. The graphic that depicts the relationship between the yield on bonds of the same credit quality but different maturities is known as the ***yield curve***. Market participants have tended to construct yield curves from observations of prices and yields in the Treasury market. Two reasons account for this tendency. First, Treasury securities are free of default risk, and differences in creditworthiness do not affect yield estimates. Second, as the largest and most active bond market, the Treasury market offers the fewest problems of illiquidity or infrequent trading. Exhibit 3.3 shows the shape of three hypothetical Treasury yield curves that have been observed from time to time in the United States. However, as noted earlier, new benchmarks are being considered by market participants because of the dwindling supply of U.S. Treasury securities. Nevertheless, the principles set forth here apply to any other benchmark selected.

From a practical viewpoint, the Treasury yield curve functions mainly as a benchmark for setting yields in many other sectors of the debt market—bank loans, mortgages, corporate debt, and international bonds. However, a Treasury yield curve based on observed yields on the Treasury market is an unsatisfactory measure of the relation between required yield and maturity. The key reason is that securities with the same maturity may actually provide different yields. Hence, it is necessary to develop more accurate and reliable estimates of the Treasury yield curve. Specifically, the key is to estimate the theoretical interest rate that the U.S. Treasury would have to pay assuming that the security it issued is a zero-coupon security. We will not explain how this is done. At this point, all that is necessary to know is that there are procedures for estimating the theoreitical interest rate or yield that the U.S. Treasury would have to pay for bonds with different maturities. These interest rates are called ***Treasury spot rates***.

Valuable information for market participants can be obtained from the Treasury spot rates. These rates are called ***forward rates***. First, we will see how these rates are obtained and then we will discuss theories about what determines forward rates. Finally, we will see how issuers can use the forward rates in making financing decisions.

Foward Rates To see how a forward rate can be computed, consider the following two Treasury spot rates. Suppose that the spot rate for a zero-coupon Treasury security maturing in one year is 4% and a zero-coupon Treasury security maturing in two years is 5%. Let's look at this situation from the perspective of an investor who wants to invest funds for two years. The investors choices are as follows:

EXHIBIT 3.3 **Three Observed Shapes for the Yield Curve**



Alternative 1: Investor buys a two-year zero-coupon Treasury security
Alternative 2: Investor buys a one-year zero-coupon Treasury security and when it matures in one year the investor buys another one-year instrument.

With Alternative 1, the investor will earn the two-year spot rate and that rate is known with certainty. In contrast, with Alternative 2, the investor will earn the one-year spot rate, but the one-year spot one year from now is unknown. Therefore, for Alternative 2, the rate that will be earned over one year is not known with certainty.

Suppose that this investor expected that one year from now the one-year spot rate will be higher than it is today. The investor might then feel Alternative 2 would be the better investment. However, this is not necessarily true. To understand why and to appreciate the need to understand why it is necessary to know what a forward rate is, let's continue with our illustration.

The investor will be indifferent to the two alternatives if they produce the same total dollars over the two-year investment horizon. Given the two-year spot rate, there is some spot rate on a one-year zero-coupon Treasury security one year from now that will make the investor indifferent between the two alternatives. We will denote that rate by f.

The value of f can be readily determined given the two-year spot rate and the one-year spot rate. If an investor placed \$100 in the two-year zero-coupon Treasury security (Alternative 1) earning 5%, the total dollars that will be generated at the end of two years is:[3]

$$\begin{aligned}\text{Total dollars at the end of two years for Alternative 1} &= \$100(1.05)^2 \\ &= \$110.25\end{aligned}$$

The proceeds from investing in the one-year Treasury security at 4% will generate the following total dollars at the end of one year:

$$\begin{aligned}\text{Total dollars at the end of two years for Alternative 2} &= \$100(1.04) \\ &= \$104\end{aligned}$$

If one year from now this amount is reinvested in a zero-coupon Treasury security maturing in one year, which we denoted f, then the total dollars at the end of two years would be:

$$\text{Total dollars at the end of two years for Alternative 2} = \$104(1 + f)$$

[3] We will discuss this compounding of returns in Chapter 7.

The investor will be indifferent between the two alternatives if the total dollars are the same. Setting the two equations for the total dollars at end of two years for the two alternatives equal we get:

$$\$110.25 = \$104(1 + f)$$

Solving the preceding equation for *f*, we get

$$f = \frac{\$110.25}{\$104} - 1 = 0.06 = 6\%$$

Here is how we use this rate of 6%. If the one-year spot rate one year from now is less than 6%, then the total dollars at the end of two years would be higher by investing in the two-year zero-coupon Treasury security (Alternative 1). If the one-year spot rate one year from now is greater than 6%, then the total dollars at the end of two years would be higher by investing in a one-year zero-coupon Treasury security and reinvesting the proceeds one year from now at the one-year spot rate at that time (Alternative 2). Of course, if the one-year spot rate one year now is 6%, the two alternatives give the same total dollars at the end of two years.

Now that we have the forward rate *f* in which we are interested and we know how that rate can be used, let's return to the question we posed at the outset. Suppose that the investor expects that one year from now, the one-year spot rate will be 5.5%. That is, the investor expects that the one-year spot rate one year from now will be higher than its current level. Should the investor select Alternative 2 because the one-year spot rate one year from now is expected to be higher? The answer is no. As we explained in the previous paragraph, if the spot rate is less than 6%, then Alternative 1 is the better alternative. Since this investor expects a rate of 5.5%, then he or she should select Alternative 1 despite the fact that he or she expects the one-year spot rate to be higher than it is today.

This is a somewhat surprising result for some investors. But the reason for this is that the market prices its expectations of future interest rates into the rates offered on investments with different maturities. This is why knowing the forward rates is critical. Some market participants believe that the forward rate is the market's consensus of future interest rates.

Similarly, borrowers need to understand what a forward rate is. For example, suppose a borrower must choose between a two-year loan and a series of two one-year loans. If the forward rate is less than the borrower's expectations of one-year rates one year from now, then the borrower will be better off with a two-year loan. If, instead, the borrower's expectations are that the one-year rate one year from now will be less than the forward rate, the borrower will be better off by choosing a series of two one-year loans.

In practice, a corporate treasurer needs to know both forward rates and what future spreads will be. Recall that a corporation pays the Treasury rate (i.e., the benchmark) plus a spread.

Forward Rates as a Hedgeable Rate A natural question about forward rates is how well they do at predicting future interest rates. Studies have demonstrated that forward rates do not do a good job in predicting future interest rates.[4] Then, why the big deal about understanding forward rates? The reason, as we demonstrated in our illustration of how to select between two alternative investments, is that the forward rates indicate how an investor's and borrower's expectations must differ from the market consensus in order to make the correct decision.

In our illustration, the one-year forward rate may not be realized. That is irrelevant. The fact is that the one-year forward rate indicated to the investor that if expectations about the one-year rate one month from now are less than 6%, the investor would be better off with Alternative 1.

For this reason, as well as others explained later, some market participants prefer not to talk about forward rates as being market consensus rates. Instead, they refer to forward rates as being ***hedgeable rates***. For example, by investing in the two-year Treasury security, the investor was able to hedge the one-year rate one year from now. Similarly, a corporation issuing a two-year security is hedging the one-year rate one year from now. (Note, however, that it is only the benchmark interest rate that is being hedged. The spread that the corporation or the issuer will pay can change.)

Determinants of the Shape of the Term Structure If we plot the term structure—the yield to maturity, or the spot rate, at successive maturities against maturity—what is it likely to look like? Exhibit 3.3 shows three shapes that have appeared with some frequency over time. Panel A shows an upward-sloping yield curve; that is, yield rises steadily as maturity increases. This shape is commonly referred to as a normal or positive yield curve. Panel B shows a downward-sloping or inverted yield curve, where yields decline as maturity increases. Finally, panel C shows a flat yield curve.

Two major theories have evolved to account for these observed shapes of the yield curve: the expectations theory and the market segmentation theory.

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[4] See Eugene F. Fama, "Forward Rates as Predictors of Future Spot Rates," *Journal of Financial Economics* (1976), pp. 361–377.

There are several forms of the expectations theory—the pure expectations theory, the liquidity theory, and the preferred habitat theory. All share a hypothesis about the behavior of short-term forward rates and also assume that the forward rates in current long-term debt contracts are closely related to the market's expectations about future short-term rates. These three theories differ, however, on whether or not other factors also affect forward rates, and how. The pure expectations theory postulates that no systematic factors other than expected future short-term rates affect forward rates; the liquidity theory and the preferred habitat theory assert that there are other factors. Accordingly, the last two forms of the expectations theory are sometimes referred to as ***biased expectations theories***.

According to the pure expectations theory, the forward rates exclusively represent the expected future rates. Thus, the entire term structure at a given time reflects the market's current expectations of the family of future short-term rates. Under this view, a rising term structure, as in Panel A of Exhibit 3.3, must indicate that the market expects short-term rates to rise throughout the relevant future. Similarly, a flat term structure reflects an expectation that future short-term rates will be mostly constant, while a falling term structure must reflect an expectation that future short rates will decline steadily.

Unfortunately, the pure expectations theory suffers from one shortcoming, which, qualitatively, is quite serious. It neglects the risks inherent in investing in bonds and like instruments. If forward rates were perfect predictors of future interest rates, then the future prices of bonds would be known with certainty. The return over any investment period would be certain and independent of the maturity of the instrument initially acquired and of the time at which the investor needed to liquidate the instrument. However, with uncertainty about future interest rates and hence about future prices of bonds, these instruments become risky investments in the sense that the return over some investment horizon is unknown.

Similarly, from a borrower or issuer's perspective, the cost of borrowing for any required period of financing would be certain and independent of the maturity of the instrument initially sold if the rate at which the borrower must refinance debt in the future is known. But with uncertainty about future interest rates, the cost of borrowing is uncertain if the borrower must refinance at some time over the periods in which the funds are initially needed.

There are two biased expectations theories that recognize the shortcomings in the pure expectations theory—the liquidity theory and the preferred habitat theory. According to the liquidity theory, the forward rates will not be an unbiased estimate of the market's expectations of future interest rates because they embody a liquidity premium. This liquidity premium reflects the risks of holding a bond for a longer time period. Thus, an upward-sloping yield curve may reflect expectations that future interest rates either (1) will rise, or (2) will be flat or even fall, but with a liquidity premium increasing fast enough with maturity so as to produce an upward-sloping yield curve.

The preferred habitat theory also adopts the view that the term structure reflects the expectation of the future path of interest rates as well as a risk premium. However, the habitat theory rejects the assertion that the risk premium must rise uniformly with maturity. Proponents of the habitat theory say that the latter conclusion could be accepted if all investors intend to liquidate their investment at the first possible date, while all borrowers are eager to borrow long, but that this is an assumption that can be rejected for a number of reasons. The argument is that different financial institutions have different investment horizons and have a preference for the maturities in which they invest. The preference is based on the maturity of their liabilities. To induce a financial institution out of that maturity sector, a premium must be paid. Thus, the forward rates include a liquidity premium and compensation for investors to move out of their preferred maturity sector. Consequently, forward rates do not reflect the market's consensus of future interest rates.

There is one more theory about the terms structure of interest rates. The ***market segmentation theory*** also recognizes that investors have preferred habitats dictated by saving and investment flows. This theory also proposes that the major reason for the shape of the yield curve lies in asset/liability management constraints (either regulatory or self-imposed) and/or creditors (borrowers) restricting their lending (financing) to specific maturity sectors. However, the market segmentation theory differs from the preferred habitat theory in that it assumes that neither investors nor borrowers are willing to shift from one maturity sector to another to take advantage of opportunities arising from differences between expectations and forward rates. Thus, for the segmentation theory, the shape of the yield curve is determined by supply of and demand for securities within each maturity sector.

Understanding Issuer Costs

We now understand that a corporation or other entity wishing to issue debt must pay the benchmark interest rate plus a risk premium. The risk premium is the spread and is affected by the various factors we discussed. In fact, when a potential issuer inquires of its investment bankers about the interest rate it would have to pay if it issued securities, the investment banker typically does *not* talk about the rate. The issuer talks about the "spread" at which the securities can be sold.

Consequently, when an issuer must decide on whether or not to issue a security, say a 15-year bond, the treasurer or chief financial officer will assess the benchmark interest rate and the spread. The forward rates along with forecasts by economists can be used to evaluate whether or not to issue a security now if rates are expected to rise or postpone issuance (and borrow short term) if rates are expected to fall. The expected change in the spread also affects the decision. The benchmark interest rate may be expected to fall but the spread increase such that the interest rate that the issuer would pay would be higher in the future. All of these elements go into the financing decision.

In Chapter 15 we will see how issuers can hedge the interest rate at which they have to pay. We will see that they can hedge the benchmark interest rate and/or hedge the spread.

Finally, the yield or cost of borrowing for an issuer of securities will depend on the benchmark interest rate plus a spread to reflect the risk premium that the market will demand. In addition, the issuance will have to pay various fees to issue a security. These fees include the payment to the SEC to register the securities, attorney fees, and fees to investment bankers. The latter fees are the underwriting spread—the difference between the price at which the securities are offered to the public by the investment banking firm and the price that the investment banking firm pays to the issuer to purchase the security—the gross spread. When an issuer evaluates its cost, it must recognize these issuance costs. In Chapter 11, we will see how the cost of funds is calculated for an issuer taking into account issuance costs. This measure is referred to as the "all in cost of funds."

SUMMARY

- The Federal Reserve System (the "Fed") is a network of banks that acts as the central banker for the United States.
- The money supply consists of cash and cash-like items. There are different definitions of what constitutes the money supply, depending on which cash-like items are included.
- The Fed affects the money supply by changing the reserve requirements, open market operations, and changing in the discount rate. Changes in the money supply affect interest rates and the availability of funds.
- Financial institutions provide various types of financial services. Financial intermediaries are a special group of financial institutions that obtain funds by issuing claims to market participants and use these funds to purchase financial assets. Intermediaries transform funds they

acquire into assets that are more attractive to the public by (1) providing maturity intermediation; (2) providing risk reduction via diversification at lower cost; (3) reducing the cost of contracting and information processing; or (4) providing a payments mechanism.

- Depository institutions (commercial banks, savings and loan associations, savings banks, and credit unions) accept various types of deposits. With the funds raised through deposits and other funding sources, they make loans to various entities and invest in securities.
- Investment bankers advise the issuer of a security on the terms of the offering, distributing the security to the public, and making a market for the security. Variations of the traditional underwriting of securities include the bought deal, distribution via an auction process, and private placements.
- Interest rates are determined by the base rate (rate on a Treasury security) plus a risk premium. The factors that affect the risk premium are (1) the perceived creditworthiness of the issuer, (2) term to maturity, (3) inclusion of options, (4) taxability of interest, and (5) expected liquidity of an issue.
- The term structure of interest rates shows the relationship between the yield on a bond and its maturity; the yield curve is the graph of the relationship between the yield on bonds of the same credit quality but different maturities.
- Valuable information for issuers and investors is provided in forward rates.
- Two major theories are offered to explain the observed shapes of the yield curve: the expectations theory (which includes the pure expectations theory, the liquidity theory, and the preferred habitat theory) and the market segmentation theory.

QUESTIONS

1. a. What is money?
   b. What is meant by M1? M2? M3?
2. What role does the Federal Reserve Bank play in determining the supply of money?
3. What is the Board of Governors and what types of decisions does this board make?
4. If the Federal Reserve uses the discount rate to encourage banks to lend funds, is this rate raised or lowered? Explain.
5. Distinguish between the transactions demand and the asset demand for money.

6. Describe the relation between the real rate of interest and the nominal rate of interest.
7. Suppose the nominal interest rate is 8% for a one-year security. If the expected real rate of interest next year is 5%, what is the inflation premium? What is the implied inflation rate?
8. Explain how a financial intermediary reduces the cost of contracting and information processing.
9. a. What are the three ways in which an investment banking firm may be involved in the issuance of a new security?
   b. What is meant by the underwriting function?
10. What is the difference between a firm commitment underwriting arrangement and a best efforts arrangement?
11. What is meant by a bought deal?
12. a. What is a preemptive right?
    b. What is a preemptive rights offering?
13. What is meant by the "base interest rate"?
14. a. Typically, how do market participants gauge the credit risk associated with a bond issue?
    b. What is the relationship between credit risk and the risk premium?
15. How does the taxability of interest affect the yield offered on a bond?
16. Suppose that the 1-year spot rate is 4.1% and the 2-year spot rate is 4.6%. What is the 1-year forward rate one year from now?
17. a. Comment on the following statement: "Forward rates are good predictors of future interest rates."
    b. Why can forward rates be viewed as hedgeable rates?
18. Consider the following yields to maturity:

Years to Maturity	Yield to Maturity
1	3.0%
2	3.5%
3	3.9%
4	4.4%
5	4.8%
6	5.2%

    a. Graph the yield to maturity against the time to maturity.
    b. Is this yield curve consistent with any of the yield curve theories? Explain.
19. A corporate treasurer is considering borrowing funds for 10 years. How can the corporate treasurer use forward rates in determining whether or not to borrow today or postpone borrowing?

20. Why are “biased” expectation theories of the term structure of interest rates biased?
21. Comment on the following: “There is no theory of the term structure of interest rates that would explain a yield curve in which interest rates increase with maturity for the first two years, decline with maturity until year 5, and then increase with maturity after year 5.”

CHAPTER 4

Introduction to Derivatives

Firms are exposed to several risks in the ordinary course of operations and when borrowing funds. For some risks, management can obtain protection from an insurance company. For example, management can insure a plant against destruction by fire by obtaining a fire insurance policy from a property and casualty insurance company. There are capital market products available to management to protect against certain risks that are not insurable by an insurance company. Such risks include risks associated with a rise in the price of commodity purchased as an input, a decline in a commodity price of a product the firm sells, a rise in the cost of borrowing funds, and an adverse exchange rate movement. The instruments that can be used to provide such protection are called ***derivative instruments***, so named because they derive their value from whatever the contract is based on. These instruments include futures contracts, forward contracts, option contracts, swap agreements, and cap and floor agreements.

There has been public concern about the use of derivative instruments by firms. This concern arises from major losses resulting from positions in derivative instruments.[1] However, an investigation of the reason for major losses would show that the losses were not due to derivatives per se, but the improper use of them by management that was either ignorant about the risks associated with using derivative instruments or management that sought to use them in a speculative manner rather than a means for managing risk. Another term for speculative purposes is trading purposes.

In this chapter we will discuss the basic features of each type of derivative instrument.

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[1] Well-publicized losses in the 1990s include Procter & Gamble's losses related to foreign exchange derivatives, Gibson Greetings losses related to interest rates swaps, and Pier 1 Imports losses due to the trading of bond futures and options.

FUTURES CONTRACTS AND FORWARD CONTRACTS

A ***futures contract*** is an agreement that requires a party to the agreement either to buy or sell something at a designated future date at a predetermined price. The something that the two parties agree will be bought and sold is referred to as the ***underlying for the contract*** or simply the ***underlying***. The basic economic function of futures markets is to provide an opportunity for market participants to hedge against the risk of adverse price movements.

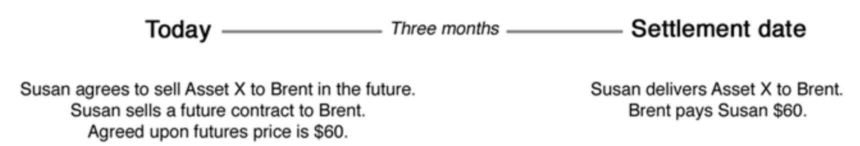
Futures contracts are products created by exchanges. Futures contracts involving traditional agricultural commodities (such as grain and livestock), imported foodstuffs (such as coffee, cocoa, and sugar), or industrial commodities are traded. Collectively, such futures contracts are known as ***commodity futures***. Futures contracts based on a financial instrument or a financial index are known as ***financial futures***. Financial futures can be classified as (1) stock index futures, (2) interest rate futures, and (3) currency futures.

Mechanics of Futures Trading

A futures contract is an agreement between a buyer (seller) and an established exchange or its clearinghouse in which the buyer (seller) agrees to take (make) delivery of the underlying at a specified price at the end of a designated period of time. The price at which the parties agree to transact in the future is called the ***futures price***. The designated date at which the parties must transact is called the ***settlement date*** or ***delivery date***.

To illustrate, suppose there is a futures contract traded on an exchange where the underlying is Asset X, and the settlement date is three months from now. Assume further that Brent buys this futures contract, and Susan sells this futures contract, and the price at which they agree to transact in the future is $60. Then $60 is the futures price. At the settlement date, Susan will deliver Asset X to Brent; Brent will give Susan $60, the futures price. This transaction is illustrated in Exhibit 4.1.

EXHIBIT 4.1 **Illustration of a Futures Contract for the Delivery of Asset X in Three Months at a Futures Price of $60**



Liquidating a Position

Most futures contracts have settlement dates in the months of March, June, September, or December. This means that at a predetermined time in the contract settlement month the contract stops trading, and a price is determined by the exchange for settlement of the contract. A party to a futures contract has two choices on liquidation of the position. First, the position can be liquidated prior to the settlement date. For this purpose, the party must take an offsetting position in the same contract. For the buyer of a futures contract, this means selling the same number of identical futures contracts; for the seller of a futures contract, this means buying the same number of identical futures contracts.

The alternative is to wait until the settlement date. At that time the party purchasing a futures contract accepts delivery of the underlying (financial instrument, currency, or commodity) at the agreed-upon price; the party that sells a futures contract liquidates the position by delivering the underlying at the agreed-upon price. For some futures contracts, settlement is made in cash only. Such contracts are referred to as ***cash-settlement contracts***.

The Role of the Clearinghouse

Associated with every futures exchange is a ***clearinghouse***, which performs several functions. One of these functions is guaranteeing that the two parties to the transaction will perform. To see the importance of this function, consider potential problems in the futures transaction described earlier from the perspective of the two parties—Brent, the buyer and Susan, the seller. Each must be concerned with the other's ability to fulfill the obligation at the settlement date. Suppose that at the settlement date the price of Asset X in the cash market is $40. Susan can buy Asset X for $40 and deliver it to Brent who, in turn, must pay her $60 (the futures price agreed upon when the two parties entered into the agreement). If Brent does not have the capacity to pay $60 or refuses to pay, however, Susan has lost the opportunity to realize a profit of $20. Suppose, instead, that the price of Asset X in the cash market is $90 at the settlement date. In this case, Brent is ready and willing to accept delivery of Asset X and pay the agreed-upon price of $60. If Susan does not have the ability or refuses to deliver Asset X, Brent has lost the opportunity to realize a profit of $30.

The clearinghouse exists to meet this problem. When a party takes a position in the futures market, the clearinghouse takes the opposite position and agrees to satisfy the terms set forth in the contract. Because of the clearinghouse, the parties to a futures contract need not worry about the financial strength and integrity of the other party that has taken the opposite side of the contract (called the ***counterparty***). After initial execution of an order, the relationship between the two parties ends. The clearinghouse interposes itself as the buyer for every sale and the seller for every purchase. Thus either party is free to liquidate a position without involving the counterparty in the original futures contract, and without worry that the counterparty may default. This is the reason why we define a futures contract as an agreement between a party and a clearinghouse associated with an exchange.

Besides its guarantee function, the clearinghouse makes it simple for parties to a futures contract to unwind their positions prior to the settlement date. Suppose that Brent wants to get out of his futures position. He will not have to seek out Susan and work out an agreement with her to terminate the original agreement. Instead, Brent can unwind his position by selling an identical futures contract. As far as the clearinghouse is concerned, its records will show that Brent has bought and sold an identical futures contract. At the settlement date, Susan will not deliver Asset X to Brent but will be instructed by the clearinghouse to deliver to someone who bought and still has an open futures position. In the same way, if Susan wants to unwind her position prior to the settlement date, she can buy an identical futures contract.

Margin Requirements

When a position is first taken in a futures contract, the investor must deposit a minimum dollar amount per contract as specified by the exchange. This amount is called the ***initial margin*** and is required as deposit for the contract. The initial margin may be in the form of an interest-bearing security. As the price of the futures contract fluctuates, the value of the investor's equity in the position changes. At the end of each trading day, the exchange determines the settlement price for the futures contract. This price is used to determine the investor's position, so that any gain or loss from the position is reflected in the investor's equity account. In financial markets, the process of recording the market value of a position is referred to as ***marking a position to market*** or simply ***marking to market***.

Maintenance margin is the minimum level (specified by the exchange) by which an investor's equity position may fall as a result of an unfavorable price movement before the investor is required to deposit additional margin. The additional margin deposited is called ***variation margin***, and it is an amount necessary to bring the equity in the account back to its initial margin level. Unlike initial margin, variation margin must be in cash, not interest-bearing instruments. Any excess margin in the account may be withdrawn by the investor. If a party to a futures contract who is required to deposit variation margin fails to do so within 24 hours, the futures position is closed out.[2]

Futures versus Forward Contracts

A ***forward contract***, just like a futures contract, is an agreement for the future delivery of the underlying at a specified price at the end of a designated period of time. Futures contracts are standardized agreements as to the delivery date (or month) and quality of the deliverable, and are traded on organized exchanges. A forward contract differs in that it is usually nonstandardized (that is, the terms of each contract are negotiated individually between buyer and seller), there is no clearinghouse, and secondary markets are often nonexistent or extremely thin. Unlike a futures contract, which is an exchange-traded product, a forward contract is an over-the-counter instrument.

Although both futures and forward contracts set forth terms of delivery, futures contracts are not intended to be settled by delivery. In fact, generally less than 2% of outstanding contracts are settled by delivery. Forward contracts, in contrast, are intended for delivery.

Futures contracts are marked to market at the end of each trading day. Consequently, futures contracts are subject to interim cash flows as additional margin may be required in the case of adverse price movements, or as cash is withdrawn in the case of favorable price movements. A forward contract may or may not be marked to market, depending on the wishes of the two parties. For a forward contract that is not marked to market, there are no interim cash flow effects because no additional margin is required.

Finally, the parties in a forward contract are exposed to credit risk because either party may default on the obligation. The risk that the counterparty may default is referred to as ***counterparty risk***. Counterparty risk is minimal in the case of futures contracts because the clearinghouse associated with the exchange guarantees the other side of the transaction.

Other than these differences, most of what we say about futures contracts applies equally to forward contracts.

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[2] Although there are initial and maintenance margin requirements for buying securities on margin, the concept of margin differs for securities and futures. When securities are acquired on margin, the difference between the price of the security and the initial margin is borrowed from the broker. The security purchased serves as collateral for the loan, and the investor pays interest. For futures contracts, the initial margin, in effect, serves as "good faith" money, an indication that the investor will satisfy the obligation of the contract. Normally no money is borrowed by the investor.

Risk and Return Characteristics of Futures Contracts

When an investor takes a position in the market by buying a futures contract, the investor is said to be in a ***long position*** or to be ***long futures***. If, instead, the investor's opening position is the sale of a futures contract, the investor is said to be in a ***short position*** or ***short futures***.

The buyer of a futures contract will realize a profit if the futures price increases; the seller of a futures contract will realize a profit if the futures price decreases. For example, suppose one month after Brent and Susan take their positions in the futures contract, the futures price of Asset X increases to $80. Brent, the buyer of the futures contract, could then sell the futures contract and realize a profit of $20 ($80 minus the futures price of $60). Effectively, at the settlement date he has agreed to buy Asset X for $60 but can sell Asset X for $80. Susan, the seller of the futures contract, will realize a loss of $20.

If the futures price falls to $45 and Susan buys the contract, she realizes a profit of $15 because she agreed to sell Asset X for $60 and now can buy it for $45. Brent would realize a loss of $15. Thus, if the futures price decreases, the buyer of the futures contract realizes a loss while the seller of a futures contract realizes a profit.

How Futures are Used to Manage Risk

We will use an example to illustrate how futures contracts can be used to manage risk. Consider a producer of crude oil and a company that uses crude oil in the operations of its business. The concern of the crude oil producer is that the price of crude oil will decline, thereby forcing it to sell crude oil at a lower price. The concern of the user of crude oil is that the price of crude oil will increase, resulting in a rise in its production costs.

Consider first the producer of crude oil. Suppose management expects that the crude oil will be available in two months and that management can sell a crude oil futures contract to deliver crude oil two months from now for $19 per barrel. The number of barrels that is expected to be sold will determine how many barrels of crude oil the firm will seek to deliver. By selling futures, management has locked in a price of $19 per barrel two months from now. Consequently, even if the price of crude oil two months from now is, say, $17 per barrel, management will receive $19 per barrel. If, instead, the price of crude oil two months from now is $20 per barrel, management has given up the opportunity to benefit from a higher price since it has agreed to accept $19 per barrel.

Now let's look at the user of crude oil. By buying a crude oil futures contract that settles in two months, management can assure that the price at which it must purchase crude oil will be no higher than $19 per barrel. So, if crude oil increases to $20 per barrel, management only needs to pay $19 per barrel. In contrast, if the price of crude oil two months from now decreases to $17 per barrel, management gave up the opportunity to benefit from a lower cost for crude oil.

In the same way that these two firms are able to use a futures contract to lock in the future price of crude oil, a firm can use futures contracts to lock in a foreign exchange rate or an interest rate.

OPTIONS

An option is a contract in which the ***writer of the option*** grants the ***buyer of the option*** the right, but not the obligation, to purchase from or sell to the writer an asset at a specified price within a specified period of time (or at a specified date). The writer, also referred to as the ***seller***, grants this right to the buyer in exchange for a certain sum of money, which is called the ***option price*** or ***option premium***. The price at which the asset may be bought or sold is called the ***exercise price*** or ***strike price***. The date after which an option is void is called the ***expiration date***. As with a futures contract, the asset that the buyer has the right to buy and the seller is obligated to sell is referred to as the underlying.

When an option grants the buyer the right to purchase the underlying from the writer (seller), it is referred to as a ***call option***, or ***call***. When the option buyer has the right to sell the underlying to the writer, the option is called a ***put option***, or ***put***.

An option is also categorized according to when the option buyer may exercise the option. There are options that may be exercised at any time up to and including the expiration date. Such an option is referred to as an ***American option***. There are options that may be exercised only at the expiration date. An option with this feature is called a ***European option***. An option that can be exercised before the expiration date but only on specified dates is called a ***Bermuda option***.

To illustrate the characteristics of an option contract, suppose that Patricia buys a call option for $2 (the option price) with the following terms:

1. The underlying is one unit of Asset X.
2. The exercise price is $60.
3. The expiration date is three months from now, and the option can be exercised any time up to and including the expiration date (that is, it is an American option).

At any time up to and including the expiration date, Patricia can decide to buy from the writer of this option one unit of Asset X, for which she will pay a price of $60. If it is not beneficial for Patricia to exercise the option, she will not. Whether Patricia exercises the option or not, the $2 she paid for the option will be kept by the option writer. If Patricia buys a put option rather than a call option, then she would be able to sell Asset X to the option writer for a price of $60.

The maximum amount that an option buyer can lose is the option price. The maximum profit that the option writer can realize is the option price. The option buyer has substantial upside return potential, while the option writer has substantial downside risk. The risk/reward relationship for option positions will be discussed later.

There are no margin requirements for the buyer of an option once the option price has been paid in full. Because the option price is the maximum amount that the investor can lose, no matter how adverse the price movement of the underlying, there is no need for margin. Because the writer of an option has agreed to accept all of the risk (and none of the reward) of the position in the underlying, the writer is generally required to put up the option price received as margin. In addition, as price changes occur that adversely affect the writer's position, the writer is required to deposit additional margin (with some exceptions) as the position is marked to market.

Exchange-Traded versus Over-the-Counter Options

Options, like other financial instruments, may be traded either on an organized exchange or in the over-the-counter (OTC) market. The advantages of an exchange-traded option are as follows. First, the exercise price and expiration date of the contract are standardized. Second, as in the case of futures contracts, the direct link between buyer and seller is severed after the order is executed because of the interchangeability of exchange-traded options. The clearinghouse associated with the exchange where the option trades performs the same function in the options market that it does in the futures market. Finally, the transactions costs are lower for exchange-traded options than for OTC options.

The higher cost of an OTC option reflects the cost of customizing the option for the many situations where a corporation seeking to use an option to manage risk needs to have a tailor-made option because the standardized exchange-traded option does not satisfy its objectives. Some commercial and investment and banking firms act as principals as well as brokers in the OTC options market. OTC options are sometimes referred to as ***dealer options***. While an OTC option is less liquid than an exchange-traded option, this is typically not of concern to the user of such an option. Most corporations who use OTC options do so as part of a financing strategy or price protection against unfavorable changes in prices of its inputs or exchange rates.

Differences between Options and Futures Contracts

Notice that, unlike in a futures contract, one party to an option contract is not obligated to transact—specifically, the option buyer has the right but not the obligation to transact. The option writer does have the obligation to perform. In the case of a futures contract, both buyer and seller are obligated to perform. Of course, a futures buyer does not pay the seller to accept the obligation, while an option buyer pays the seller an option price.

Consequently, the risk/reward characteristics of the two contracts are also different. In the case of a futures contract, the buyer of the contract realizes a dollar-for-dollar gain when the price of the futures contract increases and suffers a dollar-for-dollar loss when the price of the futures contract drops. The opposite occurs for the seller of a futures contract. Because of this relationship, futures are referred to as having a "linear payoff."

Options do not provide this symmetric risk/reward relationship. The most that the buyer of an option can lose is the option price. While the buyer of an option retains all the potential benefits, the gain is always reduced by the amount of the option price. The maximum profit that the writer may realize is the option price; this is offset against substantial downside risk. Because of this characteristic, options are referred to as having a "nonlinear payoff."

The difference in the type of payoff between futures and options is extremely important because market participants can use futures to protect against symmetric risk and options to protect against asymmetric risk.

Risk and Return Characteristics of Options

Here we illustrate the risk and return characteristics of the four basic option positions—buying a call option, selling a call option, buying a put option, and selling a put option. The illustrations assume that each option position is held to the expiration date and not exercised early. Also, to simplify the illustrations, we ignore transactions costs.[3]

[3] In addition, the illustrations do not address the cost of financing the purchase of the option price or the opportunity cost of investing the option price. Specifically, the buyer of an option must pay the seller the option price at the time the option is purchased. Thus, the buyer must finance the purchase price of the option or, assuming the purchase price does not have to be borrowed, the buyer loses the income that can be earned by investing the amount of the option price until the option is sold or exercised. In contrast, assuming that the seller does not have to use the option price as margin for the short position or can use an interest-earning asset as security, the seller has the opportunity to earn income from the proceeds of the option sale.

Buying Call Options

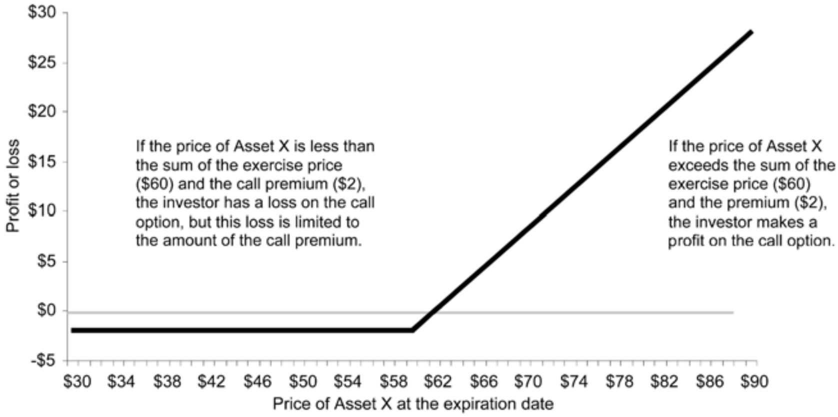
The purchase of a call option creates a position referred to as a ***long call position.*** To illustrate this position, assume that there is a call option on Asset X that expires in one month and has an exercise price of $60. The option price is $2. What is the profit or loss for the investor who purchases this call option and holds it to the expiration date?

The profit and loss from the strategy will depend on the price of Asset X at the expiration date. A number of outcomes are possible.

1. If the price of Asset X at the expiration date is less than $60 (the option price), then the investor will not exercise the option. It would be foolish to pay the option writer $60 when Asset X can be purchased in the market at a lower price. In this case, the option buyer loses the entire option price of $2. Notice, however, that this is the maximum loss that the option buyer will realize regardless of how low Asset X's price declines.
2. If Asset X's price is equal to $60 at the expiration date, there is again no economic value in exercising the option. As in the case where the price is less than $60, the buyer of the call option will lose the entire option price, $2.
3. If Asset X's price is more than $60 but less than $62 at the expiration date, the option buyer will exercise the option. By exercising, the option buyer can purchase Asset X for $60 (the exercise price) and sell it in the market for the higher price. Suppose, for example, that Asset X's price is $61 at the expiration date. The buyer of the call option will realize a $1 gain by exercising the option. Of course, the cost of purchasing the call option was $2, so $1 is lost on this position. By failing to exercise the option, the investor loses $2 instead of only $1.
4. If Asset X's price at the expiration date is equal to $62, the investor will exercise the option. In this case, the investor breaks even, realizing a gain of $2 that offsets the cost of the option, $2.
5. If Asset X's price at the expiration date is more than $62, the investor will exercise the option and realize a profit. For example, if the price is $70, exercising the option will generate a profit on Asset X of $10. Reducing this gain by the cost of the option ($2), the investor will realize a net profit from this position of $8.

Exhibit 4.2 shows in graph form the profit and loss for the buyer of the hypothetical call option. While the break-even point and the loss will depend on the option price and the exercise price, the profile shown in Exhibit 4.2 will hold for all buyers of call options. The shape indicates that the maximum loss is the option price and that there is substantial upside potential.

EXHIBIT 4.2 **Profits and Losses on the Exercise of a Call Option to Buy the Stock at $60. The investor Pays $2 for this Call Option.**



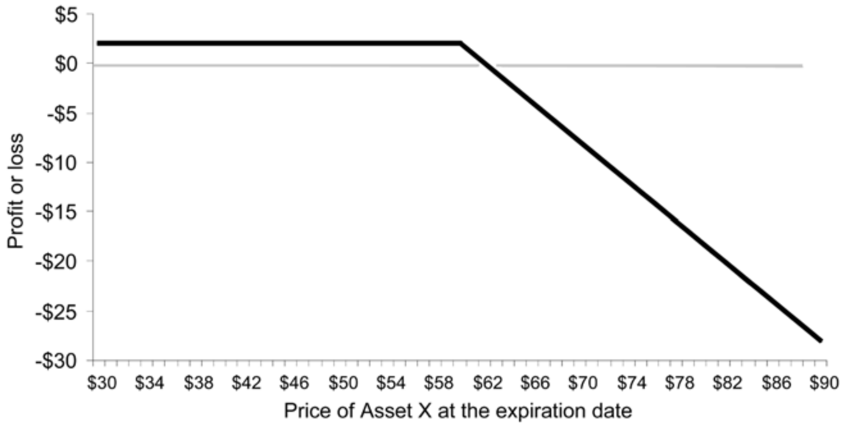
Writing (Selling) Call Options

The writer of a call option is said to be in a ***short call position***. To illustrate the option seller's (writer's) position, we use the same call option we used to illustrate buying a call option. The profit and loss profile of the short call position (that is, the position of the call option writer) is the mirror image of the profit and loss profile of the long call position (the position of the call option buyer). That is, the profit of the short call position for any given price for Asset X at the expiration date is the same as the loss of the long call position. Consequently, the maximum profit that the short call position can produce is the option price. The maximum loss is not limited because it is the highest price reached by Asset X on or before the expiration date, less the option price; this price can be indefinitely high. Exhibit 4.3 shows the profit/loss profile for a short call position.

Buying Put Options

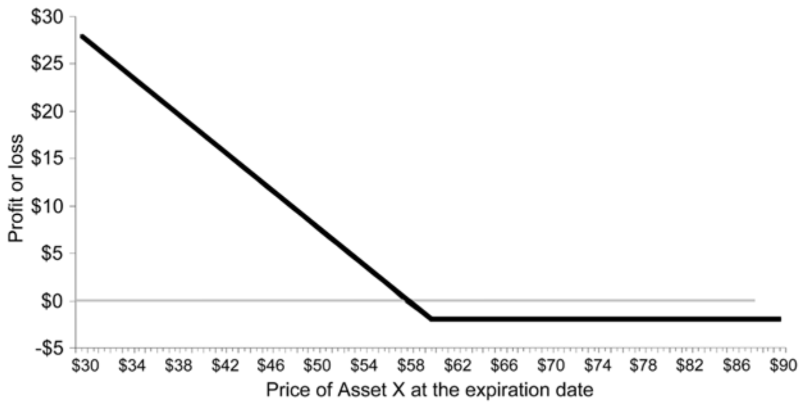
The buying of a put option creates a financial position referred to as a ***long put position***. To illustrate this position, we assume a hypothetical put option on one unit of Asset X with one month to maturity and an exercise price of $100. Assume the put option is selling for $3 and the price of Asset X at the expiration date is $60. The profit or loss for this position at the expiration date depends on the market price of Asset X. The possible outcomes are:

EXHIBIT 4.3 **Profits and Losses on the Writing of a Call Option that Allows the Call Option Buyer to Buy the Stock at $60. The Call Writer Receives $2 for this Option.**



1. If Asset X's price is greater than $60, the buyer of the put option will not exercise it because exercising would mean selling Asset X to the writer for a price that is less than the market price. A loss of $3 (the option price) will result in this case from buying the put option. Once again, the option price represents the maximum loss to which the buyer of the put option is exposed.
2. If the price of Asset X at expiration is equal to $60, the put will not be exercised, leaving the put buyer with a loss equal to the option price of $3.
3. Any price for Asset X that is less than $60 but greater than $57 will result in a loss; exercising the put option, however, limits the loss to less than the option price of $1. For example, suppose that the price is $59 at the expiration date. By exercising the option, the option buyer will realize a loss of $2. This is because the buyer of the put option can sell Asset X, purchased in the market for $59, to the writer for $60, realizing a gain of $1. Deducting the $3 cost of the option results in a loss of $2.
4. At a $57 price for Asset X at the expiration date, the put buyer will break even. The investor will realize a gain of $3 by selling Asset X to the writer of the option for $60, offsetting the cost of the option ($3).
5. If Asset X's price is below $57 at the expiration date, the long put position (the put buyer) will realize a profit. For example, suppose the price falls at expiration to $46. The long put position will produce a profit of $11: a gain of $14 for exercising the put option less the $3 option price.

EXHIBIT 4.4 **Profits and Losses on the Exercise of a Put Option to Sell the Stock at $60. The Investor Pays $2 for this Put Option.**



The profit and loss profile for the long put position is shown in graphical form in Exhibit 4.4. As with all long option positions, the loss is limited to the option price. The profit potential, however, is substantial: The theoretical maximum profit is generated if Asset X's price falls to zero. Contrast this profit potential with that of the buyer of a call option. The theoretical maximum profit for a call buyer cannot be determined beforehand because it depends on the highest price that can be reached by Asset X before or at the option expiration date.

Writing (Selling) Put Options

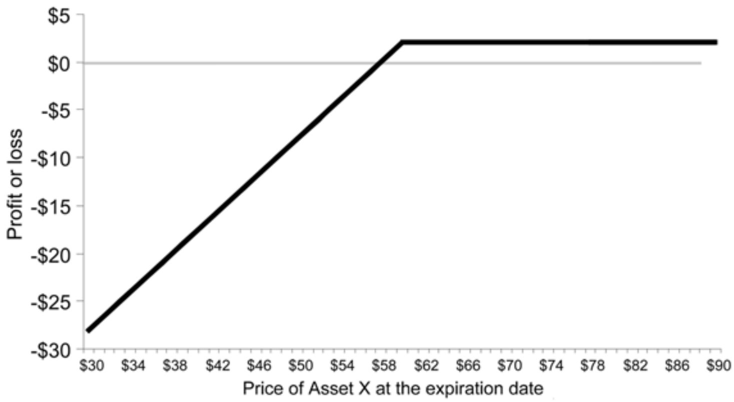
Writing a put option creates a position referred to as a ***short put position***. The profit and loss profile for a short put option is the mirror image of the long put option. The maximum profit from this position is the option price. The theoretical maximum loss can be substantial should the price of the underlying fall; at the extreme, if the price were to fall all the way to zero, the loss would be as large as the exercise price less the option price. Exhibit 4.5 graphically depicts this profit and loss profile.

To summarize, buying calls or selling puts allows the investor to gain if the price of the underlying rises. Selling calls and buying puts allows the investor to gain if the price of the underlying falls.

Basic Components of the Option Price

The option price is a reflection of the option's ***intrinsic value*** and any additional amount over its intrinsic value. The premium over intrinsic value is often referred to as the ***time premium***.

EXHIBIT 4.5 **Profits and Losses on the Writing of a Put Option that Allows the Put Option Buyer to Sell the Stock at $60. The Put Writer Receives $2 for this Option.**



Intrinsic Value of an Option

The ***intrinsic value*** of an option is the economic value of the option if it is exercised immediately, except that if there is no positive economic value that will result from exercising it immediately, then the intrinsic value is zero.

The intrinsic value of a call option is the difference between the current price of the underlying and the exercise price if positive; it is otherwise zero. For example, if the exercise price for a call option is $60 and the current asset price is $67, the intrinsic value is $7. That is, an option buyer exercising the option and simultaneously selling the underlying asset would realize $67 from the sale of the underlying, which would be covered by acquiring the underlying from the option writer for $60, thereby netting a $7 gain.

When an option has intrinsic value, it is said to be "in the money." When the exercise price of a call option exceeds the current price of the underlying, the call option is said to be "out of the money"—it has no intrinsic value. An option for which the exercise price is equal to the current price of the underlying is said to be "at the money." Both at-the-money and out-of-the-money options have an intrinsic value of zero because it is not profitable to exercise the option. Our call option with an exercise price of $60 would be: (1) in the money when the current price of the underlying is greater than $60, (2) out of the money when the current price of the underlying is less than $60, and (3) at the money when the current underlying price is equal to $60.

For a put option, the intrinsic value is equal to the amount by which the current price of the underlying is below the exercise price. For example, if the exercise price of a put option is $60 and the current price of the underlying is $52, the intrinsic value is $8. That is, the buyer of the put option who exercises the put option and simultaneously sells the underlying will net $8 by exercising. The asset will be sold to the writer for $60 and purchased in the market for $52. For our put option with an exercise price of $60, the option would be: (1) in the money when the price of the underlying is less than $60, (2) out of the money when the current price of the underlying exceeds the exercise price, (3) at the money when the exercise price is equal to the underlying's price.

Time Premium of an Option

The time premium of an option is the amount by which the option price exceeds its intrinsic value. The option buyer hopes that, at some time prior to expiration, changes in the market price of the underlying will increase the value of the rights conveyed by the option. For this prospect, the option buyer is willing to pay a premium above the intrinsic value. For example, if the price of a call option with an exercise price of $60 is $9 when the current price of the underlying is $65, the time premium of this option is $4 ($9 minus its intrinsic value of $5). Had the current price of the underlying been $50 instead of $65, then the time premium of this option would be the entire $9 because the option has no intrinsic value.

There are two ways in which an option buyer may realize the value of a position taken in the option. First is to exercise the option. The second is by selling the call option for $9. Selling the call is preferable because the exercise of an option will realize a gain of only $5—it will cause the immediate loss of any time premium. There are circumstances under which an option may be exercised prior to the expiration date; they depend on whether the total proceeds at the expiration date would be greater by holding the option or exercising and reinvesting any cash proceeds received until the expiration date.

How Options are Used for Managing Risk

We can use our illustration of the producer of crude oil and the user of crude oil to explain how buying options can be used. Suppose that there are options on crude oil. Management of the producer of crude oil wants to set a minimum price it will have to pay for crude oil two months from now. It does so by buying a put option on crude oil. The exercise price for the put option is the price that management can sell crude oil. Suppose the exercise price for a put option on crude oil that expires in two months is $19. Then if two months from now crude oil falls below $19, say to $17, then management will exercise the put option and sell the crude oil to the writer of the put option for $19. What is the effective minimum price that management will be selling crude oil? It is not the exercise price of $19. Rather, that price must be reduced by the cost of the put option (i.e., option price).

To appreciate the difference between a futures contract and an option, consider the scenario wherein two months from now the price of crude oil is $20 per barrel. In that case, management will not exercise the put option. Instead, it can sell the crude oil for $20 per barrel in the market to benefit from the higher price. The effective price it sold the crude oil for is $20 less the option price. So, with a put option management has set a minimum price for how much it will sell crude oil two months from now (exercise price less the option price) but has maintained the opportunity to benefit from a price that is higher than the exercise price. In contrast, with a futures contract on crude oil that has a futures price of $19 per barrel, management has fixed a price and cannot benefit from a higher price for crude oil two months from now.

Now let's consider the user of crude oil. Management wants to set a maximum price for crude oil two months from now. It can do so by buying a call option. For example, suppose that the exercise price for a call option that expires in two months is $19 per barrel. Then if the price of crude oil two months from now is higher than $19 per barrel, management will exercise the call option and buy crude oil for $19 per barrel. The effective maximum price it will buy crude oil for is the exercise price plus the price of the call option.

Again, let's see the difference between buying a call option and buying a futures contract. If the price of crude oil two months from now is $17 per barrel (a price that is less than the exercise price), management will not exercise the call option and, instead, buy crude oil in the market for $17. The effective purchase price is equal to $17 plus the option price. In contrast, with a futures contract to buy crude oil, management has locked in a futures price of $19 per barrel and has given up the opportunity to buy crude oil at a lower price.

SWAPS

A ***swap*** is an agreement whereby two parties (called counterparties) agree to exchange periodic payments. The dollar amount of the payments exchanged is based on some predetermined dollar principal, which is called the ***notional principal amount*** or simply ***notional amount***. The dollar amount each counterparty pays to the other is the agreed-upon periodic rate times the notional amount. The only dollars that are exchanged between the parties are the agreed-upon payments, not the notional amount.

A swap is an over-the-counter contract. Hence, the counterparties to a swap are exposed to counterparty risk.

The three types of swaps typically used by non-finance corporations are interest rate swaps, currency swaps, and commodity swaps. We illustrate these types of swaps below.

Interest Rate Swap

In an ***interest rate swap***, the counterparties swap payments in the same currency based on an interest rate. For example, one of the counterparties can pay a fixed interest rate and the other party a floating interest rate. The floating interest rate is commonly referred to as the ***reference rate***.

For example, suppose the counterparties to a swap agreement are Farm Equip Corporation (a manufacturing firm) and PNC Bank. The notional amount of this swap is $100 million and the term of the swap is five years. Every year for the next five years, Farm Equip Corporation. agrees to pay PNC Bank 9% per year, while PNC Bank agrees to pay Farm Equip Corporation the one-year London interbank offered rate (LIBOR). LIBOR is the reference rate. This means that every year, Farm Equip Corporation will pay $9 million (9% times $100 million) to PNC Bank. The amount PNC Bank will pay Farm Equip Corporation depends on LIBOR. For example, one-year LIBOR is 6%, PNC Bank will pay Farm Equip Corporation $6 million (6% times $100 million).

It is too early in this book to appreciate the motivation for the treasurer of Farm Equipment Corporation to use an interest rate swap. The motivation will be seen when we discuss financing techniques.

Currency Swaps

In a ***currency swap***, two parties agree to swap payments based on different currencies. To illustrate a currency swap, suppose two counterparties are the High Quality Electronics Corporation (a U.S. manufacturing firm) and Citibank. The notional amount is $100 million and its Swiss franc (SF) equivalent at the time the contract was entered into is SF 127 million. The swap term is eight years. Every year for the next eight years the U.S. manufacturing firm agrees to pay Citibank Swiss francs equal to 5% of the Swiss franc notional amount, or SF 6.35 million. In turn, Citibank agrees to pay High Quality Electronics 7% of the U.S. notional principal amount of $100 million, or $7 million.

Again, the motivation for the management of High Quality Electronics Corporation for using a currency swap is difficult to appreciate because we have not covered how a firm finances itself. Currency swaps are used by corporations to raise funds outside of their home currency and then swap the payments into their home currency. This allows a corporation to eliminate currency risk (i.e., unfavorable exchange rate or currency movements) when borrowing outside of its domestic currency.

Commodity Swaps

In a commodity swap, the exchange of payments by the counterparties is based on the value of a particular physical commodity. Physical commodities include precious metals, base metals, energy stores (such as natural gas or crude oil), and food (including pork bellies, wheat, and cattle). Most commodity swaps involve oil.

For example, suppose that the two counterparties to this swap agreement are Comfort Airlines Company, a commercial airline, and Prebon Energy (an energy broker). The notional amount of the contract is 1 million barrels of crude oil each year and the contract is for three years. The swap price is $19 per barrel. Each year for the next three years, Comfort Airlines Company agrees to buy 1 million barrels of crude oil for $19 per barrel. So, each year Comfort Airlines Company pays $19 million to Prebon Energy ($19 per barrel times 1 million barrels) and receives 1 million barrels of crude oil.

The motivation for Comfort Airlines of using the commodity swap is that it allows the company to lock-in a price for 1 million barrels of crude oil at $19 per barrel regardless of how high crude oil's price increases over the next three years.

Interpretation of a Swap

If we look carefully at a swap, we can see that it is not a new derivative instrument. Rather, it can be decomposed into a package of derivative instruments that we have already discussed. To see this, consider our first illustrative swap.

Every year for the next five years Farm Equip Corporation agrees to pay PNC Bank 9%, PNC Bank agrees to pay Farm Equip Corporation the reference rate, one-year LIBOR. Since the notional amount is $100 million, Farm Equip Corporation Manufacturing agrees to pay $9 million. Alternatively, we can rephrase this agreement as follows: Every year for the next five years, PNC Bank agrees to deliver something (one-year LIBOR) and to accept payment of $9 million. Looked at in this way, the counterparties are entering into multiple forward contracts: One party is agreeing to deliver something at some time in the future, and the other party is agreeing to accept delivery. The reason we say that there are multiple forward contracts is that the agreement calls for making the exchange each year for the next five years.

While a swap may be nothing more than a package of forward contracts, it is not a redundant contract for several reasons. First, in many markets where there are forward and futures contracts, the longest maturity does not extend out as far as that of a typical swap. Second, a swap is a more transactionally efficient instrument. By this we mean that in one transaction an entity can effectively establish a payoff equivalent to a package of forward contracts. The forward contracts would each have to be negotiated separately. Third, the liquidity of certain types of swaps has grown since the inception of swaps in 1981; some swaps now are more liquid than many forward contracts, particularly long-dated (i.e., long-term) forward contracts.

CAP AND FLOOR AGREEMENTS

There are agreements available in the financial market whereby one party, for a fee (premium), agrees to compensate the other if a designated reference is different from a predetermined level. The party that will receive payment if the designated reference differs from a predetermined level and pays a premium to enter into the agreement is called the buyer. The party that agrees to make the payment if the designated reference differs from a predetermined level is called the seller.

When the seller agrees to pay the buyer if the designated reference exceeds a predetermined level, the agreement is referred to as a ***cap***. The agreement is referred to as a ***floor*** when the seller agrees to pay the buyer if a designated reference falls below a predetermined level.

In a typical cap or floor, the designated reference is either an interest rate or commodity price. The predetermined level is called the ***exercise value***. As with a swap, a cap and a floor have a notional amount. Only the buyer of a cap or a floor is exposed to counterparty risk.

In general, the payment made by the seller of the cap to the buyer on a specific date is determined by the relationship between the designated reference and the exercise value. If the former is greater than the latter, then the seller pays the buyer an amount delivered as follows:

$$\text{Notional amount} \times [\text{Actual value of designated reference} - \text{Exercise value}]$$

If the designated reference is less than or equal to the exercise value, then the seller pays the buyer nothing.

For a floor, the payment made by the seller to the buyer on a specific date is determined as follows. If the designated reference is less than the exercise value, then the seller pays the buyer an amount delivered as follows:

$$\text{Notional amount} \times [\text{Exercise value} - \text{Actual value of designated reference}]$$

If the designated reference is greater than or equal to the exercise value, then the seller pays the buyer nothing.

The following example illustrates how a cap works. Suppose that the FPK Bookbinders Company enters into a five-year cap agreement with Fleet Bank with a notional amount of $50 million. The terms of the cap specify that if one-year LIBOR exceeds 8% on December 31 each year for the next five years, Fleet Bank (the seller of the cap) will pay FPK Bookbinders Company the difference between 8% (the exercise value) and LIBOR (the designated reference). The fee or premium FPK Bookbinders Company agrees to pay Fleet Bank each year is $200,000.

The payment made by Fleet Bank to FPK Bookbinders Company on December 31 for the next five years based on LIBOR on that date will be as follows. If one-year LIBOR is greater than 8%, then Fleet Bank pays $50 million × [Actual value of LIBOR – 8%]. If LIBOR is less than or equal to 8%, then Fleet Bank pays nothing.

So, for example, if LIBOR on December 31 of the first year of the cap is 10%, Fleet Bank pays FPK Bookbinders Company $1 million as shown below:

$$\$50 \text{ million} \times [10\% - 8\%] = \$1 \text{ million}$$

Interpretation of a Cap and Floor

In a cap or floor, the buyer pays a fee which represents the maximum amount that the buyer can lose and the maximum amount that the seller of the agreement can gain. The only party that is required to perform is the seller. The buyer of a cap benefits if the designated reference rises above the exercise value because the seller must compensate the buyer. The exercise value can be a reference interest rate or an exchange rate, for example. The buyer of a floor benefits if the designated reference falls below the exercise value because the seller must compensate the buyer.

In essence the payoff of these contracts is the same as that of an option. A call option buyer pays a fee and benefits if the value of the option's underlying (or equivalently, designated reference) is higher than the exercise price at the expiration date. A cap has a similar payoff. A put option buyer pays a fee and benefits if the value of the option's underlying (or equivalently, designated reference) is less than the exercise price at the expiration date. A floor has a similar payoff. An option seller is only entitled to the option price. The seller of a cap or floor is only entitled to the fee.

Motivation for a Cap or Floor

We can easily see the use of a cap or a floor. In a cap that involves an interest rate, a corporation seeking funds can use a swap to set a maximum interest rate for its borrowing cost. In a cap that involves the price of a commodity, the cap sets a maximum price for the commodity and is therefore used by a manufacturer to eliminate the price risk associated with buying that commodity. In a floor that involves a commodity, a manufacturer can use such a contract to protect against a decline in a product it sells.

SUMMARY

- The traditional purpose of derivative instruments is to provide an important opportunity to manage against the risk of adverse future price, exchange rate, or interest rate movements.
- Futures contracts are creations of exchanges, which require initial margin from parties. Each day positions are marked to market. Additional (variation) margin is required if the equity in the position falls below the maintenance margin. The clearinghouse guarantees that the parties to the futures contract will satisfy their obligations.
- A forward contract differs in several important ways from a futures contract. In contrast to a futures contract, the parties to a forward contract are exposed to the risk that the other party to the contract will fail to perform. The positions of the parties are not necessarily marked to market, so there are no interim cash flows associated with a forward contract. Finally, unwinding a position in a forward contract may be difficult.
- A buyer (seller) of a futures contract realizes a profit if the futures price increases (decreases). The buyer (seller) of a futures contract realizes a loss if the futures price decreases (increases).
- An option grants the buyer of the option the right either to buy from (in the case of a call option) or to sell to (in the case of a put option) the seller (writer) of the option the underlying at a stated price called the exercise (strike) price by a stated date called the expiration date.
- The price that the option buyer pays to the writer of the option is called the option price or option premium.
- An American option allows the option buyer to exercise the option at any time up to and including the expiration date; a European option may be exercised only at the expiration date.
- The buyer of an option cannot realize a loss greater than the option price, and has all the upside potential. By contrast, the maximum gain

that the writer (seller) of an option can realize is the option price; the writer is exposed to all the downside risk.

- The option price consists of two components: the intrinsic value and the time premium. The intrinsic value is the economic value of the option if it is exercised immediately (except that if there is no positive economic value that will result from exercising immediately, then the intrinsic value is zero). The time premium is the amount by which the option price exceeds the intrinsic value.
- In a swap, the counterparties agree to exchange periodic payments. The dollar amount of the payments exchanged is based on the notional principal amount.
- Swaps typically used by non-finance companies are interest rate swaps, currency swaps, and commodity swaps.
- A swap has the risk/return profile of a package of forward contracts.
- A cap is an agreement whereby the seller agrees to pay the buyer when a designated reference exceeds a predetermined level (the exercise value). A floor is an agreement whereby the seller agrees to pay the buyer when a designated reference is less than a predetermined level (the exercise value). The designated reference could be a specific interest rate or a commodity price.
- A cap is equivalent to a package of call options; a floor is equivalent to a package of put options.

QUESTIONS

1. The following appears in the 2000 10-K of International Business Machines:

   > "The company employs a number of strategies to manage these risks, including the use of derivative financial instruments. Derivatives involve the risk of non-performance by the counterparty."

   Explain what is meant in the last sentence of this quotation.
2. A manufacturer of furniture is concerned that the price of lumber will increase over the next three months. Explain how the manufacturer can protect against a rise in the price of lumber using lumber futures contracts.
3. The treasurer of a corporation wants to use futures contracts to protect against a decline in the price of one of the products it sells. Explain why the treasurer must be prepared to put up cash when using a futures contract.

4. The chief financial officer of the corporation you work for recently told you that he had a strong preference to use forward contracts rather than futures contracts to hedge: "You can get contracts tailor-made to suit your needs." Comment on the CFO's statement. What other factors influence the decision to use futures or forward contracts?
5. In discussing hedging instruments, you overhead the following statement: "Unlike a futures contract, a forward contract is not marked to market."
   a. Explain what is meant by "marked to market."
   b. Explain whether you agree or disagree with the above statement.
6. What is the difference between a put option and a call option?
7. What is the difference between an American option and a European option?
8. Why does an option writer need to post margin?
9. Identify two important ways in which an exchange-traded option differs from an over-the-counter option.
10. "There's no real difference between options and futures. Both are hedging tools, and both are derivative products. It's just that with options you have to pay an option price, while futures require no upfront payment except for a 'good faith' margin. I can't understand why anyone would use options." Do you agree with this statement?
11. a. What option strategy (position) can a treasurer take to protect against a rise in the cost of one of its inputs in the production process assuming that there is an option available?
    b. What option strategy (position) can a treasurer take to protect against a decline in the selling price of one of its products assuming that there is an option available?
12. How does the price of an option and the exercise price affect the minimum price that the underlying can be sold for or the maximum price that the underlying can be purchased for?
13. Suppose an investor bought both a call option and a put option on an asset. Both options have an exercise price of \$50 and both options have an option premium of \$5.
    a. Draw the profit-loss diagram for each option considered individually.
    b. Draw the profit-loss diagram for the strategy that involves buying both options.
    c. What is an investor with this combination of options hoping will happen to the price of the underlying asset?
14. a. Suppose that the price of the underlying is \$40 and that the option price is \$5. If the exercise price for a put option is \$50, what is the intrinsic value and the time premium for this option?

   b. Suppose that the price of the underlying is \$40 and that the option price is \$5. If the exercise price for a call option is \$50, what is the intrinsic value and the time premium for this option?

15. Burlingame Bank and the ABC Manufacturing Corp. enter into the following 7-year swap with a notional amount of \$75 million and the following terms: Every year for the next seven years, Burlingame Bank agrees to pay ABC Manufacturing 7% per year and receive from ABC Manufacturing LIBOR.
   a. What type of swap is this?
   b. In the first year payments are to be exchanged, suppose that LIBOR is 4%. What is the amount of the payment that the two parties must make to each other?
16. Explain why a swap is similar to a package of forward contracts.
17. Why would a corporate treasurer want to use a commodity swap to manage the price risk of a product it purchases?
18. What is the relationship between a cap and an option?

CHAPTER 5
Taxation

In assessing a company's current and future cash flows, the financial analyst requires information concerning a company's tax obligations. Unfortunately, the company's tax return is not publicly available, requiring the analyst to understand the basics of corporate taxation and to work with information disclosed in the financial statements.

The tax laws are changed almost constantly and are likely being changed as you read this chapter. Hence, no purpose would be served by covering all the details of present tax laws; they might be outdated as soon as you learn them. Instead, we discuss some of the principles behind the tax laws and in doing so provide an opportunity for you to learn some terminology, do some basic taxation calculations, and see how taxes affect a company's cash flows. We use the rates in the 2001 tax laws for demonstration purposes.

Following are the main kinds of taxes:

- ***Income taxes*** are taxes specifically levied on the basis of income.
- ***Employment taxes*** are also based on income, but specifically on wage and salary income. In the United States, employment taxes are paid by the employee and the employer, and they are designated specifically for social insurance programs (i.e., retirement and unemployment).
- ***Excise taxes*** are taxes on certain commodities, such as alcoholic beverages, tobacco products, telephone service, and gasoline. Excise taxes provide an easy way of raising revenue, and they can be imposed to discourage the use of specific products, such as tobacco.
- ***Import and export taxes*** (or tariffs) are taxes based on trade with other countries and are imposed to achieve specific economic goals in world trade.

In this chapter, we focus on income taxes and, specifically, U.S. federal corporate income taxes. However, any of the other types of taxes may have a strong influence on the cash flows of industries or firms. For example, excise taxes and import and export taxes will influence the demand for a firm's products and therefore the firm's cash flows.

THE U.S. TAX LAW

In the United States, the federal tax law is the product of all three branches of federal government. Congress passes the tax legislation that comprises the ***Internal Revenue Code*** (IRC). The ***Internal Revenue Service*** (IRS), a part of the Treasury Department, interprets these laws, adds the details, and implements them. The IRS does this by providing and processing tax forms, collecting tax payments, explaining the law in its regulations, and even providing decisions regarding the law (called rulings) in some situations. The courts are also called on to interpret the law through specific court cases, and there is now a well-developed case law related to the IRC. Together the Internal Revenue Code, IRS regulations, IRS rulings, and the case law make up federal tax law.

In forecasting future cash flows, the financial analyst needs to be aware that tax rates change frequently. The financial analyst cannot simply assume that the tax rate in existence today will be the same in five or ten years. Moreover, in comparing the after-tax performance of a firm over time, changes in tax rates must be considered.

U.S. FEDERAL TAX RATES

Exhibit 5.1 shows the 2001 U.S. federal income tax rate schedules for corporations. We can look at the schedule for a corporation to see how the income tax is computed. Each line of the schedule represents a layer of taxable income, sometimes called a "tax bracket"; the lower limit of each bracket is called its base. So the first line, for example, represents the taxable income layer with base $0 and maximum taxable income of $50,000.

Each line of the schedule also tells us the dollar amount of the tax on the base and the rate at which income above the base is taxed in that bracket. Suppose a corporation has taxable income of $12 million. Using the tax rate schedule, we see that the tax is 15% on the first $50,000, 25% on the next $25,000, 34% on the next $25,000, 39% on the next $235,000, 34% on the next $9,665,000, and 35% on the last $2,000,000, or:

EXHIBIT 5.1 **Federal Income Tax Rate Schedule for Corporations, 2001**

If taxable income is:

over …	but not over …	tax is …	of the amount over …
$0	$50,000	15%	$0
50,000	75,000	$7,500 + 25%	50,000
75,000	100,000	13,750 + 34%	75,000
100,000	335,000	22,250 + 39%	100,000
335,000	10,000,000	113,900 + 34%	335,000
10,000,000	15,000,000	3,400,000 + 35%	10,000,000
15,000,000	18,333,333	5,150,000 + 38%	15,000,000
18,333,333	—	35%	0

$$\begin{aligned}\text{Tax on } \$12{,}000{,}000 &= \$3{,}400{,}000 + 0.35(\$12{,}000{,}000 - 10{,}000{,}000)\\ &= \$3{,}400{,}000 + 700{,}000 = \$4{,}100{,}000\end{aligned}$$

The ***marginal tax rate*** is the rate at which the next dollar of income would be taxes. It is the rate that defines the tax bracket. For a corporation with income falling between $50,000 and $75,000, the marginal tax rate in 2001 is 25%; for a corporation with income between $10 million and $15 million, the marginal tax rate is 35%.

The ***average tax rate*** is the ratio of the tax paid on the taxable income. So, for example, the corporation with $12 million in taxable income paid an average tax rate of:

$$\text{Average tax rate on } \$12{,}000{,}000 = \frac{\$4{,}100{,}000}{\$12{,}000{,}000} = 0.3417 \text{ or } 34.17\%$$

Note that this average tax rate is lower than the marginal tax rate, 35%. This is true for all progressive taxes, such as the U.S. federal income tax. A ***progressive tax*** is one that levies a higher average tax rate on higher incomes.

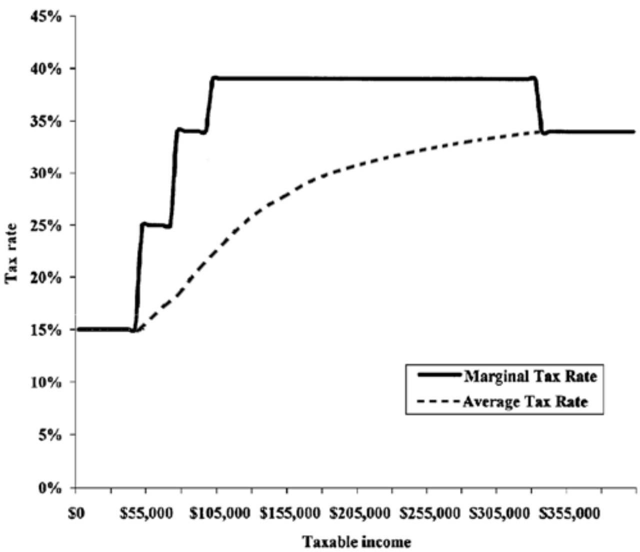
The marginal and average tax rates for a range of 2001 taxable corporate incomes are graphed in Exhibit 5.2. It is apparent from this diagram that as corporate incomes increase, the average rate approaches the marginal rate of tax. It is also apparent that the corporate income tax is progressive. Note, however, that the corporate tax rate schedule in 2001 has a "bubble" of 39% in the $100,000 to $335,000 bracket, where the rate is lower in the next higher tax bracket. These bubbles appear occasionally in the tax rate schedules mainly to increase revenues, and many times they disappear from the schedules after a year or two. They usually do not change the progressive nature of the tax.

It is important to realize that taxable income is taxed at the appropriate *marginal* rate for each bracket, and not at the average rate. Therefore, when a company's investment or financing decision is likely to affect taxable income—and hence cash flow—it will do so through the *marginal* income tax rate.

Corporate Taxable Income

There are many areas in which companies are permitted to use different methods of accounting for financial statements and tax purposes. These differences may arise from mandated methods of accounting for tax purposes (e.g., depreciation) or from the deductibility of certain expenses for the determination of income for one but not the other (e.g., goodwill). The result of these differences is a timing difference between reported tax expense and actual tax expense. If the reported tax expense exceeds the actual tax expense, the difference is a deferred tax liability and if the reported tax expense is less than the actual tax expense, the difference is a deferred tax asset. The deferred tax asset or liability therefore reflects a temporary difference between expense and revenue recognition for an accounting period.

EXHIBIT 5.2 **Marginal and Average Tax Rates from the 2001 Corporate Tax Rate Schedule**



There are many potential sources of differences between income per accounting statements and taxable income and, as we will see in Chapter 6, these differences can be useful in analyzing a company's financial situation. Examples of temporary sources of differences between accounting income and taxable income include the methods of recognition method for accruals and reserves, depreciation deductions, and tax loss carryovers. The sources of the deferred tax liability or asset are summarized in the company's income tax note to the financial statements.

Recognizing that some temporary differences persist over time, Statement of Financial Accounting Standard No. 109 requires that deferred taxes be adjusted for the expected permanent difference in tax liability per financial statements and tax books; this adjustment is referred to as a ***valuation allowance***. The result of including the valuation allowance is a deferred tax liability or asset that better reflects temporary differences between accounting and tax books.

In addition to these temporary differences, there are permanent differences between financial and tax income. For example, dividends received from other corporations are included fully in the financial income, but are permitted to be deducted in whole or part for tax purposes, which result in a permanent difference between taxable income and accounting income. Permanent differences such as this do not affect the deferred tax accounts.

The basic calculation of a corporation's taxable income is shown in Exhibit 5.3. To better understand how different features of the tax law affect a firm's taxes and, hence, its cash flows, we take a closer look at the dividends-received deduction, depreciation for tax purposes, and capital gains taxation.

The Dividends-Received Deduction

We have seen that corporate income distributed to shareholders (in the form of dividends) is taxed twice—first as corporate income and then as shareholders' income—and then if the shareholder is another corporation, that income could be taxed a third time. To minimize the chance of triple (or even quadruple) taxation of the same income, the tax laws permit a ***dividends-received deduction:*** A corporate recipient of dividends may deduct a portion of its dividend income from its taxable income.

With respect to dividend income received by corporations, the 1997 tax law, for example, specifies deductions of either 100%, 80%, or 70%, as follows:

- Deduction of 100% of dividends received may be deducted if the corporation is (1) a small business investment company operated under the Small Business Investment Act or (2) a member of an affiliated group of corporations, as in the case of a parent corporation and its wholly owned subsidiaries.

- Deduction of 80% if the dividends are received from a 20% or more owned corporation.
- Deduction of 70% if none of the conditions above applies.

Fox example, suppose the Inc. Corporation has operating income of \$2 million. Further suppose that it received \$1 million in dividends and \$500,000 in interest, and it paid \$800,000 dividends and \$600,000 interest. If the dividends received deduction is 70%, Inc's taxable income is:

Operating income	\$2,000,000
Plus: Included dividend income (30% of \$1,000,000)	300,000
Plus: Interest income	500,000
Less: Interest expense	(600,000)
Taxable income	\$2,200,000

EXHIBIT 5.3 **Corporate Taxable Income**

Gross receipts
– Cost of goods sold
Gross profit
+ Dividend income
+ Interest income
+ Gross rents
+ Gross royalties
+ Capital gain income
+ Other income
Total income
– Salaries and wages
– Repairs and maintenance
– Bad debt expense
– Rents
– Taxes and licenses
– Interest
– Charitable contributions
– Depreciation
– Depletion
– Advertising
– Pension, profit-sharing plans
– Employee benefit programs
– Other deductions
Total deductions

Taxable income

The remaining $700,000 in dividends received are not taxed—these dividends are excluded from income. The $800,000 dividends paid do not affect taxable income.

The dividends-received deduction either eliminates the tax on dividend income or reduces the effective tax rate considerably. Suppose a corporation has a marginal tax rate of 34% and the dividends it receives qualify for the 80% deduction. Then the effective tax rate on that dividend income is 20% of 34%, or 6.8%.

The dividends-received deduction increases the after-tax return of a corporation *investing* in another corporation's stock. Since corporate investors get a tax break on dividend income, they require a lower return on these securities, thus lowering the cost of capital for the corporation that *issues* these securities. The recent trend in tax law is to reduce the dividends-received deduction, increasing the multiple taxation effect and increasing the cost of capital to issuers of these securities.

While a corporation's dividend income receives special treatment, its interest income does not: Interest received by a corporation is taxed like any other income. Dividends and interest *paid* by a corporation receive different treatment as well: Interest paid by a corporation is fully deductible when computing taxable income, whereas dividends paid are not deductible. The taxation of dividend and interest received and paid enters into financial decision-making since it affects the cost of capital.

Depreciation for Tax Purposes

For accounting purposes, a firm can select a method of depreciation based on a number of factors, including the expected rate of physical depreciation of its asset and the effect on reported income. For federal income tax purposes, however, businesses are limited by law with regard to both the depreciation method and the period of time over which an asset can be depreciated.

The current depreciation tax laws are the result of an ongoing trend to create more uniformity in depreciation methods among business taxpayers while at the same time simplifying the calculations and allowing accelerated depreciation and shorter asset lives.

Currently, the two methods of depreciation available to business taxpayers are an accelerated method and straight-line. The accelerated method, referred to as the ***modified accelerated cost recovery system*** (MACRS), has four features:

1. The depreciation rate used each year is either 150% or 200% of the straight-line rate (referred to as 150 declining balance (DB) and 200 DB, respectively), depending on the type of property, applied against the undepreciated cost of the asset. Since the rate is applied against a

declining amount, this method is a declining balance method, but not the same declining balance method as that used for financial statement reporting purposes.
2. The salvage value of the asset is ignored; so the depreciable cost is the original cost and the asset's value is depreciated to zero.
3. The ***half-year convention*** is used on most property, that is, a half-year of depreciation is taken in the year the asset is acquired, no matter whether it is owned for one day or 365 days.
4. The depreciation method is switched to the straight-line method when straight-line depreciation produces a higher depreciation expense than the accelerated method.

Because the MACRS is an accelerated method, it yields greater depreciation expenses in earlier years and thus reduces taxable income and taxes relative to straight-line depreciation. However, the law allows some firms to use straight-line depreciation if they don't have the income necessary to take advantage of the faster depreciation of the MACRS. The use of MACRS for tax purposes and straight-line for financial reporting purposes, which is often the case for U.S. corporations, results in a difference in income for tax and financial accounting. This difference gives rise to deferred tax liabilities because actual taxes (calculated using MACRS depreciation) are less than reported taxes (calculated using straight-line depreciation) when MACRS results in a greater amount of depreciation, as in the earlier years of an asset's life.[1]

Congress (and the IRS) have taken much of the work out of calculating depreciation expenses for tax purposes.[2] Exhibit 5.4 outlines the depreciable life for each class of assets and the depreciation rates used for assets of each classified life. First, as panel *a* shows, depreciable lives are assigned to the various classes of assets that might be used by businesses. Second, tables are provided showing the depreciation rates to be applied to the asset's cost for each year in the life of each class of asset (panel *b* of Exhibit 5.4.)

Notice in panel *b* that each asset type is depreciated over its life plus one year: There are four years of depreciation for a 3-year asset, six years of depreciation for a 5-year asset, and so on. This is because of the half-year convention: Only half a year's depreciation is used up at the start, leaving half a year's depreciation to be taken after the asset's "life" is over for tax purposes.

[1] In Exhibit 5.4, for example, we see that most of the deferred tax liabilities arise from the depreciation of property, plant, and equipment.

[2] There are occasional changes to this system. For example, for a limited period of time (2001 through 2005), businesses are entitled to an additional 30% depreciation in the asset's first year for qualifying assets (Job Creation and Worker Assistance Act of 2002).

EXHIBIT 5.4 **Modified Accelerated Cost Recovery System (MACRS)**
Panel a. Classified Lives

3-year:	Tractor units, racehorses over two years old, special tools
5-year:	Cars, light and heavy trucks, computer and peripheral equipment, semi-conductor manufacturing equipment
7-year:	Office furniture and fixtures, railroad property
10-year:	Means of water transportation, fruit trees, nut trees
15-year:	Municipal wastewater plants, depreciable land improvements, pipelines, service station buildings
20-year:	Farm buildings, municipal sewers
27.5-year:	Residential rental property
31.5-year:	Non-residential real property, such as elevators and escalators
50-year:	Railroad grading and tunnel bores

Panel b. Depreciation Rates for 3-Year, 5-Year, 7-Year, 10-Year, 15-Year, and 20-Year Classified Assets

	Depreciation Rate (%)					
Year	3-Year	5-Year	7-Year	10-Year	15-Year	20-Year
1	33.33	20.00	14.29	10.00	5.00	3.750
2	44.45	32.00	24.49	18.00	9.50	7.219
3	14.81	19.20	17.49	14.40	8.55	6.677
4	7.41	11.52	12.49	11.52	7.70	6.177
5		11.521	8.93	9.22	6.93	5.713
6		5.76	8.92	7.37	6.23	5.285
7			8.93	6.55	5.90	4.888
8			4.46	6.55	5.90	4.522
9				6.56	5.91	4.462
10				6.55	5.90	4.461
11				3.28	5.91	4.462
12					5.90	4.461
13					5.91	4.462
14					5.90	4.461
15					5.91	4.462
16					2.95	4.461
17						4.462
18						4.461
19						4.462
20						4.461
21						2.231

These rates reflect depreciation calculated using the 200% (for 3-year, 5-year, 7-year, and 10-year property) or 150% (for 15-year and 20-year property) declining-balance method, with a switch to straight-line, using the half-year convention.

EXHIBIT 5.5 MACRS Depreciation of a $50,000 Truck, Using MACRS Rates

Year	Depreciation Rate	Depreciation Expense = Rate Times $50,000
2001	20.00%	$10,000
2002	32.00	16,000
2003	19.20	9,600
2004	11.52	5,760
2005	11.52	5,760
2006	5.76	2,880
Total	100%	$50,000

Let's see how depreciation expense is calculated using the information in Exhibit 5.5. Suppose a firm buys a truck for $50,000. According to panel *a* of the table, the truck has a 5-year class life. According to panel *b*, the first year's depreciation rate is 20%, the next year's is 32%, and so on. The results of applying these rates to the cost of the truck over six years are shown in Exhibit 5.5. The total cost is recouped over the six years, with most of the depreciation expense taken in the earlier years.

From the perspective of a financial analyst, understanding current and expected depreciation rates is important because depreciation, while not itself a cash flow, affects a corporation's taxes and hence its cash flows. If the corporation has a depreciation expense of $100 million and a 35% marginal tax rate, the benefit from the depreciation deduction for tax purposes is to reduce taxable income by $100 million and hence reduce taxes by 35% times $100 million, or $35 million. This reduction in taxes of $35 million is referred to as the ***depreciation tax-shield***. Over the life of an asset, the total dollar amount of depreciation is the same regardless of the rate of depreciation. However, changes in depreciation rates affect the ***timing*** of the depreciation tax-shield and hence their value today.

Capital Gains

We tend to use the term "capital gain" loosely to mean an increase in the value of an asset. however, in tax law a ***capital gain*** is specifically a realized gain that results when an asset is sold for more than was paid for it. Because tax rates are progressive, taxing capital gains in one lump in one year at higher rates seems unfair, so Congress has traditionally granted special treatment—via lower effective tax rates—to capital gains.

Special treatment for capital gains has come in either of two ways: (1) an exclusion of a portion of the gain or (2) a cap on the tax rate applied to capital gains. A cap is a "ceiling" on the tax rate applied to capital gains and is lower than the tax rate applied to other income. In 2001, for example, the tax rate cap on capital gains was 35% for corporations.

Suppose that in 2001 the Taxit Corporation has ordinary taxable income (that is, taxable income not including capital gains) of \$50,000 and a capital gain of \$10,000. Taxit's tax bracket is 25%, which is below 2001's corporate capital gains rate of 35%. So Taxit's tax on its \$60,000 of income is:

$$\text{Tax on } \$60{,}000 = \$7{,}500 + 0.25(\$60{,}000 - \$50{,}000) = \$10{,}000$$

Suppose instead that Taxit has ordinary income of \$200,000 and a capital gain of \$10,000. Taxit's tax is:

$$\begin{aligned} \text{Tax} &= \underbrace{\$22{,}250 + 0.39\ (\$200{,}000 - 100{,}000)}_{\text{tax on ordinary income}} + \underbrace{0.35\ (\$10{,}000)}_{\text{tax on capital gain income}} \\ &= \$61{,}250 + 3{,}500 \\ &= \$64{,}750 \end{aligned}$$

The other way of giving special treatment to capital gains for tax purposes is the exclusion. A capital gains exclusion excludes a portion, say 60%, of the capital gain from taxation and taxes the remainder at the ordinary tax rate. Consider Taxit Corporation's income. If 60% of its capital gain is excluded, only 60% of the \$10,000, or \$6,000 is included in taxable income.

After a while, Congress caught on that for a depreciable asset, a part of the gain was really the result of "over-depreciating" it (for tax purposes) during its life; that is, depreciation expenses taken over the life of the asset (which reduced taxable income and taxes) do not represent the actual amount the asset depreciated in value. So, Congress inserted provisions in the tax laws that require breaking the gain into two parts:

1. The ***recapture of depreciation***, the difference between (*a*) the lower of the original cost or the sales price and (*b*) the under-depreciated portion of the asset's cost for tax purposes.
2. The capital gain, which is the sales price less the original cost.

The recapture portion of the gain is taxed at ordinary rates, and the capital gain portion is given special treatment (so effectively, it is taxed at less than ordinary rates).

Suppose Reclaim Inc. bought a depreciable asset ten years ago for \$100,000, and its book value (cost less accumulated depreciation) for tax purposes is now \$30,000. This means that the firm has taken \$70,000 of depreciation expense over the ten years and has reduced its taxable income by that amount. If it now sells this asset for $125,000, it has a capital gain of $25,000:

Sales price	$125,000
Cost	100,000
Capital gain	$25,000

But Reclaim has also recaptured its entire depreciation expense by selling the asset. The tax code requires that recaptured depreciation be added to ordinary income and, thus, taxed at the ordinary income tax rate. Reclaim would have to pay ordinary income tax on the recaptured $70,000 of depreciation and capital gains tax on $25,000.

Original cost	$100,000
Less book value	30,000
Recapture (taxed as ordinary income)	$70,000

If only part of the asset's depreciation is recaptured when it is sold, only the recaptured part is taxed, and there would be no capital gain. The recaptured portion is the difference between sales price and book value. For example, if Reclaim sold the asset for $75,000, instead of $125,000, it would have:

Sales price	$75,000
Less book value	30,000
Recapture (taxed as ordinary income)	$45,000

As you can see, taxes, depreciation, and capital gains are all mutually related. Furthermore, they all become considerations in investment decisions, which almost always deal in some way with the purchase and sale of assets, and in cash flow, which is directly affected by tax law.

TAX CREDITS

From time to time Congress allows business credits against calculated income tax. One such credit that has popped up now and then in the tax law is the ***investment tax credit*** (ITC). The ITC may or may not exist at the time you read this chapter.

A tax credit is a direct reduction of the computed income tax. Suppose, for example, that the tax code allows an ITC of 10%. If a company invests $100 million, say, in new machinery, it is entitled to a direct reduction in taxes based on the cost of the machinery: 10% of $100 million, or $10 million.

The ITC is not the only tax credit that Congress has offered businesses. At one time or another there have been energy tax credits, targeted job credits, alcohol fuel credits, disabled access credits, and more.

Tax Credit versus Tax Deduction

Deductions and credits both reduce taxes payable. A ***deduction*** reduces taxable income and thus indirectly reduces the taxes paid. A tax ***credit*** is subtracted from the taxes paid, and thus directly reduces taxes.

For example, suppose a corporation has $100 million in taxable income, without considering a potential deduction or credit, and for simplicity assume a flat tax rate of 40%. Let's look at the effect on the firm's taxes of a $10 million deduction compared to a $10 million tax credit:

	No Deduction, No Credit	Deduction, No Credit	Credit, No Deduction
Taxable income without deduction	$100	$100	$100
Deduction	0	10	0
Taxable income	$100	$90	$100
Tax rate	0.40	0.40	0.40
Tax before credit	$40	$36	$40
Credit	0	0	10
Tax	$40	$36	$30

The benefit from the deduction is $4, whereas the benefit from the credit is $10.

NET OPERATING LOSS CARRYBACKS AND CARRYOVERS

A ***net operating loss*** is an excess of business deductions over business gross income in a tax year. The Internal Revenue Code allows businesses to carry back a net operating loss to preceding years and to carry forward the loss to future years to reduce the taxes payable for those years. The current tax law, for example, permits net operating losses of corporations to be carried back three years from the year of the loss and carried over (forward through time) 15 years.

Here's how carrybacks and carryovers work. Suppose that in the year 2000, a corporation has a $100 million net operating loss. To simplify the calculations, let's also assume that the corporate tax rate is a flat 40% of income. Suppose further that the corporation paid taxes on income as follows in the three years prior to 2000:

Year	Taxable Income	Taxes Paid
1997	$10,000,000	$4,000,000
1998	50,000.000	20,000,000
1999	50,000,000	20,000,000

To use the 2000 loss, the corporation begins by carrying it back to the earliest year (1997 in this example), applying it to reduce that year's taxable income, and then recomputing the tax. Any loss that is left over is carried to the next year, and so on. The 2000 tax law allows a 3-year carryback, so the computation would look like this:

Year	Taxable Income	Amount of Loss Applied	Refigured Taxable Income	Refigured Taxes	Refund
1997	$10,000,000	$10,000,000	$0	$0	$4,000,000
1998	50,000,000	50,000,000	0	0	20,000,000
1999	50,000,000	40,000,000	10,000,000	4,000,000	16,000,000
		$100,000,000			$40,000,000

The corporation would then apply for a $40 million refund of 1997-to-1999 taxes on the basis of its 2000 loss.

What if the corporation's loss was larger than the sum of the previous three years' taxable incomes? Then the corporation could carryover any unused portion of the loss to future tax years, applying it to taxable income in the tax returns for those years. As an example, assume the corporation's loss was $200 million, instead of $100 million. The corporation would be able to apply $110 million of that to taxable income and then could carryover the remaining loss of $90 million. The corporation would apply as much as possible to its 2001 taxable income, carryover any remainder to 2002, and so on, until either the loss was exhausted or the time limit prescribed in the IRC—currently 15 years—was reached.

STATE AND LOCAL TAXES

In addition to the federal income tax, individuals and corporations may also be assessed state and local income taxes. State and local tax structures are, for the most part, dependent upon the federal tax system. With some exceptions and an occasional adjustment to taxable income, state and local taxes are levied as a percentage of the federal income.

EXHIBIT 5.6 KPMG International's Corporate Tax Rate Survey, January 2002

Country	Corporate Tax Rate January 1, 2001 (%)	Corporate Tax Rate January 1, 2002 (%)
Argentina	35	35
Australia	34	30
Brazil	34	34
Canada	42.1	38.6
Czech Republic	31	31
France	35.33	35.33
Germany	38.36	38.36
Italy	40.25	40.25
Japan	42	42
Switzerland	24.7	24.5
United Kingdom	30	30
United States	40	40

Source: http://www.tax.kpmg.net

State and local taxes can be significant, with rates ranging from 1% to 12%, depending on the locality in which the corporation conducts its business. For example, in fiscal year 2001, the Walt Disney Company paid federal taxes—with a top rate of 35%—and state taxes—with an effective rate of 7.5%—together an effective marginal tax rate of 42.5%.[3]

TAXATION OUTSIDE OF THE UNITED STATES

The basic corporate income tax imposed by central governments is a fixed percentage or an increasing percentage of the statutorily determined corporate income. Countries typically tax resident corporations on worldwide income regardless of whether the income is repatriated. Nonresident corporations, that is, corporations whose corporate seat and place of management are outside the country, are typically subject only to corporate taxes derived from within the country.

The rate varies significantly from country to country. The range of corporate tax rates is shown in Exhibit 5.6. These tax rates, which are

[3] *The Walt Disney Company 2001 Annual Report,* p. 69. Because state taxes are deductible for federal income tax purposes, the state tax rate reflects this benefit and, hence, is lower than the statutory state corporate tax rate.

from KPMG International's *Corporate Tax Rate Survey* for January 2002, are estimates of the corporate tax burden, considering both national and local tax rates. Several countries impose no tax or minimal tax rates. These countries are referred to as ***tax havens***.

The basic tax rates shown in Exhibit 5.6 may be misleading for several reasons. These reasons are given in Chapter 26.

Consequently, when an analyst computes the average tax rate for a U.S. firm with significant operations overseas, this rate can vary greatly by the allocation of its activities throughout the world.

SUMMARY

- Tax rates change often and the financial analyst needs to consider the changing tax environment in making evaluation of a firm's future cash flows.
- The dividend income that a corporation receives from another corporation is effectively taxed at a lower rate than other income because of the dividends received deduction. Interest income of a corporation does not receive special treatment.
- Dividends paid by a corporation are not deductible in arriving at taxable income. Interest paid by a corporation is deductible for tax purposes.
- Depreciation for tax purposes is prescribed by the tax code. The method of depreciation for tax purposes may differ from the method used for financial statement accounting purposes.
- Special tax provisions for capital gains effectively reduce the tax paid on these gains. However, tax provisions regarding how much of a gain on a sale of an asset is given special treatment requires breaking out the gain into two components: recaptured depreciation (depreciation taken in the past but not really reflective of the asset's decline in value) and capital gain (the appreciation in the asset's value).
- Net operating loss carryovers effectively smooth out the taxes of a business in those cases where taxable income varies significantly from year to year.
- Tax rates vary by country and a comparison of basic tax rates among countries is complicated.

QUESTIONS

1. Distinguish between an average tax rate and a marginal tax rate. Under what circumstance are the two the same?

2. Using the tax rate schedule provided in Exhibit 5.1, what is the amount of tax, the marginal tax rate, and the average tax rate for a corporation with the following taxable income: a. $35,000 b. $120,000 c. $300,000 d. $1,000,000 e. $2,000,000
3. What is the role of the dividends-received deduction?
4. The PARENT Corporation received $3 million in dividends from the SUB Corporation. PARENT'S income is taxed at a flat rate of 40%. How much tax must PARENT Corporation pay on these dividends if the relationship between the two companies for purposes of the dividends-received deduction is:
   a. SUB and PARENT have no affiliation.
   b. SUB is 10%-owned by PARENT.
   c. SUB is wholly owned by PARENT.
5. DIV Corporation received $5 million in dividends and had $10 million in other taxable income. DIV's income is taxed at a flat rate of 30%.
   a. What is DIV's tax bill if there is no dividends-received deduction?
   b. What is DIV's tax bill if a 60% dividends-received deduction is allowed?
   c. What is DIV's tax bill if a 70% dividends-received deduction is allowed?
   d. What is DIV's tax bill if a 80% dividends-received deduction is allowed?
6. The NOL Company had a loss of $1 million for 2001. The firm had income and paid taxes in the four prior years of:

Year	Taxable Income	Taxes Paid (30% of Taxable Income)
1997	$2,000,000	$600,000
1998	500,000	150,000
1999	300,000	90,000
2000	100,000	30,000

   Suppose the tax law allows losses to be carried back three years and forward 15 years.
   a. How much of a refund of prior taxes can NOL receive?
   b. How much of the loss, if any, can be carried forward to future years?
7. The Loser Corporation had a loss of $200,000 in 2000. The firm had income and paid taxes in the three prior years of:

Year	Taxable Income	Taxes Paid (40% of Taxable Income)
1997	$100,000	$40,000
1998	200,000	80,000
1999	100,000	40,000

Suppose the tax law allows losses to be carried back three years and forward 15 years.

a. How much of a refund of prior taxes can Loser receive?

b. How much of the loss, if any, can be carried forward to future years?

8. The Mayberry Company purchased equipment for $100,000. Assume that this equipment qualifies as a five-year asset under the MACRS.

   a. What is the depreciation expense for tax purposes for each year the equipment is depreciated?

   b. If Mayberry's marginal tax rate is 40%, what is the depreciation tax shield for each year?

9. The USA Company purchased equipment for $1 million. Assume this equipment qualifies as a seven-year asset under the MACRS. What is the depreciation expense for tax purposes for each year the equipment is depreciated?

10. In 2000, NI Corporation had sales of $1 million, cost of goods sold of $600,000, and depreciation of $100,000. The corporation received $200,000 in dividends, paid $100,000 in dividends, and bought equipment for $300,000. Its tax rate was 30%, and the dividends-received deduction was 80%.

    a. What was the taxable income of NI Corporation?

    b. How much must NI pay in taxes?

11. In 1999, TI Corporation had sales of $2 million, cost of goods sold of $1 million, and depreciation of $500,000. The firm received $300,000 in dividends and $100,000 in interest income, and paid $150,000 in dividends and $200,000 in interest. It bought equipment for $300,000. The firm's tax rate was 30%, and the dividends-received deduction was 70%.

    a. What was the taxable income of TI Corporation?

    b. How much must TI pay in taxes?

CHAPTER 6

Financial Statements

Financial statements are summaries of the operating, financing, and investment activities of a business. Financial statements should provide information useful to both investors and creditors in making credit, investment, and other business decisions. And this usefulness means that investors and creditors can use these statements to predict, compare, and evaluate the amount, timing, and uncertainty of potential cash flows. In other words, financial statements provide the information needed to assess a company's future earnings and therefore the cash flows expected to result from those earnings. In this chapter, we discuss the four basic financial statements: the balance sheet, the income statement, the statement of cash flows, and the statement of shareholders' equity. The analysis of financial statements is provided in Part Six of this book.

ACCOUNTING PRINCIPLES AND ASSUMPTIONS

The accounting data in financial statements are prepared by the firm's management according to a set of standards, referred to as ***generally accepted accounting principles*** (GAAP).

The financial statements of a company whose stock is publicly traded must, by law, be audited at least annually by independent public accountants (i.e., accountants who are not employees of the firm). In such an audit, the accountants examine the financial statements and the data from which these statements are prepared and attest—through the published auditor's opinion—that these statements have been prepared according to GAAP. The auditor's opinion focuses on whether the statements conform to GAAP and that there is adequate disclosure of any material change in accounting principles.

The financial statements are created using several assumptions that affect how we use and interpret the financial data:

- *Transactions are recorded at historical cost*. Therefore, the values shown in the statements are not market or replacement values, but rather reflect the original cost (adjusted for depreciation, in the case of depreciable assets).
- *The appropriate unit of measurement is the dollar.* While this seems logical, the effects of inflation, combined with the practice of recording values at historical cost, may cause problems in using and interpreting these values.
- *The statements are recorded for predefined periods of time*. Generally, statements are produced to cover a chosen fiscal year or quarter, with the income statement and the statement of cash flows spanning a period's time and the balance sheet and statement of shareholders' equity as of the end of the specified period. But because the end of the fiscal year is generally chosen to coincide with the low point of activity in the firm's operating cycle, the annual balance sheet and statement of shareholders' equity may not be representative of values for the year.
- *Statements are prepared using accrual accounting and the matching principle*. Most businesses use accrual accounting, where income and revenues are matched in timing such that income is recorded in the period in which it is earned and expenses are reported in the period in which they are incurred to generate revenues. The result of the use of accrual accounting is that reported income does not necessarily coincide with cash flows. Because the financial analyst is concerned ultimately with cash flows, he or she often must understand how reported income relates to a company's cash flows.
- *It is assumed that the business will continue as a going concern*. The assumption that the business enterprise will continue indefinitely justifies the appropriateness of using historical costs instead of current market values because these assets are expected to be used up over time instead of sold.
- *Full disclosure requires providing information beyond the financial statements*. The requirement that there be full disclosure means that, in addition to the accounting numbers for such accounting items as revenues, expenses, and assets, narrative and additional numerical disclosures are provided in notes accompanying the financial statements. An analysis of financial statements is therefore not complete without this additional information.
- *Statements are prepared assuming conservatism*. In cases in which more than one interpretation of an event is possible, statements are prepared using the most conservative interpretation.

EXHIBIT 6.1 **Fictitious Corporation Balance Sheets for Years Ending December 31, in Thousands**

	2003	2002
ASSETS		
Cash	$400	$200
Marketable securities	200	0
Accounts receivable	600	800
Inventories	1,800	1,000
Total current assets	$3,000	$2,000
Gross plant and equipment	$11,000	$10,000
Accumulated depreciation	(4,000)	(3,000)
Net plant and equipment	7,000	7,000
Intangible assets	1,000	1,000
Total assets	$11,000	$10,000
LIABILITIES AND SHAREHOLDERS' EQUITY		
Accounts payable	$500	$400
Other current liabilities	500	200
Long-term debt	4,000	5,000
Total liabilities	$5,000	$5,600
Common stock, $1 par value;		
Authorized 2,000,000 shares		
Issued 1,500,000 and 1,200,000 shares	1,500	1,200
Additional paid-in capital	1,500	800
Retained earnings	3,000	2,400
Total shareholders' equity	6,000	4,400
Total liabilities and shareholders' equity	$11,000	$10,000

The financial statements and the auditors' findings are published in the firm's annual and quarterly reports sent to shareholders and the 10K and 10Q filings with the Securities and Exchange Commission (SEC). Also included in the reports, among other items, is a discussion by management, providing an overview of company events. The annual reports are much more detailed and disclose more financial information than the quarterly reports.

THE BALANCE SHEET

The ***balance sheet*** is a summary of the assets, liabilities, and equity of a business at a particular point in time—usually the end of the firm's fiscal year (see Exhibit 6.1). The balance sheet is also known as the ***statement***

of financial condition or the ***statement of financial position***. The values shown for the different accounts on the balance sheet are not purported to reflect current market values; rather, they reflect historical costs.

Assets are the resources of the business enterprise, such as plant and equipment, that are used to generate future benefits. If a company owns plant and equipment that will be used to produce goods for sale in the future, the company can expect these assets (the plant and equipment) to generate cash inflows in the future.

Liabilities are obligations of the business. They represent commitments to creditors in the form of future cash outflows. When a firm borrows, say, by issuing a long-term bond, it becomes obligated to pay interest and principal on this bond as promised.

Equity, also called ***shareholders' equity*** or ***stockholders' equity***, reflects ownership. The equity of a firm represents the part of its value that is not owed to creditors and therefore is left over for the owners. In the most basic accounting terms, equity is the difference between what the firm owns—its assets—and what it owes its creditors—its liabilities.

The balance sheets for Fictitious Corporation, shown in Exhibit 6.1, provide an example. At the end of the 1999 accounting year, the firm has $11 million in assets, financed by $5 million in liabilities and $6 million in equity.

ASSETS

There are two major categories of assets: current assets and noncurrent assets, where noncurrent assets include plant assets, intangibles, and investments. Assets that do not fit neatly into these categories may be recorded as either ***other assets***, ***deferred charges***, or ***other noncurrent assets***.

Current Assets

Current assets (also referred to as ***circulating capital*** and ***working assets***) are assets that could reasonably be converted into cash within one operating cycle or one year, whichever takes longer. An operating cycle begins when the firm invests cash in the raw materials used to produce its goods or services and ends with the collection of cash for the sale of those same goods or services. For example, if Fictitious manufactures and sells candy products, its operating cycle begins when it purchases the raw materials for the products (e.g., sugar) and ends when it receives cash for selling the candy to retailers. Because the operating cycle of most businesses is less than one year, we tend to think of current assets as those assets that can be converted into cash in one year.

Current assets consist of cash, marketable securities, accounts receivable, and inventories. ***Cash*** comprises both currency—bills and coins—and assets that are immediately transformable into cash, such as deposits in bank accounts. ***Marketable securities*** are securities that can be readily sold when cash is needed. Every company needs to have a certain amount of cash to fulfill immediate needs, and any cash in excess of immediate needs is usually invested temporarily in marketable securities. Investments in marketable securities are simply viewed as a short-term place to store funds; marketable securities do not include those investments in other companies' stock that are intended to be long term. Some financial reports combine cash and marketable securities into one account referred to as ***cash and cash equivalents*** or ***cash and marketable securities***.

Accounts receivable are amounts due from customers who have purchased the firm's goods or services but haven't yet paid for them. To encourage sales, many firms allow their customers to "buy now and pay later," perhaps at the end of the month or within 30 days of the sale. Accounts receivable therefore represents money that the firm expects to collect soon. Because not all accounts are ultimately collected, the gross amount of accounts receivable is adjusted by an estimate of the uncollectible accounts, the ***allowance for doubtful accounts***, resulting in a ***net accounts receivable*** figure.

Inventories represent the total value of the firm's raw materials, work-in-process, and finished (but as yet unsold) goods. A manufacturer of toy trucks would likely have plastic and steel on hand as raw materials, work-in-process consisting of truck parts and partly completed trucks, and finished goods consisting of trucks packaged and ready for shipping. There are three basic methods of accounting for inventory, including:

- FIFO (first in, first out), which assumes that the first items purchased are the first items sold,
- LIFO (last in, first out), which assumes that the last items purchased are the first items sold, and
- Average cost, which assumes that the cost of items sold is the average of the cost of all items purchased.

The choice of inventory accounting method is significant because it affects values recorded on both the balance sheet and the income statement, as well as tax payments and cash flows.

EXHIBIT 6.2 Current Assets for Wal-Mart Stores, Procter & Gamble, and Walt Disney Company (2001)

	Wal-Mart Stores		Procter & Gamble		Walt Disney Company	
Current Asset	in Millions	% of Total	in Millions	% of Total	in Millions	% of Total
Cash and cash equivalents	$2,161	7.6%	$2,306	21.2%	$618	8.8%
Accounts receivable	2,000	7.1	3,328	30.5	3,965	56.4
Inventory	22,614	80.1	3,384	31.1	671	9.5
Other	1,471	5.2	1,871	17.2	1,775	25.3
Total	$28,246	100.0%	$10,889	100.0%	$7,029	100.0%

Source: The 2001 10-K reports for the respective companies.

Another current asset account that a company may have is prepaid expenses. ***Prepaid expenses*** are amounts that have been paid but not as yet consumed. A common example is the case of a company paying insurance premiums for an extended period of time (say, a year), but for which only a portion (say, three months) is applicable to the insurance coverage for the current fiscal year; the remaining insurance that is prepaid as of the end of the year is considered an asset. Prepaid expenses may be reported as part of ***other current liabilitie***s.

Companies' investment in current assets depends, in large part, on the industry in which they operate. Consider the breakdown of current assets by asset type for three companies for 2001 reported in Exhibit 6.2. Retailers, such as Wal-Mart, have a relatively large investment in inventory, whereas manufacturing firms, such as consumer product manufacturer Procter & Gamble, have substantial investments in both accounts receivable and inventory. Companies that generate a large portion of their operating revenues from patents, copyrights, and other such intangibles (e.g., film libraries) tend to have a relatively larger investment in accounts receivable, as we can see with the Walt Disney Company.

Noncurrent Assets

Noncurrent assets are assets that are not current assets; that is, it is not expected that noncurrent assets can be converted into cash within an operating cycle. Noncurrent assets include physical assets, such as plant and equipment, and nonphysical assets, such as intangibles.

Plant assets are the physical assets, such as the equipment, machinery, and buildings, that are used in the operation of the business. We describe a firm's current investment in plant assets by using three values: gross plant assets, accumulated depreciation, and net plant assets. ***Gross plant and equipment***, or gross plant assets, is the sum of the original costs of all equipment, buildings, and machinery the firm uses to produce its goods and services. ***Depreciation***, as you will see in the next chapter, is a charge that accounts for the using up of an asset over the length of an accounting period; it is a means for allocating the asset's cost over its useful life. ***Accumulated depreciation*** is the sum of all the depreciation charges taken so far for all the company's assets. ***Net plant and equipment***, or ***net plant assets***, is the difference between gross plant assets and accumulated depreciation. The net plant and equipment amount is hence the value of the assets—historical cost less any depreciation—according to the accounting books and is therefore often referred to as the ***book value*** of the assets.

Intangible assets are the current value of nonphysical assets that represent long-term investments of the company. Such intangible assets include patents, copyrights, and goodwill. The cost of some intangible assets is amortized ("spread out") over the life of the asset. ***Amortization*** is akin to depreciation: The asset's cost is allocated over the life of the asset; the reported value is the original cost of the asset, less whatever has been amortized. The number of years over which an intangible asset is amortized depends on the particular asset and its perceived useful life. For example, a ***patent*** is the exclusive right to produce and sell a particular, uniquely defined good and has a legal life of 17 years, though the useful life of a patent—the period in which it adds value to the company—may be much less than 17 years. Therefore the company may choose to amortize a patent's cost over a period less than 17 years. As another example, a ***copyright*** is the exclusive right to publish and sell a literary, artistic, or musical composition, and is granted for 50 years beyond the author's life, though its useful life in terms of generating income for the company may be much less than 50 years. More challenging is determining the appropriate amortization period for goodwill. ***Goodwill*** was created when one company buys another company at a price that exceeds the acquired company's fair market value of its assets.

A company may have additional noncurrent assets, depending on their particular circumstances. A company may have a noncurrent asset referred to as ***investments***, which are assets that are purchased with the intention of holding them for a long term, but which do not generate revenue or are not used to manufacture a product. Examples of investments include equity securities of another company and real estate that is held for speculative purposes. Other noncurrent assets include ***long-term prepaid expenses***, arising from prepayment for which a benefit is received over an extended period of time, and ***deferred tax assets***, arising from timing differences between reported income and tax income, whereby reported income exceeds taxable income.

Long-term investment in securities of other companies may be recorded at cost or market value, depending on the type of investment; investments held to maturity are recorded at cost, whereas investments held as trading securities or available for sale are recorded at market value. Whether the unrealized gains or losses affect earnings on the income statement depend on whether the securities are deemed trading securities or available for sale.[1]

LIABILITIES

Liabilities, a firm's obligations to its creditors, are made up of current liabilities, long-term liabilities, and deferred taxes.

Current Liabilities

Current liabilities are obligations that must be paid within one operating cycle or one year, whichever is longer. Current liabilities include:

- ***Accounts payable***, which are obligations to pay suppliers. They arise from goods and services that have been purchased but not yet paid.
- ***Accrued expenses***, which are obligations such as wages and salaries payable to the employees of the business, rent, and insurance.
- ***Current portion of long-term debt*** or the ***current portion of capital leases***. Any portion of long-term indebtedness—obligations extending beyond one year—due within the year.
- Short-term loans from a bank or notes payable within a year.

The reliance on short-term liabilities and the type of current liabilities depends, in part, on the industry in which the firm operates. Consider the breakdown of current liabilities for three firms for 2001 reported in Exhibit 6.3. These three companies differ quite a bit in their use of the different types of current liabilities, with Wal-Mart more reliant on accounts payable (i.e., trade credit) and Disney using accounts payable the least.

[1] If the securities are considered trading securities, the unrealized gains or losses (that is, the changes in market value each period) affect earnings; if the securities are available-for-sale, unrealized gains or losses bypass the income statement and only affect shareholders' equity on the balance sheet.

EXHIBIT 6.3 **Current Liabilities for Wal-Mart Stores, Procter & Gamble, and the Walt Disney Company**

Current Liability	Wal-Mart Stores in Millions	Wal-Mart Stores % of Total	Procter & Gamble in Millions	Procter & Gamble % of Total	Walt Disney Company in Millions	Walt Disney Company % of Total
Accounts payable	$24,134	88.5%	$7,613	77.3%	$4,603	74.0%
Short-term and current long-term debt	3,148	11.5	2,233	22.7	829	13.3
Other current liabilities	0	0.0	0	0.0	787	12.7
Total	$27,282	100.0%	$9,846	100.0%	$6,219	100.0%

Source: The 2001 10-K reports for the respective companies.

Long-Term Liabilities

Long-term liabilities are obligations that must be paid over a period beyond one year. They include notes, bonds, capital lease obligations, and pension obligations. Notes and bonds both represent loans on which the borrower promises to pay interest periodically and to repay the principal amount of the loan.

A ***lease*** obligates the lessee—the one leasing and using the leased asset—to pay specified rental payments for a period of time. Whether the lease obligation is recorded as a liability or is expensed as lease payments made depends on whether the lease is a capital lease or an operating lease. The rules for classifying a lease as a capital lease or an operating lease and the accounting treatment of each are explained in Chapter 27.

A company's pension and post-retirement benefit obligations may give rise to long-term liabilities. The pension benefits are commitments by the company to pay specific retirement benefits, whereas post-retirement benefits include any other retirement benefit besides pensions, such as health care. Basically, if the fair value of the pension plan's assets exceeds the ***projected benefit obligation*** (the estimated present value of projected pension costs), the difference is recorded as a long-term asset. If, on the other hand, the plan's assets are less than the projected benefit obligation, the difference is recorded as a long-term liability. In a similar manner, the company may have an asset or a liability corresponding to post-retirement benefits.

Deferred Taxes

Along with long-term liabilities, the analyst may encounter another account, deferred taxes. ***Deferred taxes*** are taxes that will have to be paid to the federal and state governments based on accounting income, but are not due yet. Deferred taxes arise when different methods of accounting are used for financial statements and for tax purposes. These differences are temporary and are the result of different timing of revenue or expense recognition for financial statement reporting and tax purposes. The deferred tax liability arises when the actual tax liability is less than the tax liability shown for financial reporting purposes (meaning that the firm will be paying the difference in the future), whereas the deferred tax asset, mentioned earlier, arises when the actual tax liability is greater than the tax liability shown for reporting purposes.

EQUITY

Equity is the owner's interest in the company. For a corporation, ownership is represented by common stock and preferred stock. Shareholders' equity is also referred to as the ***book value of equity***, since this is the value of equity according to the records in the accounting books.

The value of the ownership interest of preferred stock is represented in financial statements as its ***par value***, which is also the dollar value on which dividends are figured. For example, if you own a share of preferred stock that has a \$100 par value and a 9% dividend rate, you receive \$9 in dividends each year. Further, your ownership share of the company is \$100. Preferred shareholders' equity is the product of the number of preferred shares outstanding and the par value of the stock; it is shown that way on the balance sheet.

The remainder of the equity belongs to the common shareholders. It consists of three parts: ***common stock outstanding*** (listed at par or at stated value), additional paid-in capital, and retained earnings. The par value of common stock is an arbitrary figure; it has no relation to market value or to dividends paid on common stock. Some stock has no par value, but may have an arbitrary value, or ***stated value***, per share. Nonetheless, the total par value or stated value of all outstanding common shares is usually entitled "capital stock" or "common stock." Then, to inject reality into the equity part of the balance sheet, an entry called ***additional paid-in capital*** is added; this is the amount received by the corporation for its common stock in excess of the par or stated value. If a firm sold 10,000 shares of \$1 par value common stock at \$40 a share, its equity accounts would show:

Common stock, \$1 par value	\$10,000
Additional paid-in capital	\$390,000

In Exhibit 6.1, Fictitious' common stock represents the stock's par value and the amount paid in excess of par value is recorded as additional paid-in capital. Some corporations eliminate this arbitrary division of accounts and instead report the entire amount paid for the common stock as ***capital stock*** or ***common stock***.

If some of the stock is bought back by the firm, the amount it pays for its own stock is recorded as ***treasury stock***. Because these shares are not owned by shareholders, common shareholders' equity is reduced by the cost of the treasury stock.

There are actually four different labels that can be applied to the number of shares of a corporation on a balance sheet:

- The number of shares ***authorized*** by the shareholders.
- The number of shares ***issued*** and sold by the corporation, which can be less than the number of shares authorized.
- The number of shares currently ***outstanding***, which can be less than the number of shares issued if the corporation has bought back (repurchased) some of its issued stock.
- The number of shares of ***treasury stock***, which is stock that the company has repurchased.

The outstanding stock is reported in the stock accounts, and adjustments must be made for any treasury stock. In the case of Fictitious Corporation, shown in Exhibit 6.1, in 2003 there were 2 million authorized shares, 1.5 million issued shares, and (since there was no treasury stock) 1.5 million shares outstanding.

As another example, consider the numbers of shares for the Walt Disney Company. For the fiscal year ended September 30, 2001, Disney had 3.6 billion shares authorized, 2.1 billion shares issued, and 2.019 billion shares outstanding.

The number of shares actually issued by Disney is well below the number of shares the company is authorized to issue; as of the end of 2001, Disney could issue 3.6 – 2.1 = 1.5 billion common shares without shareholder approval.

The bulk of the equity interest in a company is in its retained earnings. ***Retained earnings*** is the accumulated net income of the company, less any dividends that have not been paid, over the life of the corporation. Retained earnings are *not* strictly cash and any correspondence to cash is coincidental. Any cash generated by the firm that has not been paid out in dividends has been reinvested in the firm's assets—to finance accounts receivable, inventories, equipment, and so forth.

The book value of equity—the sum total of retained earnings, common stock, and (if applicable) preferred stock—represents the equity interest of the corporation's owners, stated in terms of historical costs. However, historical costs often bear little resemblance to the value of equity stated in terms of market values. Consider the case of several companies at the end of their fiscal 2001 year:

Company	Book Value of Equity in Millions	Market Value of Equity (in Millions)
Amazon	$(1,440)	$4,038
Coca-Cola	11,366	117,224
General Electric	54,824	397,830
Microsoft	47,289	385,659
Sprint	11,714	17,847
Wal-Mart Stores	35,102	267,091

Source: Book values of equity are drawn from the company's 2001 annual report. Market value, as of the end of the company's fiscal year-end, is from Yahoo! Finance, biz.yahoo.com.

In most cases, the market value of equity exceeds the company's book value by a wide margin, as typified by Coca-Cola, General Electric, and Wal-Mart. Yet there are cases in which the book value of equity is negative (as illustrated by Amazon), which bears no relation to the company's market value of equity. And in other, relatively uncommon cases such as Sprint, the market value of equity is close to the company's book value.

THE INCOME STATEMENT

An ***income statement*** is a summary of the revenues and expenses of a business over a period of time, usually either one month, three months, or one year. This statement is also referred to as the ***profit and loss statement***. It shows the results of the firm's operating and financing decisions during that time. Income statements for Fictitious Corporation are presented in Exhibit 6.4.

The operating decisions of the company—those that apply to production and marketing—generate ***sales*** or ***revenues*** and incur the ***cost of goods sold*** (also referred to as the ***cost of sales*** or the ***cost of products sold***). The difference between sales and cost of goods sold is ***gross profit***. Operating decisions also result in administrative and general expenses, such as advertising fees and office salaries. Deducting these expenses from gross profit leaves ***operating profit***, which is also referred to as ***earnings before interest and taxes*** (EBIT), ***operating income***, or ***operating earnings***. Operating decisions take the firm from sales to EBIT on the income statement. Exhibit 6.4 shows that Fictitious Corporation generated sales of $10 million in 2003, which produced an operating profit of $2 million.

The results of financing decisions are reflected in the remainder of the income statement. When interest expenses and taxes, which are both influenced by financing decisions, are subtracted from EBIT, the result is net income. Net income is, in a sense, the amount available to owners of the firm. If the firm has preferred stock, the preferred stock dividends are deducted from net income to arrive at ***earnings available to common shareholders***. If the firm does not have preferred stock (as is the case with Fictitious and most nonfictitious corporations), net income is equivalent to earnings available for common shareholders. The board of directors may then distribute all or part of this as common stock dividends, retaining the remainder to help finance the firm. As shown in Exhibit 6.4, Fictitious Corporation had a net income for 2003 of $1.2 million. Of this, $600,000 was paid to common shareholders. The remaining $600,000 went into retained earnings.

EXHIBIT 6.4 **Fictitious Corporation Income Statements for Years Ending December 31 (in Thousands)**

	2003	2002
Sales	$10,000	$9,000
Cost of goods sold	(6,500)	(6,000)
Gross profit	$3,500	$3,000
Lease expense	(1,000)	(1,000)
Administrative expense	(500)	(500)
Earnings before interest and taxes (EBIT)	$2,000	$2,000
Interest	(400)	(500)
Earnings before taxes	$1,600	$1,500
Taxes	(400)	(500)
Net income	$1,200	$1,000
Preferred dividends	(100)	(100)
Earnings available to common shareholders	$1,100	$900
Common dividends	(500)	(400)
Retained earnings	$600	$500

The entry "retained earnings" in the balance sheet is the record of accumulated earnings, less any dividends paid since the inception of the corporation. The entry "retained earnings" in the income statement is the amount of earnings retained (that is, not paid out) during that period. As you can see, Fictitious retained $600,000 of its 2003 earnings (Exhibit 6.4), increasing its retained earnings from $2.4 million in 2002 to $3 million in 2003 (Exhibit 6.1).

Companies must report comprehensive income prominently within their financial statements. ***Comprehensive income*** is a net income amount that includes all revenues, expenses, gains, and losses items and is based on the idea that all results of the firm—whether operating or nonoperating—should be reflected in the earnings of the company. This is referred to as the ***all-inclusive income concept***. The all-inclusive income concept requires that these items be recognized in the financial statements as part of comprehensive income.

It is important to note that net income does not represent the actual cash flow from operations and financing. Rather, it is a summary of operating performance measured over a given time period, using specific accounting procedures. Depending on these accounting procedures, net income may or may not correspond to cash flow.

THE STATEMENT OF CASH FLOWS

The ***statement of cash flows*** is a summary over a period of time of a firm's cash flows from operating, investment, and financing activities. The statement of cash flows for Fictitious is shown in Exhibit 6.5.

The firm's statement of cash flows lists separately its operating cash flows, investing cash flows, and financing cash flows. By analyzing these individual flows, current and potential owners and creditors can examine such aspects of the business as:

- The source of financing for business operations, whether through internally generated funds or external sources of funds.
- The ability of the company to meet debt obligations (interest and principal payments).
- The ability of the company to finance expansion through operating cash flow.
- The ability of the company to pay dividends to shareholders.
- The flexibility the business has in financing its operations.

A firm that generates cash flows only by selling off its assets (obtaining cash flows from investments) or by issuing more securities (obtaining cash flows from financing) cannot keep that up for very long. For future prosperity the firm must be able to generate cash flows from its operations.

Cash Flows from Operating Activities

The cash flow from operating activities is the most complex of the three. Ideally, we could obtain it directly, by summing all cash receipts (inflows) and disbursements (outflows) for the periods covered by the statement. However, in spite of its usefulness, this sum is, in practice, burdensome to prepare. Instead, the cash flow from operations is generally obtained indirectly. Using the indirect method, we begin with net income as reported on the income statement and adjust it for each change in current assets and current liabilities and each noncash operating item; what remains is the cash flow from (used for) operations, as shown in Exhibit 6.5.

EXHIBIT 6.5 **Fictitious Company Statement of Cash Flows, Years Ended December 31, in Thousands**

	2003	2002
Cash flow from (used for) operating activities		
Net income	$1,200	$1,000
Add or deduct adjustments to cash basis:		
Change in accounts receivables	$200	$(200)
Change in accounts payable	100	400
Change in marketable securities	(200)	200
Change in inventories	(800)	(600)
Change in other current liabilities	300	0
Depreciation	1,000	1,000
	600	800
Cash flow from operations	$1,800	$1,800
Cash flow from (used for) investing activities		
Purchase of plant and equipment	$(1,000)	$0
Cash flow from (used for) investing activities	$(1,000)	$0
Cash flow from (used for) financing activities		
Sale of common stock	$1,000	$0
Repayment of long-term debt	(1,000)	(1,500)
Payment of preferred dividends	(100)	(100)
Payment of common dividends	(500)	(400)
Cash flow from (used for) financing activities	(600)	(1,900)
Increase (decrease) in cash flow	$200	$(100)
Cash at the beginning of the year	200	300
Cash at the end of the year	$400	$200

EXHIBIT 6.6 **Adjustment of Net Income for Changes in Working Capital Accounts to Arrive at Cash Flow from Operations**

Change in Working Capital Account	Adjustment to Net Income
An increase in a current asset account	Deduct the change
A decrease in a current asset account	Add the change
An increase in a current liability account	Add the change
A decrease in a current liability account	Deduct the change

The basic adjustments to net income for changes in current assets and current liabilities are summarized in Exhibit 6.6. Income is adjusted for noncash revenues and expenses, such as depreciation, by adding them because they have been deducted in the computation for net income but do not require cash to be paid out.

We adjust net income for changes in current assets and liabilities because those changes represent the difference between accrual accounting and cash accounting. For example, an increase in the inventories account is the result of an investment of cash to generate sales in the near future. Exhibit 6.1 shows that Fictitious Corporation invested $800,000 in inventories during 2003 ($1 million in 2002 versus $1.8 million in 2003). Since that investment was an operating cash flow, we must *subtract* it from net income. As another example, Exhibit 6.1 shows that accounts receivable decreased by $200,000. That decrease in a current asset represents a flow of cash to the firm—the return of cash invested in accounts receivable. So the $200,000 must be *added* to net income to obtain cash flow. These adjustments are shown in the "Cash flow from operating activities" section of Exhibit 6.5, along with the other adjustments required to obtain Fictitious Corporation's operating activities.

Cash Flows from Investing and Financing Activities

The computation of the cash flows from investing and financing activities is straightforward. The ***cash flow from (used for) investing activities*** includes cash flow due to investments in plant assets, the disposal of plant assets, acquisitions of other companies, and divestitures of subsidiaries. For Fictitious Corporation, the $1 million invested in plant and equipment shows up as a net outflow on the statement of cash flows.

The ***cash flow from (used for) financing activities*** includes cash flows due to the sale or repurchase of common or preferred stock, the issuing or retirement of long-term debt securities, and the payment of common and preferred dividends.

The flows attributed to these activities are shown in Exhibit 6.5 for Fictitious Corporation. By design, the statement of cash flows is a reconciliation of the cash flows from the firm's three cash sources: operations, investing, and financing. It takes us from net income to the change in the cash account over the accounting period. For example, for Fictitious Corporation the net change in the cash balance during 1999 is an increase of $200,000 as shown in the first line of Exhibit 6.1. Exhibit 6.5 shows us that this increase is the result of net cash flows during 1999 of $1.8 million from operations, *less* $1 million from investing activities, *less* $600,000 from financing activities.

Consider another example. Suppose the Pretend Corporation has the following financial results:

- Net income of $40,000
- Increase in current assets of $5,000
- Increase in current liabilities of $2,000
- Sale of $10,000 of plant and equipment
- Purchase $20,000 of plant of equipment
- Depreciation of $12,000
- Repurchase $20,000 of common stock
- Dividends on common stock of $2,000

What is the Pretend's cash flow? The first step is to adjust net income for the changes in the working capital accounts: a downward adjustment of $5,000 for the increase in current assets and an upward adjustment of $2,000 for the increase in current liabilities. Adding depreciation, the cash flow from operating activities is $49,000. The cash flows from investing activities consists of the flow for Pretend's sale and purchase of plant and equipment. The cash flow from financing activities involves Pretend's repurchase of common stock and its payment of common dividends. The statement of cash flows for Pretend Corporation is shown in Exhibit 6.7.

The financial analyst can use the statement of cash flows to learn more about a company's financial health. Consider the cash flows shown in Exhibit 6.8 for different companies in 2001. Wal-Mart Stores, Dell, Disney, and Intel have cash flows that are typical of healthy, growing companies: funds are generated internally (that is, through operating activity) and funds are applied to investing activities. Motorola is generating funds from both operations and investments (that is, selling off assets). Wal-Mart Stores and Walt Disney are generating sufficient funds to fund their investment activity and reduce their dependence on externally-raised funds (as indicated by the cash flow used for financing activities). Dell and Disney are able to generate sufficient cash flows through operating activities to reduce dependence on external financing.

EXHIBIT 6.7 **Pretend Corporation Statement of Cash Flows**

Cash flow from operations	
Net income	$40,000
Increase in current assets	(5,000)
Increase in current liabilities	2,000
Depreciation	12,000
Cash flow from operations	$49,000
Cash flow from investing activities	
Sale of plant and equipment	$10,000
Purchase of plant and equipment	(20,000)
Cash flow used for investing activities	$(10,000)
Cash flow from financing activities	
Repurchase of common stock	$(20,000)
Dividends on common stock	(2,000)
Cash flow used for financing activities	$(22,000)
Increase in cash flow	$17,000

EXHIBIT 6.8 **Cash Flows from (Used for) Operating, Investment, and Financing Activities (In millions)**

Company	Cash Flows from (for) Operating Activities	Cash Flows from (for) Investment Activities	Cash Flows from (for) Financing Activities	Increase (Decrease) in Cash and Cash Equivalents
Dell Computer	$4,195	$(757)	$(2,305)	$1,101
Disney	3,048	(2,015)	(1,257)	(224)
Motorola	2,124	2,477	(1,802)	2,781
Intel	8,654	(195)	(3,465)	4,994
Wal-Mart Stores	10,260	(7,146)	113	107

Source: Statement of cash flows from the 2001 10-K reports for the respective companies.

EXHIBIT 6.9 Fictitious Corporation Statement of Shareholders' Equity

	Shares	Common Stock	Retained Earnings	Total
Balance at December 31, 2001	1,200	$2,000	$1,900	$3,900
Common stock sold (repurchased)	—	—	—	—
Net income		—	900	900
Cash dividend declared		—	(400)	(400)
Balance at December 31, 2002	1,200	$2,000	$2,400	$4,400
Common stock sold (repurchased)	300	1,000	—	1,000
Net income		—	1,100	1,100
Cash dividend declared		—	(500)	(500)
Balance at December 31, 2003	1,500	$3,000	$3,000	$6,000

THE STATEMENT OF SHAREHOLDERS' EQUITY

Additional information about equity can be found in the ***statement of shareholders' equity***, which is a breakdown of the amounts and changes in equity accounts. This statement serves as a connecting link between the balance sheet and the income statement, providing the analyst with more detail on changes in the individual equity accounts.

Whereas the balance sheet provides information on the number of shares outstanding at the specific point in time, the statement of shareholders' equity provides more detail on any changes, including shares issued to satisfy the exercise of stock options and repurchased shares. This statement can be expanded to accommodate treasury stock, if appropriate. The statement of shareholders' equity for Fictitious is shown in Exhibit 6.9.

The statement of shareholders' equity can provide a useful brief history of not only the effects of options, but also of items that may bypass the income statement.

NOTES TO FINANCIAL STATEMENTS

The financial statements of a corporation contain information beyond that presented in the balance sheet, the income statement, the statement of cash flows, and the statement of shareholders' equity. This additional information is presented in the notes to these financial statements. The first note summarizes the company's accounting policies including the methods of inventory accounting, methods of depreciation, and foreign currency translation. Depending on the circumstances of the company and the nature of its business, there may be additional notes providing, for example, supplemental balance sheet data, information on mergers or acquisitions, lease arrangements, or information on joint ventures.

SUMMARY

- The annual report of a company provides financial data, in the form of financial statements and notes, management discussion, and the auditor's opinion.
- The financial statements (the balance sheet, income statement, statement of cash flows, and statement of shareholders' equity), along with the accompanying notes, provide information necessary to assess the operating performance and the financial condition of the firm. Using this information, in conjunction with an understanding of accounting, analysts can see where a business has been, which may tell us something about where it is going.
- The balance sheet provides information about the value of accounts at a point in time, generally at the end of the fiscal year or the end of the fiscal quarter.
- The income statement provides information about the operating performance of a company over a period of time (typically a fiscal year or fiscal quarter).
- The statement of cash flows provides data on the cash flows of the company over time and the sources of these cash flows.
- The statement of shareholders' equity details the changes in the equity accounts over a period of time.
- The notes to financial statements provide more detail on many accounts.

QUESTIONS

1. What is meant by generally accepted accounting principles?
2. What is meant by accrual accounting and the matching principle?
3. Describe the type of information provided in each of the three financial statements:
   a. The balance sheet.
   b. The income statement.
   c. The statement of cash flows.

4. Comment on the following two statements:
   a. "Asset values reported in the balance sheet are shown at market value."
   b. "The cash of a company is equal to its retained earnings."
5. Distinguish between accounts receivable and accounts payable.
6. Define each of the following: (a) current assets, (b) intangible assets, (c) deferred taxes, (d) retained earnings, and (e) earnings before interest and taxes.
7. Distinguish between accounts receivable and accounts payable.
8. Complete the following balance sheet:

Cash	$100	Accounts payable	$200
Inventory	______	Notes payable	300
Gross plant and equipment	1,800	Long-term debt	_____
Accumulated depreciation	______	Common equity	1,000
Net plant and equipment	1,500		
Total assets	$2,000	Total liabilities and equity	_____

9. Calculate the amount of retained earnings from the following information:

- Common stock dividends are 40% of earnings available to common shareholders.
- Earnings before taxes are $3,000.
- Preferred stock dividends are $200.
- Taxes are 30% of earnings.

5. 10. Construct the statement of cash flows given the following information:

- $10,000 in new long-term debt is issued.
- $30,000 of common stock is repurchased.
- Common stock dividends are $15,000.
- Current assets are increased by $20,000.
- Current liabilities are decreased by $40,000.
- Depreciation is $10,000.
- Net income is $100,000.
- Plant and equipment purchased during the period are $30,000.

CHAPTER 7

Mathematics of Finance

The notion that money has a time value is one of the most basic concepts in investment analysis. Making decisions today regarding future cash flows requires understanding that the value of money does not remain the same throughout time.

A dollar today is worth less than a dollar some time in the future for two reasons.

> *Reason No. 1:* Cash flows occurring at different points in time have different values relative to any one point in time.
>
> One dollar one year from now is not as valuable as one dollar today. After all, you can invest a dollar today and earn interest so that the value it grows to next year is greater than the one dollar today. This means we have to take into account the *time value of money* to quantify the relation between cash flows at different points in time.
>
> *Reason No. 2:* Cash flows are uncertain.
>
> Expected cash flows may not materialize. Uncertainty stems from the nature of forecasts of the timing and/or the amount of cash flows. We do not know for certain when, whether, or how much cash flows will be in the future. This uncertainty regarding future cash flows must somehow be taken into account in assessing the value of an investment.

Translating a current value into its equivalent future value is referred to as *compounding*. Translating a future cash flow or value into its equivalent value in a prior period is referred to as *discounting*. This chapter outlines the basic mathematical techniques used in compounding and discounting.

Suppose someone wants to borrow $100 today and promises to pay back the amount borrowed in one month. Would the repayment of only the $100 be fair? Probably not. There are two things to consider. First, if the lender didn't lend the $100, what could he or she have done with it? Second, is there a chance that the borrower may not pay back the loan? So, when considering lending money, we must consider the opportunity cost (i.e., what could have been earned or enjoyed), as well as the uncertainty associated with getting the money back as promised.

Let's say that someone is willing to lend the money, but that they require repayment of the $100 plus some compensation for the opportunity cost *and* any uncertainty the loan will be repaid as promised. The amount of the loan, the $100, is the *principal*. The compensation required for allowing someone else to use the $100 is the *interest*.

Looking at this same situation from the perspective of time and value, the amount that you are willing to lend today is the loan's *present value*. The amount that you require to be paid at the end of the loan period is the loan's *future value*. Therefore, the future period's value is comprised of two parts:

$$\text{Future Value} = \text{Present value} + \text{Interest}$$

The interest is compensation for the use of funds for a specific period. It consists of (1) compensation for the length of time the money is borrowed and (2) compensation for the risk that the amount borrowed will not be repaid exactly as set forth in the loan agreement.

DETERMINING THE FUTURE VALUE

Suppose you deposit $1,000 into a savings account at the Surety Savings Bank and you are promised 10% interest per period. At the end of one period you would have $1,100. This $1,100 consists of the return of your principal amount of the investment (the $1,000) and the interest or return on your investment (the $100). Let's label these values:

- $1,000 is the value today, the present value, PV
- $1,100 is the value at the end of one period, the future value, FV
- 10% is the rate interest is earned in one period, the interest rate, i

To get to the future value from the present value:

$$FV = \underset{\text{principal}}{\underset{\uparrow}{PV}} + \underset{\text{interest}}{\underset{\uparrow}{(PV \times i)}}$$

This is equivalent to:

$$FV = PV(1 + i)$$

In terms of our example,

$$FV = \$1{,}000 + (\$1{,}000 \times 0.10) = \$1{,}000(1 + 0.10) = \$1{,}100$$

If the $100 interest is withdrawn at the end of the period, the principal is left to earn interest at the 10% rate. Whenever you do this, you earn *simple interest*. It is simple because it repeats itself in exactly the same way from one period to the next as long as you take out the interest at the end of each period and the principal remains the same. If, on the other hand, both the principal and the interest are left on deposit at the Surety Savings Bank, the balance earns interest on the previously paid interest, referred to as *compound interest*. Earning interest on interest is called compounding because the balance at any time is a combination of the principal, interest on principal, and *interest on accumulated interest* (or simply, *interest on interest*).

If you compound interest for one more period in our example, the original $1,000 grows to $1,210.00:

$$\begin{aligned} \text{FV} &= \text{principal} + \text{first period interest} + \text{second period interest} \\ &= \$1{,}000.00 + (\$1{,}000.00 \times 0.10) + (\$1{,}100.00 \times 0.10) \\ &= \$1{,}210.00 \end{aligned}$$

The present value of the investment is $1,000, the interest earned over two years is $210, and the future value of the investment after two years is $1,210.

The relation between the present value and the future value after two periods, breaking out the second period interest into interest on the principal and interest on interest, is:

$$\text{FV} = \underset{\substack{\uparrow \\ \text{principal}}}{\text{PV}} + \underset{\substack{\uparrow \\ \text{first period's} \\ \text{interest on} \\ \text{the principal}}}{(PV \times i)} + \underset{\substack{\uparrow \\ \text{second period's} \\ \text{interest on} \\ \text{the principal}}}{(PV \times i)} \quad \underset{\substack{\uparrow \\ \text{second period's} \\ \text{interest on the first} \\ \text{period's interest}}}{(PV \times i \times i)}$$

or, collecting the *PV*'s from each term and applying a bit of elementary algebra,

$$FV = PV(1 + 2i + i^2) = PV\,(1 + i)^2$$

The balance in the account two years from now, $1,210, is comprised of three parts:

1. the principal, \$1,000,
2. interest on principal, \$100 in the first period plus \$100 in the second period
3. interest on interest, 10% of the first period's interest, or \$10

To determine the future value with compound interest for *more* than two periods, we follow along the same lines:

$$FV = PV(1 + i)^N \tag{7-1}$$

The value of N is the number of *compounding periods*, where a compounding period is the unit of time after which interest is paid at the rate i. A period may be any length of time: a minute, a day, a month, or a year. The important thing is to make sure the same compounding period is reflected throughout the problem being analyzed. The term "$(1 + i)^N$" is referred to as the *compound factor.* It is the rate of exchange between present dollars and dollars N compounding periods into the future. Equation (7-1) is the *basic valuation equation*—the foundation of financial mathematics. It relates a value at one point in time to a value at another point in time, considering the compounding of interest.

The relation between present and future values for a principal of \$1,000 and interest of 10% per period through 10 compounding periods is shown graphically in Exhibit 7.1. For example, the value of \$1,000, earning interest at 10% per period, is \$2,593.70 ten periods into the future:

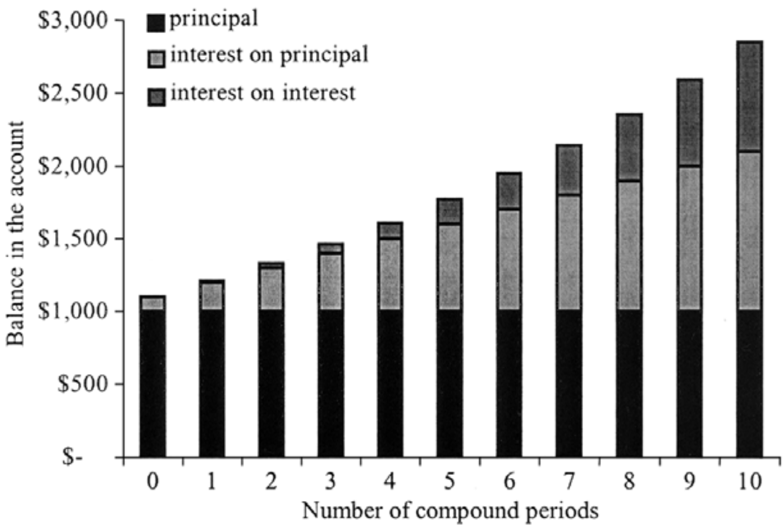
$$FV = \$1{,}000\ (1 + 0.10)^{10} = \$1{,}000\ (2.5937) = \$2{,}593.70$$

As you can see in this exhibit, the \$2,593.70 balance in the account at the end of 10 periods is comprised of three parts:

1. the principal, \$1,000
2. interest on the principal of \$1,000, which is \$100 per period for 10 periods or \$1,000
3. interest on interest totaling \$593.70

We can express the change in the value of the savings balance (i.e., the difference between the ending value and the beginning value) as a growth rate. A *growth rate* is the rate at which a value appreciates (a positive growth) or depreciates (a negative growth) over time. Our \$1,000 grew at a rate of 10% per year over the 10-year period to \$2,593.70. The average annual growth rate of our investment of \$1,000 is 10%—the value of the savings account balance increased 10% per year.

EXHIBIT 7.1 **The Value of $1,000 Invested 10 Years in an Account that Pays 10% Compounded Interest per Year**



We could also express the appreciation in our savings balance in terms of a return. A *return* is the income on an investment, generally stated as a change in the value of the investment over each period divided by the amount of the investment at the beginning of the period. We could also say that our investment of $1,000 provides an average annual return of 10% per year. The average annual return is *not* calculated by taking the change in value over the entire 10-year period ($2,593.70 – $1,000) and dividing it by $1,000. This would produce an *arithmetic average return* of 159.37% over the 10-year period, or 15.937% per year. But the arithmetic average ignores the process of compounding. The correct way of calculating the average annual return is to use a *geometric average return*:

$$i = \sqrt[N]{\frac{FV}{PV}} - 1 \tag{7-2}$$

which is a rearrangement of equation (7-1). Using the values from the example,

$$i = \sqrt[10]{\frac{\$2,593.70}{\$1,000.00}} - 1 = \left(\frac{\$2,593.70}{\$1,000.00}\right)^{1/10} - 1 = 1.10 - 1 = 10\%$$

Therefore, the annual return on the investment—sometimes referred to as the *compound average annual return* or the *true return*—is 10% per year.

Here is another example for calculating a future value. A common investment product of a life insurance company is a guaranteed investment contract (GIC). With this investment, an insurance company guarantees a specified interest rate for a period of years. Suppose that the life insurance company agrees to pay 6% annually for a 5-year GIC and the amount invested by the policyholder is $10 million. The amount of the liability (that is, the amount this life insurance company has agreed to pay the GIC policyholder) is the future value of $10 million when invested at 6% interest for five years. In terms of equation (7-1), PV = \$10,000,000, i = 6%, and N = 5, so that the future value is:

$$FV = \$10{,}000{,}000\ (1 + 0.06)^5 = \$13{,}382{,}256.$$

Compounding More than One Time Per Year

An investment may pay interest more than one time per year. For example, interest may be paid semiannually, quarterly, monthly, weekly, or daily, even though the stated rate is quoted on an annual basis. If the interest is stated as, say, 10% per year, compounded semiannually, the nominal rate—often referred to as the *annual percentage rate* or APR—is 10%. The basic valuation equation handles situations in which there is compounding more frequently than once a year if we translate the nominal rate into a rate *per compounding period*. Therefore, an APR of 10% with compounding semiannually is 5% per period—where a period is six months—and the number of periods in one year is 2.

Consider a deposit of $50,000 in an account for five years that pays 8% interest, compounded quarterly. The interest rate per period, i, is 8%/4 = 2% and the number of compounding periods is 5 × 4 = 20. Therefore the balance in the account at the end of five years is:

$$FV = \$50{,}000(1 + 0.02)^{20} = \$50{,}000(1.4859474) = \$74{,}297.37$$

As shown in Exhibit 7.2, through 50 years with both annual and quarterly compounding, the investment's value increases at a faster rate with the increased frequency of compounding.

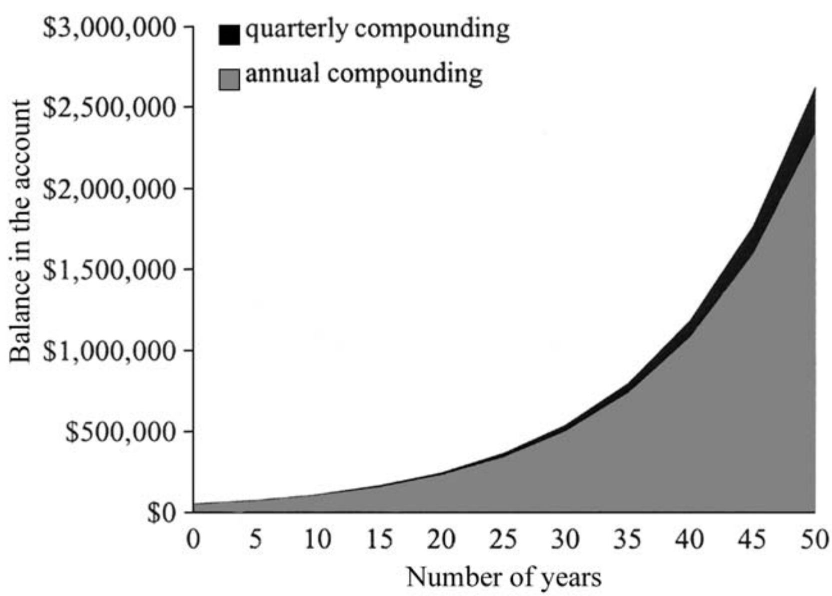
The last example illustrates the need to correctly identify the "period" because this dictates the interest rate per period and the number of compounding periods. Because interest rates are often quoted in terms of an APR, we need to be able to translate the APR into an interest rate per period and to adjust the number of periods. To see how this works, let's use an example of a deposit of $1,000 in an account that pays interest at a rate of 12% per year, with interest compounded for different compounding frequencies. How much is in the account after, say, five years depends on the compounding frequency:

Compounding Frequency	Period	Rate per Compounding Period, *i*	Number of Periods in 5 Years, *N*	*FV* at the End of Five Years
Annual	one year	12%	5	$1,762.34
Semiannual	six months	6%	10	1,790.85
Quarterly	three months	3%	20	1,806.11
Monthly	one month	1%	60	1,816.70

As you can see, both the rate per period, *i*, and the number of compounding periods, *N*, are adjusted and depend on the frequency of compounding. Interest can be compounded for any frequency, such as daily or hourly.

Let's work through another example for compounding with compounding more than once a year. Suppose we invest $200,000 in an investment that pays 4% interest per year, compounded quarterly. What will be the future value of this investment at the end of 10 years?

EXHIBIT 7.2 **Value of $50,000 Invested in the Account that Pays 8% Interest Per Year: Quarterly versus Annual Compounding**



The given information is $i = 4\%/4 = 1\%$ and $N = 10 \times 4 = 40$ quarters. Therefore,

$$FV = \$200{,}000(1 + 0.01)^{40} = \$297{,}772.75$$

Continuous Compounding

The extreme frequency of compounding is *continuous compounding*—interest is compounded instantaneously. The factor for compounding continuously for one year is e^{APR}, where e is 2.71828..., the base of the natural logarithm. And the factor for compounding continuously for two years is $e^{\text{APR}}e^{\text{APR}}$ or $e^{2\text{APR}}$. The future value of an amount that is compounded continuously for N years is:

$$FV = PVe^{N(\text{APR})} \tag{7-3}$$

where APR is the annual percentage rate and $e^{N(\text{APR})}$ is the compound factor.

If \$1,000 is deposited in an account for five years with interest of 12% per year, compounded continuously,

$$FV = \$1{,}000e^{5(0.12)} = \$1{,}000(e^{0.60}) = \$1{,}000(1.82212) = \$1{,}822.12$$

Comparing this future value with that if interest is compounded annually at 12% per year for 5 years, \$1,762.34, we see that the effects of this extreme frequency of compounding.

Multiple Rates

In our discussion thus far, we have assumed that the investment will earn the same periodic interest rate, i. We can extend the calculation of a future value to allow for different interest rates or growth rates for different periods. Suppose an investment of \$10,000 pays 9% during the first year and 10% during the second year. At the end of the first period, the value of the investment is \$10,000(1 + 0.09), or \$10,900. During the second period, this \$10,900 earns interest at 10%. Therefore the future value of this \$10,000 at the end of the second period is:

$$FV = \$10{,}000(1 + 0.09)(1 + 0.10) = \$11{,}990$$

We can write this more generally as:

$$FV = PV(1 + i_1)\,(1 + i_2)\,(1 + i_3)\,.\,.\,.\,(1 + i_N) \tag{7-4}$$

where i_N is the interest rate for period N.

Consider a $50,000 investment in a one-year bank certificate of deposit (CD) today and rolled over annually for the next two years into one-year CDs. The future value of the $50,000 investment will depend on the one-year CD rate each time the funds are rolled over. Assuming that the one-year CD rate today is 5% and that it is expected that the one-year CD rate one year from now will be 6%, and the one-year CD rate two years from now will be 6.5%, then we know:

$$FV = \$50{,}000(1 + 0.05)(1 + 0.06)(1 + 0.065) = \$59{,}267.25$$

Continuing this example, what is the average annual interest rate over this period? We know that the future value is $59,267.25, the present value is $50,000, and $N = 3$:

$$i = \sqrt[3]{\frac{\$59{,}267.25}{\$50{,}000.00}} - 1 = \sqrt[3]{1.185345} = 5.8315\%$$

which is also:

$$i = \sqrt[3]{(1 + 0.05)(1 + 0.06)(1 + 0.065)} - 1 = 5.8315\%$$

DETERMINING THE PRESENT VALUE

Now that we understand how to compute future values, let's work the process in reverse. Suppose that for borrowing a specific amount of money today, the Yenom Company promises to pay lenders $5,000 two years from today. How much should the lenders be willing to lend Yenom in exchange for this promise? This dilemma is different than figuring out a future value. Here we are given the future value and have to figure out the present value. But we can use the same basic idea from the future value problems to solve present value problems.

If you can earn 10% on other investments that have the same amount of uncertainty as the $5,000 Yenom promises to pay, then:

- the future value, $FV = \$5{,}000$
- the number of compounding periods, $N = 2$
- the interest rate, $i = 10\%$

We also know the basic relation between the present and future values:

$$FV = PV(1 + i)^N$$

Substituting the known values into this equation:

$$\$5{,}000 = PV(1 + 0.10)^2$$

To determine how much you are willing to lend now, PV, to get \$5,000 one year from now, FV, requires solving this equation for the unknown present value:

$$PV = \frac{\$5{,}000}{(1+0.10)^2} = \$5{,}000\left(\frac{1}{1+0.10}\right)^2 = \$5{,}000(0.82645) = \$4{,}132.25$$

Therefore, you would be willing to lend \$4,132.25 to receive \$5,000 one year from today if your opportunity cost is 10%. We can check our work by reworking the problem from the reverse perspective. Suppose you invested \$4,132.25 for two years and it earned 10% per year. What is the value of this investment at the end of the year?

We know: PV = \$4,132.25, N = 10% or 0.10, and i = 2.

Therefore the future value is:

$$FV = PV(1 + i)^N = \$4{,}132.25\ (1 + 0.10)^2 = \$5{,}000.00$$

Compounding translates a value in one point in time into a value at some future point in time. The opposite process translates future values into present values: Discounting translates a value back in time. From the basic valuation equation:

$$FV = PV\ (1 + i)^N$$

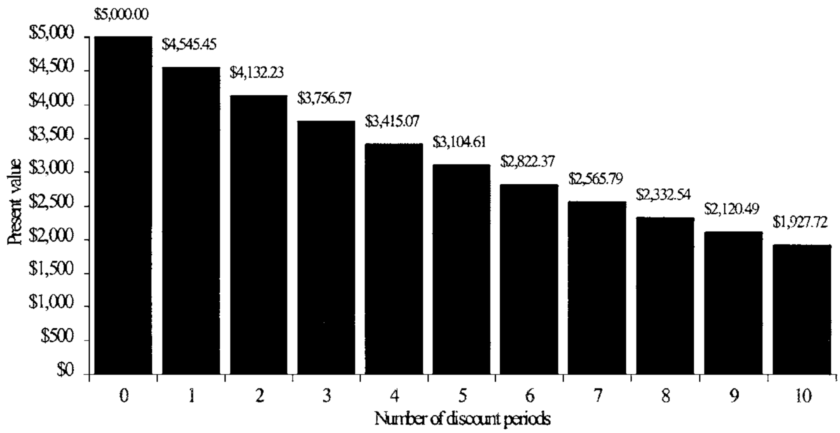
we divide both sides by $(1 + i)^N$ and exchange sides to get the present value,

$$PV = \frac{FV}{(1+i)^N} \qquad (7\text{-}5)$$

$$\text{or } PV = FV\left(\frac{1}{1+i}\right)^N \text{ or } PV = FV\left[\frac{1}{(1+i)^N}\right]$$

The term in brackets [ ] is referred to as the *discount factor* since it is used to translate a future value to its equivalent present value. The present value of \$5,000 for discount periods ranging from 0 to 10 is shown in Exhibit 7.3.

EXHIBIT 7.3 Present Value of $5,000 Discounted at 10%



If the frequency of compounding is greater than once a year, we make adjustments to the rate per period and the number of periods as we did in compounding. For example, if the future value five years from today is $100,000 and the interest is 6% per year, compounded semiannually, $i = 6\%/2 = 3\%$ and $N = 5 \times 2 = 10$, and the present value is:

$$PV = \$100{,}000(1 + 0.03)^{10} = \$100{,}000(1.34392) = \$134{,}392$$

Here is an example of calculating a present value. Suppose that the goal is to have $75,000 in an account by the end of four years. And suppose that interest on this account is paid at a rate of 5% per year, compounded semiannually. How much must be deposited in the account today to reach this goal? We are given $FV = \$75{,}000$, $i = 5\%/2 = 2.5\%$ per six months, and $N = 4 \times 2 = 8$ six-month periods. The amount of the required deposit is therefore:

$$PV = \frac{\$75{,}000}{(1 + 0.025)^8} = \$61{,}555.99$$

Compound and Discount Factor Tables

There are different ways to translate values forward and backward in time. The basic way is through equations (7-1) and (7-5), using whichever values of PV, FV, N, or i given, and solving for the present or future value required by the problem.

Another way is to use tables of discount factors and compound factors. A table of compound factors for periods ranging from 1 to 20 and for rates of interest from 1% to 15% is provided in Exhibit 7.4. Similarly, a table of discount factors for the same range of periods and interest rates is provided in Exhibit 7.5. The compound factor to use for a problem is determined by choosing the table value corresponding to the row for the number of periods and the column for the interest rate per period given in the problem. A discount factor is determined in a like manner.

To see how to use a table of factors, let's find the compound factors for several combinations of periods and interest rates. The compound factor for 10 periods and an interest rate of 5% per period is 1.6289. The compound factor for five periods and an interest rate of 10% per period is 1.6105. The compound factor for three periods and an interest rate of 6% per period is 1.1910.

The table of compound factors can also be used for situations where you need to determine the number of periods or the interest rate. For example, suppose that you are asked to find out how long it takes to double your money if the interest rate per period is 8%. Doubling your money would mean that the future value is twice the present value. Using the equation:

$$FV = PV\,(1 + i)^N$$

and inserting the known values:

$$2.0000 = 1.0000(1 + 0.08)^N \text{ or } 2.0000 = (1 + 0.08)^N$$

the compound factor is 2.0000.

The compound factor for 8% per period over some unknown number of periods is 2.0000. Looking at the top panel, going down the 8% interest rate column, we see that the factor closest to 2.0000 is nine periods (compound factor = 1.9990). Therefore, it takes nine periods to double your money if interest is compounded at 8% per period.

Consider another example. If you want to invest \$1,000 for six periods, at what interest rate must the account pay compounded interest in order for you to have \$1,500 after six periods? We know

$$\begin{aligned} FV &= PV(1 + i)^N \\ \$1{,}500 &= \$1{,}000(1 + i)^6 \\ 1.5000 &= (1 + i)^6 \end{aligned}$$

EXHIBIT 7.4 Table of Compound Factors

Number of	Compounding Rate											
Periods	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%
1	1.0100	1.0200	1.0300	1.0400	1.0500	1.0600	1.0700	1.0800	1.0900	1.1000	1.1100	1.1200
2	1.0201	1.0404	1.0609	1.0816	1.1025	1.1236	1.1449	1.1664	1.1881	1.2100	1.2321	1.2544
3	1.0303	1.0612	1.0927	1.1249	1.1576	1.1910	1.2250	1.2597	1.2950	1.3310	1.3676	1.4049
4	1.0406	1.0824	1.1255	1.1699	1.2155	1.2625	1.3108	1.3605	1.4116	1.4641	1.5181	1.5735
5	1.0510	1.1041	1.1593	1.2167	1.2763	1.3382	1.4026	1.4693	1.5386	1.6105	1.6851	1.7623
6	1.0615	1.1262	1.1941	1.2653	1.3401	1.4185	1.5007	1.5869	1.6771	1.7716	1.8704	1.9738
7	1.0721	1.1487	1.2299	1.3159	1.4071	1.5036	1.6058	1.7138	1.8280	1.9487	2.0762	2.2107
8	1.0829	1.1717	1.2668	1.3686	1.4775	1.5938	1.7182	1.8509	1.9926	2.1436	2.3045	2.4760
9	1.0937	1.1951	1.3048	1.4233	1.5513	1.6895	1.8385	1.9990	2.1719	2.3579	2.5580	2.7731
10	1.1046	1.2190	1.3439	1.4802	1.6289	1.7908	1.9672	2.1589	2.3674	2.5937	2.8394	3.1058
11	1.1157	1.2434	1.3842	1.5395	1.7103	1.8983	2.1049	2.3316	2.5804	2.8531	3.1518	3.4785
12	1.1268	1.2682	1.4258	1.6010	1.7959	2.0122	2.2522	2.5182	2.8127	3.1384	3.4985	3.8960
13	1.1381	1.2936	1.4685	1.6651	1.8856	2.1329	2.4098	2.7196	3.0658	3.4523	3.8833	4.3635
14	1.1495	1.3195	1.5126	1.7317	1.9799	2.2609	2.5785	2.9372	3.3417	3.7975	4.3104	4.8871
15	1.1610	1.3459	1.5580	1.8009	2.0789	2.3966	2.7590	3.1722	3.6425	4.1772	4.7846	5.4736
16	1.1726	1.3728	1.6047	1.8730	2.1829	2.5404	2.9522	3.4259	3.9703	4.5950	5.3109	6.1304
17	1.1843	1.4002	1.6528	1.9479	2.2920	2.6928	3.1588	3.7000	4.3276	5.0545	5.8951	6.8660
18	1.1961	1.4282	1.7024	2.0258	2.4066	2.8543	3.3799	3.9960	4.7171	5.5599	6.5436	7.6900
19	1.2081	1.4568	1.7535	2.1068	2.5270	3.0256	3.6165	4.3157	5.1417	6.1159	7.2633	8.6128
20	1.2202	1.4859	1.8061	2.1911	2.6533	3.2071	3.8697	4.6610	5.6044	6.7275	8.0623	9.6463
21	1.2324	1.5157	1.8603	2.2788	2.7860	3.3996	4.1406	5.0338	6.1088	7.4002	8.9492	10.8038
22	1.2447	1.5460	1.9161	2.3699	2.9253	3.6035	4.4304	5.4365	6.6586	8.1403	9.9336	12.1003
23	1.2572	1.5769	1.9736	2.4647	3.0715	3.8197	4.7405	5.8715	7.2579	8.9543	11.0263	13.5523
24	1.2697	1.6084	2.0328	2.5633	3.2251	4.0489	5.0724	6.3412	7.9111	9.8497	12.2392	15.1786
25	1.2824	1.6406	2.0938	2.6658	3.3864	4.2919	5.4274	6.8485	8.6231	10.8347	13.5855	17.0001
26	1.2953	1.6734	2.1566	2.7725	3.5557	4.5494	5.8074	7.3964	9.3992	11.9182	15.0799	19.0401
27	1.3082	1.7069	2.2213	2.8834	3.7335	4.8223	6.2139	7.9881	10.2451	13.1100	16.7386	21.3249
28	1.3213	1.7410	2.2879	2.9987	3.9201	5.1117	6.6488	8.6271	11.1671	14.4210	18.5799	23.8839
29	1.3345	1.7758	2.3566	3.1187	4.1161	5.4184	7.1143	9.3173	12.1722	15.8631	20.6237	26.7499
30	1.3478	1.8114	2.4273	3.2434	4.3219	5.7435	7.6123	10.0627	13.2677	17.4494	22.8923	29.9599

EXHIBIT 7.5 Table of Discount Factors

Number of	Discount Rate											
Periods	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%
1	0.9901	0.9804	0.9709	0.9615	0.9524	0.9434	0.9346	0.9259	0.9174	0.9091	0.9009	0.8929
2	0.9803	0.9612	0.9426	0.9246	0.9070	0.8900	0.8734	0.8573	0.8417	0.8264	0.8116	0.7972
3	0.9706	0.9423	0.9151	0.8890	0.8638	0.8396	0.8163	0.7938	0.7722	0.7513	0.7312	0.7118
4	0.9610	0.9238	0.8885	0.8548	0.8227	0.7921	0.7629	0.7350	0.7084	0.6830	0.6587	0.6355
5	0.9515	0.9057	0.8626	0.8219	0.7835	0.7473	0.7130	0.6806	0.6499	0.6209	0.5935	0.5674
6	0.9420	0.8880	0.8375	0.7903	0.7462	0.7050	0.6663	0.6302	0.5963	0.5645	0.5346	0.5066
7	0.9327	0.8706	0.8131	0.7599	0.7107	0.6651	0.6227	0.5835	0.5470	0.5132	0.4817	0.4523
8	0.9235	0.8535	0.7894	0.7307	0.6768	0.6274	0.5820	0.5403	0.5019	0.4665	0.4339	0.4039
9	0.9143	0.8368	0.7664	0.7026	0.6446	0.5919	0.5439	0.5002	0.4604	0.4241	0.3909	0.3606
10	0.9053	0.8203	0.7441	0.6756	0.6139	0.5584	0.5083	0.4632	0.4224	0.3855	0.3522	0.3220
11	0.8963	0.8043	0.7224	0.6496	0.5847	0.5268	0.4751	0.4289	0.3875	0.3505	0.3173	0.2875
12	0.8874	0.7885	0.7014	0.6246	0.5568	0.4970	0.4440	0.3971	0.3555	0.3186	0.2858	0.2567
13	0.8787	0.7730	0.6810	0.6006	0.5303	0.4688	0.4150	0.3677	0.3262	0.2897	0.2575	0.2292
14	0.8700	0.7579	0.6611	0.5775	0.5051	0.4423	0.3878	0.3405	0.2992	0.2633	0.2320	0.2046
15	0.8613	0.7430	0.6419	0.5553	0.4810	0.4173	0.3624	0.3152	0.2745	0.2394	0.2090	0.1827
16	0.8528	0.7284	0.6232	0.5339	0.4581	0.3936	0.3387	0.2919	0.2519	0.2176	0.1883	0.1631
17	0.8444	0.7142	0.6050	0.5134	0.4363	0.3714	0.3166	0.2703	0.2311	0.1978	0.1696	0.1456
18	0.8360	0.7002	0.5874	0.4936	0.4155	0.3503	0.2959	0.2502	0.2120	0.1799	0.1528	0.1300
19	0.8277	0.6864	0.5703	0.4746	0.3957	0.3305	0.2765	0.2317	0.1945	0.1635	0.1377	0.1161
20	0.8195	0.6730	0.5537	0.4564	0.3769	0.3118	0.2584	0.2145	0.1784	0.1486	0.1240	0.1037
21	0.8114	0.6598	0.5375	0.4388	0.3589	0.2942	0.2415	0.1987	0.1637	0.1351	0.1117	0.0926
22	0.8034	0.6468	0.5219	0.4220	0.3418	0.2775	0.2257	0.1839	0.1502	0.1228	0.1007	0.0826
23	0.7954	0.6342	0.5067	0.4057	0.3256	0.2618	0.2109	0.1703	0.1378	0.1117	0.0907	0.0738
24	0.7876	0.6217	0.4919	0.3901	0.3101	0.2470	0.1971	0.1577	0.1264	0.1015	0.0817	0.0659
25	0.7798	0.6095	0.4776	0.3751	0.2953	0.2330	0.1842	0.1460	0.1160	0.0923	0.0736	0.0588
26	0.7720	0.5976	0.4637	0.3607	0.2812	0.2198	0.1722	0.1352	0.1064	0.0839	0.0663	0.0525
27	0.7644	0.5859	0.4502	0.3468	0.2678	0.2074	0.1609	0.1252	0.0976	0.0763	0.0597	0.0469
28	0.7568	0.5744	0.4371	0.3335	0.2551	0.1956	0.1504	0.1159	0.0895	0.0693	0.0538	0.0419
29	0.7493	0.5631	0.4243	0.3207	0.2429	0.1846	0.1406	0.1073	0.0822	0.0630	0.0485	0.0374
30	0.7419	0.5521	0.4120	0.3083	0.2314	0.1741	0.1314	0.0994	0.0754	0.0573	0.0437	0.0334

Therefore, the compound factor is 1.5. Using Exhibit 7.4, we see going across the row corresponding to six periods that the compound factor is 1.5000 at (approximately) a 7% interest rate. Therefore, if you save \$1,000 in an account that provides 7% per period compounded interest for 6 periods, you will have a balance of approximately \$1,500 after six periods.

To see how to use Exhibit 7.5, let's find the discount factors for several combinations of periods and interest rates. The discount factor for ten periods and an interest rate of 5% per period is 0.6139. The discount factor for five periods and an interest rate of 10% per period is 0.6209. The discount factor for three periods and an interest rate of 6% per period is 0.8396. Just as we did for the compound factors, these discount factors can be used to solve for *N*, given a value of the discount factor and an interest rate, or to solve for the interest rate, given the value for the discount factor and the number of discounting periods.

If we look at equations (7-1) and (7-5) and think about them for a moment, it becomes apparent that inverting the values in one table produces the values in the other. For example, using the corresponding factors for $N = 10$ and $r = 5\%$, we see this inverse relation:

$$\begin{aligned}\text{Compound factor} &= 1/\text{Discount factor}\\ 1.6289 &= 1/0.6139\end{aligned}$$

Likewise,

$$\begin{aligned}\text{Discount factor} &= 1/\text{Compound factor}\\ 0.6139 &= 1/1.6289\end{aligned}$$

The compound and discount factors are inversely related to one another for any pair of *N* and *i* values.

Using a Financial Calculator

The financial math of discounting and compounding can also be performed using a financial calculator. The basic idea is to input the known values and let the calculator solve for the one unknown value—the financial math is programmed into the calculator. To use a financial calculator effectively, you need to understand how to input the known values. For example, in most financial calculators the present value is input as a negative value. Consider using the financial calculator to solve the following problem: You invest \$5,000 today in an account that pays 8% interest. What is the balance in the account at the end of five years?

The known values are the following: $PV = \$5,000$, $i = 8\%$, and $N = 5$. The one unknown is the future value. Using several popular financial calculators, we can readily solve this problem to arrive at the answer of \$7,346.64:

Hewlett-Packard 10B	Hewlett-Packard 12C	Hewlett-Packard 17B	Texas Instruments BA-II Plus
5000 ± PV 8 I/YR 5 N FV	5000 CHS PV 8 i 5 n FV	FIN TVM 5000 ± PV 8 I%YR 5 N FV	5000 ± PV 8 I 5 N FV

DETERMINING THE UNKNOWN INTEREST RATE

As we saw earlier in our discussion of growth rates, we can rearrange the basic equation to solve for *i*:

$$i = \sqrt[N]{\frac{FV}{PV}} - 1 = \left(\frac{FV}{PV}\right)^{1/N} - 1$$

As an example, suppose that the value of an investment today is \$100 and the expected value of the investment in five years is expected to be \$150. What is the annual rate of appreciation in value of this investment over the five-year period?

$$\begin{aligned} i &= \sqrt[5]{\frac{\$150}{\$100}} - 1 \\ &= \sqrt[5]{1.5} - 1 = 0.0845 \text{ or } 8.45\% \text{ per year} \end{aligned}$$

As we saw earlier, we can approximate the interest rate using Exhibit 7.4 or Exhibit 7.5. From the formulas for the present value and future value, you can see that the compounding factor is the ratio of the future value to the present value, whereas the discounting factor is the ratio of the present value to the future value. That is,

$$\text{Compounding factor} = (1 + i)^N = \frac{FV}{PV}$$

and

$$\text{Discounting factor} = \left(\frac{1}{1+i}\right)^N = \frac{PV}{FV}$$

In this example,

$$\frac{FV}{PV} = 1.500 \quad \text{and} \quad \frac{PV}{FV} = 0.6667$$

In Exhibit 7.4, the factor closest to 1.5 in the row corresponding to five periods is in the column for a 9% interest rate. In Exhibit 7.5, the factor closest to 0.6667 in the row corresponding to five periods is in the 9% interest rate column. Therefore, investing $100 today will produce $150 five years from now if the investment appreciates approximately 9% per year, as before.

There are many applications in which managers need to determine the rate of change in values over a period of time. If values are increasing over time, we refer to the rate of change as the growth rate. To make comparisons easier, we usually specify the growth rate as a rate per year.

For example, if we wish to determine the rate of growth in these values, we solve for the unknown interest rate. Consider the growth rate of dividends for Bell Atlantic. Bell Atlantic paid dividends of $1.18 per share in 1990 and $1.55 in 1998. We have dividends for two different points in time: 1990 and 1998. Using equation (7-3), with 1990 dividends as the present value, 1998 dividends as the future value, and $N = 8$:

$$\text{Growth rate in Bell Atlantic's dividends 1990–1998}$$
$$= \sqrt[8]{\frac{\$1.55}{\$1.18}} - 1 = 3.468\%$$

Therefore, Bell Atlantic's dividends grew at a rate of almost 3.5% per year over this eight-year period.

DETERMINING THE NUMBER OF COMPOUNDING PERIODS

Given the present and future values, calculating the number of periods when we know the interest rate is a bit more complex than calculating the interest rate when we know the number of periods. Nevertheless, we can develop an equation for determining the number of periods, beginning with the valuation formula given by equation (7-1) and rearranging to solve for N,

$$N = \frac{\ln FV - \ln PV}{\ln(1+i)} \tag{7-6}$$

where ln indicates the natural logarithm, which is the log of the base *e*.[1]

Suppose that the present value of an investment is $100 and you wish to determine how long it will take for the investment to double in value if the investment earns 6% per year, compounded annually:

$$N = \frac{\ln 200 - \ln 100}{\ln 1.06} = \frac{5.2983 - 4.6052}{0.0583}$$
$$= 11.8885 \text{ or approximately 12 years}$$

You'll notice that we round off to the next whole period. To see why, consider this last example. After 11.8885 years, we have doubled our money if interest were paid 88.85% the way through the twelfth year. But, we stated earlier that interest is paid *at the end of each period*—not part of the way through. At the end of the eleventh year, our investment is worth $189.93, and at the end of the twelfth year, our investment is worth $201.22. So, our investment's value doubles by the twelfth period—with a little extra, $1.22.

The factors presented in Exhibits 7.4 and 7.5 can be used to approximate the number of periods. The approach is similar to the way we approximated the interest rate. The compounding factor in this example is 2.0000 and the discounting factor is 0.5000 (that is, FV/PV = 2.0000 and PV/FV = 0.5000). Using Exhibit 7.4, following down the column corresponding to the interest rate of 6%, the compound factor closest to 2.0000 is for 12 periods. Likewise, using Exhibit 7.5, following down the column corresponding to the interest rate of 6%, the discount factor closest to 0.5000 is for 12 periods.

THE TIME VALUE OF A SERIES OF CASH FLOWS

Managers regularly need to determine the present or future value of a *series* of cash flows rather than simply a single cash flow. The principles of determining the future value or present value of a series of cash flows are the same as for a single cash flow, yet the math becomes a bit more cumbersome.

Suppose that the following deposits are made in a Thrifty Savings and Loan account paying 5% interest, compounded annually:

[1] *e* is approximately equal to 2.718. The natural logarithm function can be found on most calculators, usually indicated by "ln".

Time when Deposit is Made	Amount of Deposit
Today	$1,000
At the end of the first year	2,000
At the end of the second year	1,500

What is the balance in the savings account at the end of the second year if no withdrawals are made and interest is paid annually?

Let's simplify any problem like this by referring to today as the end of period 0, and identifying the end of the first and each successive period as 1, 2, 3, and so on. Represent each end-of-period cash flow as "*CF*" with a subscript specifying the period to which it corresponds. Thus, CF_0 is a cash flow today, CF_{10} is a cash flow at the end of period 10, and CF_{25} is a cash flow at the end of period 25, and so on.

Representing the information in our example using cash flow and period notation:

Period	Cash Flow	End of Period Cash Flow
0	CF_0	$1,000
1	CF_1	$2,000
2	CF_2	$1,500

The future value of the series of cash flows at the end of the second period is calculated as follows:

Period	End of Period Cash Flow	Number of Periods Interest is Earned	Compounding Factor	Future Value
0	$1,000	2	1.1025	$1,102.50
1	2,000	1	1.0500	2,100.00
2	1,500	0	1.0000	1,500.00
				$4,702.50

The last cash flow, $1,500, was deposited at the very end of the second period—the point of time at which we wish to know the future value of the series. Therefore, this deposit earns no interest. In more formal terms, its future value is precisely equal to its present value.

Today, the end of period 0, the balance in the account is $1,000 since the first deposit is made but no interest has been earned. At the end of period 1, the balance in the account is $3,050, made up of three parts:

1. the first deposit, \$1,000
2. \$50 interest on the first deposit
3. the second deposit, \$2,000

The balance in the account at the end of period 2 is \$4,702.50, made up of five parts:

1. the first deposit, \$1,000
2. the second deposit, \$2,000
3. the third deposit, \$1,500
4. \$102.50 interest on the first deposit, \$50 earned at the end of the first period, \$52.50 more earned at the end of the second period
5. \$100 interest earned on the second deposit at the end of the second period

These cash flows can also be represented in a time line. A *time line* is used to help graphically depict and sort out each cash flow in a series. The time line for this example is shown in Exhibit 7.6. From this example, you can see that the future value of the entire series is the sum of each of the compounded cash flows comprising the series. In much the same way, we can determine the future value of a series comprising any number of cash flows. And if we need to, we can determine the future value of a number of cash flows before the end of the series.

For example, suppose you are planning to deposit \$1,000 today and at the end of each year for the next ten years in a savings account paying 5% interest annually. If you want to know the future value of this series after four years, you compound each cash flow for the number of years it takes to reach four years. That is, you compound the first cash flow over four years, the second cash flow over three years, the third over two years, the fourth over one year, and the fifth you don't compound at all because you will have just deposited it in the bank at the end of the fourth year.

EXHIBIT 7.6 Time Line for the Future Value of a Series of Uneven Cash Flows Deposited to Earn 5% Compounded Interest Per Period

End of period	0		1	2
Time				
Cash flows	$CF_0 = \$1{,}000.00$		$CF_1 = \$2{,}000.00$	$CF_2 = \$1{,}500.00$
			➥ $\$2{,}000.00(1.05) =$	2,100.00
		➥ $\$1{,}000.00(1.05)^2 =$		1,102.50
				$FV = \$4{,}702.50$

To determine the present value of a series of future cash flows, each cash flow is discounted back to the present, where the beginning of the first period, today, is designated as 0. As an example, consider the Thrifty Savings & Loan problem from a different angle. Instead of calculating what the deposits and the interest on these deposits will be worth in the future, let's calculate the present value of the deposits. The present value is what these future deposits are worth today.

In the series of cash flows of $1,000 today, $2,000 at the end of period 1, and $1,500 at the end of period 2, each are discounted to the present, 0, as follows:

Period	End of Period Cash Flow	Number of Periods of Discounting	Discount Factor	Present Value
0	$1,000	0	1.00000	$1,000.00
1	$2,000	1	0.95238	1,904.76
2	$1,500	2	0.90703	1,360.54
				FV = $4,265.30

The present value of the series is the sum of the present value of these three cash flows, $4,265.30. For example, the $1,500 cash flow at the end of period 2 is worth $1,428.57 at the end of the first period and is worth $1,360.54 today.

The present value of a series of cash flows can be represented in notation form as:

$$PV = CF_0\left(\frac{1}{1+i}\right)^0 + CF_1\left(\frac{1}{1+i}\right)^1 + CF_2\left(\frac{1}{1+i}\right)^2 + \ldots + CF_N\left(\frac{1}{1+i}\right)^N$$

For example, if there are cash flows today and at the end of periods 1 and 2, today's cash flow is not discounted, the first period cash flow is discounted one period, and the second period cash flow is discounted two periods.

We can represent the present value of a series using summation notation as shown below:

$$PV = \sum_{t=0}^{N} CF_t\left(\frac{1}{1+i}\right)^t \qquad (7\text{-}7)$$

This equation tells us that the present value of a series of cash flows is the sum of the products of each cash flow and its corresponding discount factor.

We can also use the cash flow program in a financial calculator to solve for the present value of an uneven series of cash flows:

Hewlett-Packard 10B	Hewlett-Packard 12C	Hewlett-Packard 17B	Texas Instruments BA-II Plus
1000 CFj	1000 CF_0	FIN CFLO	CF
2000 CFj	2000 CFj	1000 INPUT	1000 ENTER
1500 CFj	1500 CFj	1 INPUT	↑ 1 ENTER
5 I/YR	5 i	2000 INPUT	↑ 2000 ENTER
■ NPV	f NPV	1 INPUT	↑ 1 ENTER
		1500 INPUT	↑ 1500 ENTER
		1 INPUT	↑ 1 ENTER
		CALC	CPT NPV 5 I/Y ↑ CPT
		5 I%	
		NPV	

Shortcuts: Annuities

There are valuation problems that require us to evaluate a series of level cash flows—each cash flow is the same amount as the others—received at regular intervals. Let's suppose you expect to deposit \$2,000 at the end of each of the *next* four years (2000, 2001, 2002, and 2003) in an account earning 8% compounded interest. How much will you have available at the end of 2003, the fourth year?

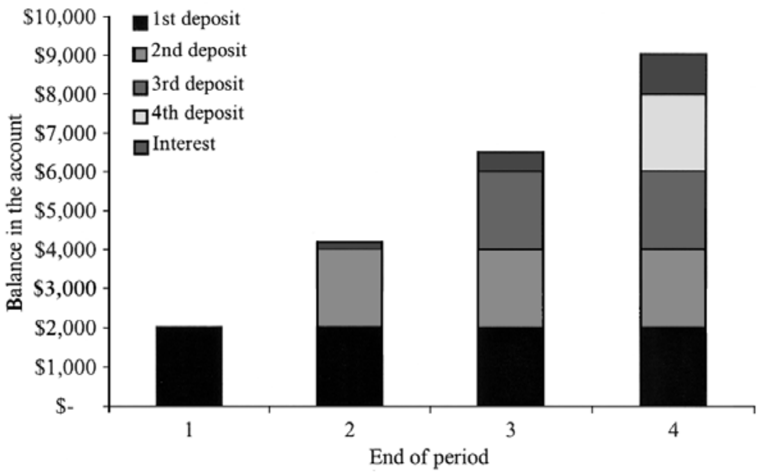
As we just did for the future value of a series of uneven cash flows, we can calculate the future value (as of the end of 2003) of each \$2,000 deposit, compounding interest at 8%:

$$\begin{aligned} FV &= \$2{,}000(1+0.08)^3 + \$2{,}000(1+0.08)^2 + \$2{,}000(1+0.08)^1 \\ &\quad + \$2{,}000(1+0.08)^0 \\ &= \$2{,}519.40 + \$2{,}332.80 + \$2{,}160.00 + \$2{,}000 = \$9{,}012.20 \end{aligned}$$

Exhibit 7.7 shows the contribution of each deposit and the accumulated interest at the end of each period.

- At the end of 1998, there is \$2,000.00 in the account since you have just made your first deposit.
- At the end of 1999, there is \$4,160.00 in the account: two deposits of \$2,000 each, plus \$160 interest (8% of \$2,000).
- At the end of 2000, there is \$6,492.80 in the account: three deposits of \$2,000.00 each, plus accumulated interest of \$492.80 [\$160.00 + (0.08 × \$4,000) + (0.08 × \$160)].

EXHIBIT 7.7 **Balance in an Account in which Deposits of $2,000 Each are Made Each Year. The Balance in the Account Earns 8%.**



- At the end of the fourth year, you would have $9,012.20 available: four deposits of $2,000 each, plus $1,012.20 accumulated interest [$160.00 + $492.80 + (0.08 × $6,000) + (0.08 × ($160.00 + 492.80)].

Notice that in our calculations, each deposit of $2,000 is multiplied by a factor that corresponds to an interest rate of 8% and the number of periods that the deposit has been in the savings account. Since the deposit of $2,000 is common to each multiplication, we can simplify the math a bit by multiplying the $2,000 by the sum of the factors to get the same answer:

$$FV = \$2{,}000(1.2597) + \$2{,}000(1.1664) + \$2{,}000(1.0800) + \$2{,}000(1.0000) = \$9{,}012.20$$

A series of cash flows of equal amount, occurring at even intervals is referred to as an annuity. Determining the value of an annuity, whether compounding or discounting, is simpler than valuing uneven cash flows. If each CF_t is equal (that is, all the cash flows are the same value) and the first one occurs at the end of the first period ($t = 1$), we can express the future value of the series as:

$$FV = \sum_{t=1}^{N} CF_t(1+i)^{N-t}$$

N is last and t indicates the time period corresponding to a particular cash flow, starting at 1 for an ordinary annuity. Since CF_t is shorthand for: CF_1, CF_2, CF_3, . . ., CF_N, and we know that $CF_1 = CF_2 = CF_3 = \ldots CF_N$, let's make things simple by using CF to indicate the same value for the periodic cash flows. Rearranging the future value equation we get:

$$FV = CF \sum_{t=1}^{N} (1+i)^{N-t} \tag{7-8}$$

This equation tells us that the future value of a level series of cash flows, occurring at regular intervals beginning one period from today (notice that t starts at 1), is equal to the amount of cash flow multiplied by the sum of the compound factors.

In a like manner, the equation for the present value of a series of level cash flows beginning after one period simplifies to:

$$PV = \sum_{t=1}^{N} CF_t \left(\frac{1}{1+i}\right)^t = CF \sum_{t=1}^{N} \left(\frac{1}{1+i}\right)^t$$

or

$$PV = CF \sum_{t=1}^{N} \frac{1}{(1+i)^t} \tag{7-9}$$

This equation tells us that the present value of an annuity is equal to the amount of one cash flow multiplied by the sum of the discount factors.

Equations (7-8) and (7-9) are the valuation—future and present value—formulas for an *ordinary annuity.* An ordinary annuity is a special form of annuity, where the first cash flow occurs *at the end of the first period*.

To calculate the future value of an annuity we multiply the amount of the annuity (that is, the amount of one periodic cash flow) by the sum of the compound factors. The sum of these compounding factors for a given interest rate, i, and number of periods, N, is referred to as the *future value annuity factor.* Likewise, to calculate the present value of an annuity we multiply one cash flow of the annuity by the sum of the discount factors. The sum of the discounting factors for a given i and N is referred to as the *present value annuity factor.*

Suppose you wish to determine the future value of a series of deposits of \$1,000, deposited each year in the No Fault Vault Bank for five years, with the first deposit made at the end of the first year. If the NFV

Bank pays 5% interest on the balance in the account at the end of each year and no withdrawals are made, what is the balance in the account at the end of the five years?

Each $1,000 is deposited at a different time, so it contributes a different amount to the future value. For example, the first deposit accumulates interest for four periods, contributing $1,215.50 to the future value (at the end of period 5), whereas the last deposit contributes only $1,000 to the future value since it is deposited at exactly the point in time when we are determining the future value, hence there is no interest on this deposit.

The future value of an annuity is the sum of the future value of each deposit:

Period	Amount of Deposit	Number of Periods Interest is Earned	Compounding Factor	Future Value
1	$1,000	4	1.2155	$1,215.50
2	1,000	3	1.1576	1,157.60
3	1,000	2	1.1025	1,102.50
4	1,000	1	1.0500	1,050.00
5	1,000	0	1.0000	1,000.00
Total			5.5256	$5,525.60

The future value of the series of $1,000 deposits, with interest compounded at 5%, is $5,525.60. Since we know the value of one of the level period flows is $1,000, and the future value of the annuity is $5,525.60, and looking at the sum of the individual compounding factors, 5.5256, we can see that there is an easier way to calculate the future value of an annuity. If the sum of the individual compounding factors for a specific interest rate and a specific number of periods were available, all we would have to do is multiply that sum by the value of one cash flow to get the future value of the entire annuity.

In this example, the shortcut is multiplying the amount of the annuity, $1,000, by the sum of the compounding factors, 5.5256:

$$FV = \$1{,}000 \times 5.5256 = \$5{,}525.60$$

For large numbers of periods, summing the individual factors can be a bit clumsy—with possibilities of errors along the way. An alternative formula for the sum of the compound factors—that is, the future value annuity factor—is:

$$\text{Future value annuity factor} = \frac{(1+i)^N - 1}{i} \tag{7-10}$$

In the last example, $N = 5$ and $i = 5\%$:

$$\text{Future value annuity factor} = \frac{(1 + 0.05)^5 - 1}{0.05} = \frac{1.2763 - 1.000}{0.05} = 5.5256$$

Let's use the long method to find the present value of the series of five deposits of $1,000 each, with the first deposit at the end of the first period. Then we'll do it using the shortcut method. The calculations are similar to the future value of an ordinary annuity, except we are taking each deposit back in time, instead of forward:

Period	Amount of Deposit	Discounting Periods	Discounting Factor	Present Value
1	$1,000	1	0.9524	$952.40
2	1,000	2	0.9070	907.00
3	1,000	3	0.8638	863.80
4	1,000	4	0.8227	822.70
5	1,000	5	0.7835	783.50
Total			4.3294	$4,329.40

The present value of this series of five deposits is $4,329.40.

This same value is obtained by multiplying the annuity amount of $1,000 by the sum of the discounting factors, 4.3294:

$$PV = \$1{,}000 \times 4.3294 = \$4{,}329.40$$

Another, more convenient way of solving for the present value of an annuity is to rewrite the factor as:

$$\text{Present value annuity factor} = \frac{1 - \dfrac{1}{(1 + i)^N}}{i} \tag{7-11}$$

If there are many discount periods, this formula is a bit easier to calculate. In our last example,

$$\text{Present value annuity factor} = \frac{\left[1 - \dfrac{1}{(1 + 0.05)^5}\right]}{0.05} = \frac{1 - 0.7835}{0.05} = 4.3295$$

which is different from the sum of the factors, 4.3294, due to rounding.

We can turn this present value of an annuity problem around to look at it from another angle. Suppose you borrow \$4,329.40 at an interest rate of 5% per period and are required to pay back this loan in five installments ($N = 5$): one payment per period for five periods, starting one period from now. The payments are determined by equating the present value with the product of the cash flow and the sum of the discount factors:

$$\begin{aligned} PV &= CF(\text{sum of discount factors}) \\ &= CF\sum_{t=1}^{5}\frac{1}{(1+0.05)^t} \\ &= CF(0.9524 + 0.9070 + 0.8638 + 0.8227 + 0.7835) \\ &= CF(4.3294) \end{aligned}$$

substituting the known present value,

$$\$4{,}329.40 = CF(4.3294)$$

and rearranging to solve for the payment:

$$CF = \$4{,}329.40/4.3290 = \$1{,}000.00$$

We can convince ourselves that five installments of \$1,000 each can pay off the loan of \$4,329.40 by carefully stepping through the calculation of interest and the reduction of the principal:

Beginning of Periods Loan Balance	Payment	Interest (Principal × 5%)	Reduction in Loan Balance (Payment – Interest)	End of Period Loan Balance
\$4,329.40	\$1,000.00	\$216.47	\$783.53	\$3,545.87
3,545.87	1,000.00	177.29	822.71	2,723.16
2,723.16	1,000.00	136.16	863.84	1,859.32
1,859.32	1,000.00	92.97	907.03	952.29
952.29	1,000.00	47.61	952.29*	0

* The small difference between calculated reduction (\$952.38) and reported reduction is due to rounding differences.

For example, the first payment of \$1,000 is used to: (1) pay interest on the loan at 5% (\$4,329.40 × 0.05 = \$216.47) and (2) pay down the principal or loan balance (\$1,000.00 – 216.47 = \$783.53 paid off). Each successive payment pays off a greater amount of the loan—as the principal amount of the loan is reduced, less of each payment goes to paying off interest and more goes to reducing the loan principal. This analysis of the repayment of a loan is referred to as loan amortization. *Loan amortization* is the repayment of a loan with equal payments, over a specified period of time. As we can see from the example of borrowing $4,329.40, each payment can be broken down into its interest and principal components.

Shortcuts: Tables and Calculators

Annuity factor tables simplify the task of valuing annuities. Exhibit 7.8 is a table of future value of annuity factors for interest rates and periods from 1% to 20% and from 1 to 20 payments, respectively. Exhibit 7.9 is the corresponding table for present value of annuity factors. For example, the future value annuity factor from Exhibit 7.8 for five periodic payments and an interest rate of 10% is 6.1051 and the present value annuity factor from Exhibit 7.9 for five periodic payments and an interest rate of 10% is 3.7908; the factor for 10 periodic payments and an interest rate of 5% is 7.7217.

Like the tables of compound and discount factors, we can use the annuity factor tables to solve for N (given i and the appropriate factor) or for i (given N and the appropriate factor). Suppose that we deposit $1,000 at the end of each year in an account that pays 6% compounded annual interest. How many years must we make deposits in the account to have a balance of $7,000? We can work this using the future value annuity factor in Exhibit 7.8. We know that the future value is $7,000, the interest rate is 6%, and the periodic payments are $1,000. We substitute this information into the formula for the future value of an annuity and solve for the future value annuity factor:

$$FV = CF\left[\sum_{t=1}^{N}(1+i)^{N-t}\right]$$

where the term in brackets is the future value annuity factor. That is,

$$\text{Future value annuity factor} = \sum_{t=1}^{N}(1+i)^{N-t}$$

Substituting the known future value and the known value of CF:

$$\$7{,}000 = \$1{,}000\ (\text{future value annuity factor})$$

we know that the future value annuity factor is 7.0000.

EXHIBIT 7.8 Table of Factors for the Future Value of a $1 Annuity

Number of	Compounding Rate											
Cash Flows	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500	2.0600	2.0700	2.0800	2.0900	2.1000	2.1100	2.1200
3	3.0301	3.0604	3.0909	3.1216	3.1525	3.1836	3.2149	3.2464	3.2781	3.3100	3.3421	3.3744
4	4.0604	4.1216	4.1836	4.2465	4.3101	4.3746	4.4399	4.5061	4.5731	4.6410	4.7097	4.7793
5	5.1010	5.2040	5.3091	5.4163	5.5256	5.6371	5.7507	5.8666	5.9847	6.1051	6.2278	6.3528
6	6.1520	6.3081	6.4684	6.6330	6.8019	6.9753	7.1533	7.3359	7.5233	7.7156	7.9129	8.1152
7	7.2135	7.4343	7.6625	7.8983	8.1420	8.3938	8.6540	8.9228	9.2004	9.4872	9.7833	10.0890
8	8.2857	8.5830	8.8923	9.2142	9.5491	9.8975	10.2598	10.6366	11.0285	11.4359	11.8594	12.2997
9	9.3685	9.7546	10.1591	10.5828	11.0266	11.4913	11.9780	12.4876	13.0210	13.5795	14.1640	14.7757
10	10.4622	10.9497	11.4639	12.0061	12.5779	13.1808	13.8164	14.4866	15.1929	15.9374	16.7220	17.5487
11	11.5668	12.1687	12.8078	13.4864	14.2068	14.9716	15.7836	16.6455	17.5603	18.5312	19.5614	20.6546
12	12.6825	13.4121	14.1920	15.0258	15.9171	16.8699	17.8885	18.9771	20.1407	21.3843	22.7132	24.1331
13	13.8093	14.6803	15.6178	16.6268	17.7130	18.8821	20.1406	21.4953	22.9534	24.5227	26.2116	28.0291
14	14.9474	15.9739	17.0863	18.2919	19.5986	21.0151	22.5505	24.2149	26.0192	27.9750	30.0949	32.3926
15	16.0969	17.2934	18.5989	20.0236	21.5786	23.2760	25.1290	27.1521	29.3609	31.7725	34.4054	37.2797
16	17.2579	18.6393	20.1569	21.8245	23.6575	25.6725	27.8881	30.3243	33.0034	35.9497	39.1899	42.7533
17	18.4304	20.0121	21.7616	23.6975	25.8404	28.2129	30.8402	33.7502	36.9737	40.5447	44.5008	48.8837
18	19.6147	21.4123	23.4144	25.6454	28.1324	30.9057	33.9990	37.4502	41.3013	45.5992	50.3959	55.7497
19	20.8109	22.8406	25.1169	27.6712	30.5390	33.7600	37.3790	41.4463	46.0185	51.1591	56.9395	63.4397
20	22.0190	24.2974	26.8704	29.7781	33.0660	36.7856	40.9955	45.7620	51.1601	57.2750	64.2028	72.0524
21	23.2392	25.7833	28.6765	31.9692	35.7193	39.9927	44.8652	50.4229	56.7645	64.0025	72.2651	81.6987
22	24.4716	27.2990	30.5368	34.2480	38.5052	43.3923	49.0057	55.4568	62.8733	71.4027	81.2143	92.5026
23	25.7163	28.8450	32.4529	36.6179	41.4305	46.9958	53.4361	60.8933	69.5319	79.5430	91.1479	104.6029
24	26.9735	30.4219	34.4265	39.0826	44.5020	50.8156	58.1767	66.7648	76.7898	88.4973	102.1742	118.1552
25	28.2432	32.0303	36.4593	41.6459	47.7271	54.8645	63.2490	73.1059	84.7009	98.3471	114.4133	133.3339
26	29.5256	33.6709	38.5530	44.3117	51.1135	59.1564	68.6765	79.9544	93.3240	109.1818	127.9988	150.3339
27	30.8209	35.3443	40.7096	47.0842	54.6691	63.7058	74.4838	87.3508	102.7231	121.0999	143.0786	169.3740
28	32.1291	37.0512	42.9309	49.9676	58.4026	68.5281	80.6977	95.3388	112.9682	134.2099	159.8173	190.6989
29	33.4504	38.7922	45.2189	52.9663	62.3227	73.6398	87.3465	103.9659	124.1354	148.6309	178.3972	214.5828
30	34.7849	40.5681	47.5754	56.0849	66.4388	79.0582	94.4608	113.2832	136.3075	164.4940	199.0209	241.3327

EXHIBIT 7.9 Table of Factors for the Present Value of a $1 Annuity

Number of	Discount Rate											
Cash Flows	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%
1	0.9901	0.9804	0.9709	0.9615	0.9524	0.9434	0.9346	0.9259	0.9174	0.9091	0.9009	0.8929
2	1.9704	1.9416	1.9135	1.8861	1.8594	1.8334	1.8080	1.7833	1.7591	1.7355	1.7125	1.6901
3	2.9410	2.8839	2.8286	2.7751	2.7232	2.6730	2.6243	2.5771	2.5313	2.4869	2.4437	2.4018
4	3.9020	3.8077	3.7171	3.6299	3.5460	3.4651	3.3872	3.3121	3.2397	3.1699	3.1024	3.0373
5	4.8534	4.7135	4.5797	4.4518	4.3295	4.2124	4.1002	3.9927	3.8897	3.7908	3.6959	3.6048
6	5.7955	5.6014	5.4172	5.2421	5.0757	4.9173	4.7665	4.6229	4.4859	4.3553	4.2305	4.1114
7	6.7282	6.4720	6.2303	6.0021	5.7864	5.5824	5.3893	5.2064	5.0330	4.8684	4.7122	4.5638
8	7.6517	7.3255	7.0197	6.7327	6.4632	6.2098	5.9713	5.7466	5.5348	5.3349	5.1461	4.9676
9	8.5660	8.1622	7.7861	7.4353	7.1078	6.8017	6.5152	6.2469	5.9952	5.7590	5.5370	5.3282
10	9.4713	8.9826	8.5302	8.1109	7.7217	7.3601	7.0236	6.7101	6.4177	6.1446	5.8892	5.6502
11	10.3676	9.7868	9.2526	8.7605	8.3064	7.8869	7.4987	7.1390	6.8052	6.4951	6.2065	5.9377
12	11.2551	10.5753	9.9540	9.3851	8.8633	8.3838	7.9427	7.5361	7.1607	6.8137	6.4924	6.1944
13	12.1337	11.3484	10.6350	9.9856	9.3936	8.8527	8.3577	7.9038	7.4869	7.1034	6.7499	6.4235
14	13.0037	12.1062	11.2961	10.5631	9.8986	9.2950	8.7455	8.2442	7.7862	7.3667	6.9819	6.6282
15	13.8651	12.8493	11.9379	11.1184	10.3797	9.7122	9.1079	8.5595	8.0607	7.6061	7.1909	6.8109
16	14.7179	13.5777	12.5611	11.6523	10.8378	10.1059	9.4466	8.8514	8.3126	7.8237	7.3792	6.9740
17	15.5623	14.2919	13.1661	12.1657	11.2741	10.4773	9.7632	9.1216	8.5436	8.0216	7.5488	7.1196
18	16.3983	14.9920	13.7535	12.6593	11.6896	10.8276	10.0591	9.3719	8.7556	8.2014	7.7016	7.2497
19	17.2260	15.6785	14.3238	13.1339	12.0853	11.1581	10.3356	9.6036	8.9501	8.3649	7.8393	7.3658
20	18.0456	16.3514	14.8775	13.5903	12.4622	11.4699	10.5940	9.8181	9.1285	8.5136	7.9633	7.4694
21	18.8570	17.0112	15.4150	14.0292	12.8212	11.7641	10.8355	10.0168	9.2922	8.6487	8.0751	7.5620
22	19.6604	17.6580	15.9369	14.4511	13.1630	12.0416	11.0612	10.2007	9.4424	8.7715	8.1757	7.6446
23	20.4558	18.2922	16.4436	14.8568	13.4886	12.3034	11.2722	10.3711	9.5802	8.8832	8.2664	7.7184
24	21.2434	18.9139	16.9355	15.2470	13.7986	12.5504	11.4693	10.5288	9.7066	8.9847	8.3481	7.7843
25	22.0232	19.5235	17.4131	15.6221	14.0939	12.7834	11.6536	10.6748	9.8226	9.0770	8.4217	7.8431
26	22.7952	20.1210	17.8768	15.9828	14.3752	13.0032	11.8258	10.8100	9.9290	9.1609	8.4881	7.8957
27	23.5596	20.7069	18.3270	16.3296	14.6430	13.2105	11.9867	10.9352	10.0266	9.2372	8.5478	7.9426
28	24.3164	21.2813	18.7641	16.6631	14.8981	13.4062	12.1371	11.0511	10.1161	9.3066	8.6016	7.9844
29	25.0658	21.8444	19.1885	16.9837	15.1411	13.5907	12.2777	11.1584	10.1983	9.3696	8.6501	8.0218
30	25.8077	22.3965	19.6004	17.2920	15.3725	13.7648	12.4090	11.2578	10.2737	9.4269	8.6938	8.0552

Examining the future value annuity table for the 6% interest rate, we don't find a factor of 7.0000, but we do see one very close, 6.9753, which corresponds to $N = 6$ payments. In this example, six payments of $1,000 each year produces $7,000 at the end of the sixth year.

We can also use financial calculators to do the work. The present value of the series of five $1,000 cash flows, using a 5% interest rate, is:

Hewlett-Packard 10B	Hewlett-Packard 12C	Hewlett-Packard 17B	Texas Instruments BA-II Plus
1000 PMT	1000 PMT	FIN TVM	1000 PMT
5 I/YR	5 i	5000 PMT	5 I
5 N	5 n	5 I%YR	5 N
PV	PV	5 N	PV
		PV	

The future value of an annuity is calculated in a like manner.

VALUING CASH FLOWS WITH DIFFERENT TIME PATTERNS

Valuing a Perpetual Stream of Cash Flows

There are some circumstances where cash flows are expected to continue forever. For example, a corporation may promise to pay dividends on preferred stock forever, or, a company may issue a bond that pays interest every six months, forever. How do you value these cash flow streams? Recall that when we calculated the present value of an annuity, we took the amount of one cash flow and multiplied it by the sum of the discount factors that corresponded to the interest rate and number of payments. But what if the number of payments extends forever—into infinity?

A series of cash flows that occur at regular intervals, forever, is a *perpetuity*. Valuing a perpetual cash flow stream is just like valuing an ordinary annuity. It looks like this:

$$PV = CF_1\left(\frac{1}{1+i}\right)^1 + CF_2\left(\frac{1}{1+i}\right)^2 + CF_3\left(\frac{1}{1+i}\right)^3 + \ldots + CF_\infty\left(\frac{1}{1+i}\right)^\infty$$

Simplifying, recognizing that the cash flows CF_t are the same in each period, and using summation notation,

$$PV = CF \sum_{t=1}^{\infty} \left(\frac{1}{1+i} \right)^t$$

As the number of discounting periods approaches infinity, the summation approaches $1/i$. To see why, consider the present value annuity factor for an interest rate of 10%, as the number of payments goes from 1 to 200:

Number of Discounting Periods, *N*	Present Value Annuity Factor
1	0.9091
10	6.1446
40	9.7791
100	9.9993
200	9.9999

For greater numbers of payments, the factor approaches 10, or 1/0.10. Therefore, the present value of a perpetual annuity is very close to:

$$PV = \frac{CF}{i} \tag{7-12}$$

Suppose you are considering an investment that promises to pay $100 each period forever, and the interest rate you can earn on alternative investments of similar risk is 5% per period. What are you willing to pay today for this investment?

$$PV = \frac{\$100}{0.05} = \$2{,}000$$

Therefore, you would be willing to pay $2,000 today for this investment to receive, in return, the promise of $100 each period forever.

Let's look at the value of a perpetuity another way. Suppose that you are given the opportunity to purchase an investment for $5,000 that promises to pay $50 at the end of every period forever. What is the periodic interest per period—the return—associated with this investment?

We know that the present value is PV = $5,000 and the periodic, perpetual payment is CF = $50. Inserting these values into the formula for the present value of a perpetuity:

$$\$5{,}000 = \frac{\$50}{i}$$

Solving for i,

$$i = \frac{\$50}{\$5{,}000} = 0.01 \text{ or } 1\% \text{ per period}$$

Therefore, an investment of $5,000 that generates $50 per period provides 1% compounded interest per period.

Valuing an Annuity Due

The ordinary annuity cash flow analysis assumes that cash flows occur at the end of each period. However, there is another fairly common cash flow pattern in which level cash flows occur at regular intervals, but the first cash flow occurs immediately. This pattern of cash flows is called an *annuity due*. For example, if you win the Florida Lottery Lotto grand prize, you will receive your winnings in 20 installments (after taxes, of course). The 20 installments are paid out annually, beginning immediately. The lottery winnings are therefore an annuity due.

Like the cash flows we have considered thus far, the future value of an annuity due can be determined by calculating the future value of each cash flow and summing them. And, the present value of an annuity due is determined in the same way as a present value of any stream of cash flows.

Let's consider first an example of the future value of an annuity due, comparing the values of an ordinary annuity and an annuity due, each comprising three cash flows of $500, compounded at the interest rate of 4% per period. The calculation of the future value of both the ordinary annuity and the annuity due at the end of three periods is:

Ordinary annuity	*Annuity due*
$FV = \$500 \sum_{t=1}^{3} (1 + 0.04)^{3-t}$	$FV_{\text{due}} = \$500 \sum_{t=1}^{3} (1 + 0.04)^{3-t+1}$

The future value of each of the $500 payments in the annuity due calculation is compounded for *one more period* than for the ordinary annuity. For example, the first deposit of $500 earns interest for two periods in the ordinary annuity situation [$\$500\ (1 + 0.04)^2$], whereas the first $500 in the annuity due case earns interest for three periods [$\$500\ (1 + 0.04)^3$].

In general terms,

$$FV_{\text{due}} = CF \sum_{t=1}^{N} (1 + i)^{N-t+1} \tag{7-13}$$

which is equal to the future value of an ordinary annuity multiplied by a factor of $1 + i$:

$$FV_{due} = CF[\text{Future value annuity factor (ordinary) for } N \text{ and } i](1 + i)$$

The present value of the annuity due is calculated in a similar manner, adjusting the ordinary annuity formula for the different number of discount periods:

$$PV_{due} = CF\sum_{t=1}^{N} \frac{1}{(1+i)^{t-1}} \tag{7-14}$$

Since the cash flows in the annuity due situation are each discounted one less period than the corresponding cash flows in the ordinary annuity, the present value of the annuity due is greater than the present value of the ordinary annuity for an equivalent amount and number of cash flows. Like the future value an annuity due, we can specify the present value in terms of the ordinary annuity factor:

$$PV_{due} = CF[\text{Present value annuity factor (ordinary) for } N \text{ and } i](1 + i)$$

Financial calculators make your calculations easier by automatically adjusting the present or future value annuity factor if you specify the "begin" or "due" mode. For example, if you are using the HP12C calculator and want to calculate the future value of $500 to be received at the beginning of each of three periods, you first put the calculator in the annuity due mode [$\boxed{\text{g}}$ $\boxed{\text{BEG}}$], then specify the cash flow (the $500), the number of payments (3), and the interest rate (4%).

Valuing a Deferred Annuity

A *deferred annuity* has a stream of cash flows of equal amounts at regular periods starting at some time *after* the end of the first period. When we calculated the present value of an annuity, we brought a series of cash flows back to the beginning of the first period—or, equivalently the end of the period 0. With a deferred annuity, we determine the present value of the ordinary annuity and then discount this present value to an earlier period.

To illustrate the calculation of the present value of an annuity due, suppose you deposit $20,000 per year in an account for 10 years, starting today, for a total of 10 deposits. What will be the balance in the account at the end of 10 years if the balance in the account earns 5% per year? The future value of this annuity due is:

$$FV_{\text{due, 10}} = \$20{,}000 \sum_{t=1}^{10} (1 + 0.05)^{10 - t + 1}$$
$$= \$20{,}000 \begin{pmatrix} \text{Future value annuity factor (ordinary)} \\ \text{for 10 periods and 5\%} \end{pmatrix} (1 + 0.05)$$
$$= \$20{,}000(12.5779)(1 + 0.05) = \$264{,}135.74$$

Suppose you want to deposit an amount today in an account such that you can withdraw \$5,000 per year for four years, with the first withdrawal occurring five years from today. We can solve this problem in two steps:

Step 1: Solve for the present value of the withdrawals.
Step 2: Discount this present value to the present.

The first step requires determining the present value of a four-cash flow ordinary annuity of \$5,000. This calculation provides the present value as of the end of the fourth year (one period prior to the first withdrawal):

$$PV_4 = \$5{,}000 \sum_{t=1}^{4} \frac{1}{(1 + 0.04)^t}$$
$$= \$5{,}000(\text{present value annuity factor } N = 4,\ i = 4\%)$$
$$= \$18{,}149.48$$

This means that there must be a balance in the account of \$18,149.48 at the end of the fourth period to satisfy the withdrawals of \$5,000 per year for four years.

The second step requires discounting the \$18,149.48—the savings goal—to the present, providing the deposit today that produces the goal:

$$PV_0 = \frac{\$18{,}149.48}{(1 + 0.04)^4} = \$15{,}514.25$$

The balance in the account throughout the entire eight-year period is shown in Exhibit 7.10, with the balance indicated both before and after the \$5,000 withdrawals.

Let's look at a more complex deferred annuity. Consider making a series of deposits, beginning today, to provide for a steady cash flow beginning at some future time period. If interest is earned at a rate of 4% compounded per year, what amount must be deposited in a savings account each year for four years, starting today, so that \$1,000 may be withdrawn each year for five years, beginning five years from today? As with any deferred annuity, we need to perform this calculation in steps:

Step 1: Calculate the present value of the \$1,000 per year five-year ordinary annuity as of the end of the fourth year:

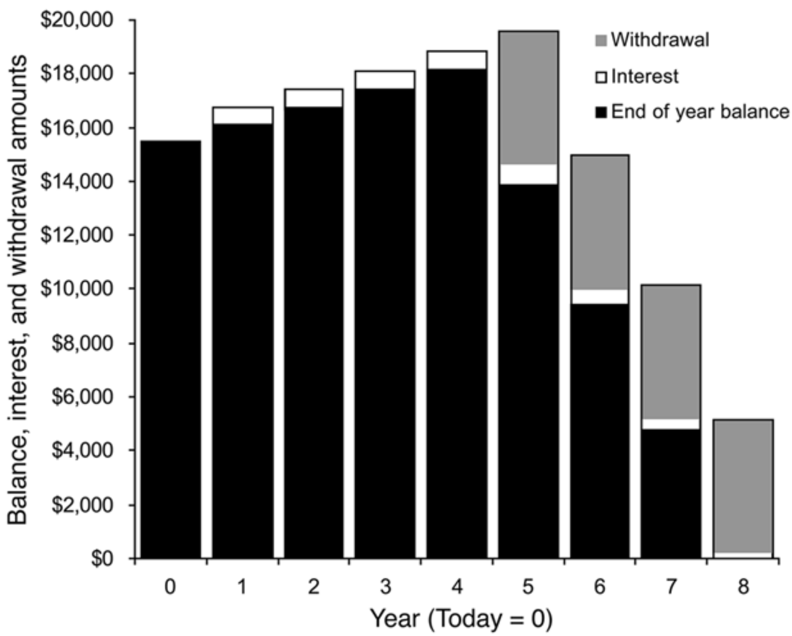
The present value of the annuity deferred to the end of the fourth period is

$$PV_4 = \$1{,}000 \sum_{t=1}^{5} \frac{1}{(1+0.04)^t} = \$1{,}000(4.4518) = \$4{,}451.80$$

Therefore, there must be \$4,451.80 in the account at the end of the fourth year to permit five \$1,000 withdrawals at the end of each of the years 5, 6, 7, 8, and 9.

Step 2: Calculate the cash flow needed to arrive at the future value of that annuity due comprising four annual deposits earning 4% compounded interest, starting today.

EXHIBIT 7.10 **Balance in the Account that Requires a Deposit Today (Year 0) that Permits Withdrawals of \$5,000 Each Starting at the End of Year 5**



The present value of the annuity at the end of the fourth year, \$4,451.80, is the future value of the annuity due of four payments of an unknown amount. Using the formula for the future value of an annuity due,

$$\$4{,}451.80 = CF\sum_{t=1}^{4}(1+0.04)^{4-t+1} = CF(4.2465)(1.04)$$

and rearranging,

$$CF = \$4{,}451.80/4.4164 = \$1{,}008.02$$

Therefore, by depositing \$1,008.02 today and the same amount on the same date each of the next three years, we will have a balance in the account of \$4,451.80 at the end of the fourth period. With this period 4 balance, we will be able to withdraw \$1,000 at the end of the following five periods.

THE CALCULATION OF INTEREST RATES

A common problem in finance is comparing alternative financing or investment opportunities when the interest rates are specified in a way that makes it difficult to compare terms. The Truth in Savings Act requires institutions to provide the annual percentage yield for savings accounts. As a result of this law, consumers can compare the yields on different savings arrangements. But this law does not apply beyond savings accounts. One investment may pay 10% interest compounded semiannually, whereas another investment may pay 9% interest compounded daily. One financing arrangement may require interest compounding quarterly, whereas another may require interest compounding monthly. To compare investments or financing with different frequencies of compounding, we must first translate the stated interest rates into a common basis. There are two ways to convert interest rates stated over different time intervals so that they have a common basis: the annual percentage rate and the effective annual interest rate.

One obvious way to represent rates stated in various time intervals on a common basis is to express them in the same unit of time—so we annualize them. The annualized rate is the product of the stated rate of interest per compound period and the number of compounding periods in a year. Let i be the rate of interest per period and n be the number of compounding periods in a year. The annualized rate, also referred to as the *nominal interest rate* or the *annual percentage rate* (APR) is:

$$\text{APR} = i \times n$$

Consider the following example. Suppose the Lucky Break Loan Company has simple loan terms: Repay the amount borrowed, plus 50%, in six months. Suppose you borrow \$10,000 from Lucky. After six months, you must pay back the \$10,000 plus \$5,000. The annual percentage rate on financing with Lucky is the interest rate per period (50% for six months) multiplied by the number of compound periods in a year (two six-month periods in a year). For the Lucky Break financing arrangement:

$$\text{APR} = 0.50 \times 2 = 1.00 \text{ or } 100\% \text{ per year}$$

But what if you cannot pay Lucky back after six months? Lucky will let you off this time, but you must pay back the following at the end of the next six months:

- the \$10,000 borrowed,
- the \$5,000 interest from the first six months, and
- the 50% of interest on both the unpaid \$10,000 and the unpaid \$5,000 interest (\$15,000 (0.50) = \$7,500).

So, at the end of the year, knowing what is good for you, you pay off Lucky:

Amount of the original loan	\$10,000
Interest from first six months	5,000
Interest on second six months	7,500
Total payment at end of the year	\$22,500

Using the Lucky Break method of financing, you have to pay \$12,500 interest to borrow \$10,000 for one year's time. Because you have to pay \$12,500 interest to borrow \$10,000 over one year's time, you pay not 100% interest, but rather 125% interest per year (\$12,500/\$10,000 = 1.25 = 125%). What's going on here? It looks like the APR in the Lucky Break example ignores the compounding (interest on interest) that takes place after the first six months. And that's the way it is with all APRs. The APR ignores the effect of compounding. Therefore, this rate understates the true annual rate of interest if interest is compounded at any time prior to the end of the year. Nevertheless, APR is an acceptable method of disclosing interest on many lending arrangements, since it is easy to understand and simple to compute. However, because it ignores compounding, it is not the best way to convert interest rates to a common basis.

Effective versus Annualized Rates of Interest

Another way of converting stated interest rates to a common basis is the effective rate of interest. The effective annual rate (EAR) is the true economic return for a given time period—it takes into account the compounding of interest—and is also referred to as the effective rate of interest.

Using our Lucky Break example, we see that we must pay \$12,500 interest on the loan of \$10,000 for one year. Effectively, we are paying 125% annual interest. Thus, 125% is the effective annual rate of interest. In this example, we can easily work through the calculation of interest and interest on interest. But for situations where interest is compounded more frequently, we need a direct way to calculate the effective annual rate. We can calculate it by resorting once again to our basic valuation equation:

$$FV = PV\,(1 + i)^n$$

Next, we consider that a return is the change in the value of an investment over a period and an annual return is the change in value over a year. Using our basic valuation equation, the relative change in value is the difference between the future value and the present value, divided by the present value:

$$\text{EAR} = \frac{FV - PV}{PV} = \frac{PV(1 + i)^n}{PV}$$

Canceling PV from both the numerator and the denominator,

$$\text{EAR} = (1 + i)^n - 1 \tag{7-15}$$

Let's look how the EAR is affected by the compounding. Suppose that the Safe Savings and Loan promises to pay 6% interest on accounts, compounded annually. Since interest is paid once, at the end of the year, the effective annual return, EAR, is 6%. If the 6% interest is paid on a semiannual basis—3% every six months—the effective annual return is larger than 6% since interest is earned on the 3% interest earned at the end of the first six months. In this case, to calculate the EAR, the interest rate per compounding period—six months—is 0.03 (that is, 0.06/2) and the number of compounding periods in an annual period is 2:

$$\text{EAR} = (1 + 0.03)^2 - 1 = 1.0609 - 1 = 0.0609 \text{ or } 6.09\%$$

Extending this example to the case of quarterly compounding with a nominal interest rate of 6%, we first calculate the interest rate per period, i, and the number of compounding periods in a year, n:

i = 0.06/4 = 0.015 per quarter
n = 4 quarters in a year

The EAR is:

$$\text{EAR} = (1 + 0.015)^4 - 1 = 1.0614 - 1 = 0.0614 \text{ or } 6.14\%$$

As we saw earlier in this chapter, the extreme frequency of compounding is continuous compounding. Continuous compounding is when interest is compounded at the smallest possible increment of time. In continuous compounding, the rate per period becomes extremely small:

$$i = \frac{\text{APR}}{\infty}$$

And the number of compounding periods in a year, n, is infinite. The EAR is therefore:

$$\text{EAR} = e^{\text{APR}} - 1 \tag{7-16}$$

where e is the natural logarithmic base.

For the stated 6% annual interest rate compounded continuously, the EAR is:

$$\text{EAR} = e^{0.06} - 1 = 1.0618 - 1 = 0.0618 \text{ or } 6.18\%$$

The relation between the frequency of compounding for a given stated rate and the effective annual rate of interest for this example indicates that the greater the frequency of compounding, the greater the EAR.

Frequency of Compounding	Calculation	Effective Annual Rate
Annual	$(1 + 0.060)^1 - 1$	6.00%
Semiannual	$(1 + 0.030)^2 - 1$	6.09%
Quarterly	$(1 + 0.015)^4 - 1$	6.14%
Continuous	$e^{0.06} - 1$	6.18%

Figuring out the effective annual rate is useful when comparing interest rates for different investments. It doesn't make sense to compare the APRs for different investments having a different frequency of compounding within a year. But since many investments have returns stated in terms of APRs, we need to understand how to work with them.

To illustrate how to calculate effective annual rates, consider the rates offered by two banks, Bank A and Bank B. Bank A offers 9.2% compounded semiannually and Bank B other offers 9% compounded daily. We can compare these rates using the EARs. Which bank offers the highest interest rate? The effective annual rate for Bank A is $(1 + 0.046)^2 - 1 =$ 9.4%. The effective annual rate for Bank B is $(1 + 0.000247)^{365} - 1 =$ 9.42%. Therefore, Bank B offers the higher interest rate.

SUMMARY

- We can translate a present value into a value in the future through compounding. We can translate a future value into an equivalent value today through discounting. Financial mathematics consists of the mathematical tools we use to perform compounding and discounting.
- The basic valuation equation, $FV = PV\ (1 + i)^N$, is used to translate present values into future values and to translate future values into present values. This basic relationship includes interest compounding—that is, interest earnings on interest already earned.
- Using the basic valuation equation, we can translate any number of cash flows into a present or future value. When faced with a series of cash flows, we must value each cash flow individually, and then sum these individual values to arrive at the present value of the future value of the series. Our work can be cut a bit shorter if these cash flows are equal and occur at periodic intervals of time, referred to as an annuity.
- Tables containing present value factors, future value factors, present value annuity factors, and future value annuity factors can be used to reduce the computations involved in financial math.
- We can use financial mathematics to value many different patterns of cash flows, including perpetuities, annuity due, and deferred annuities. Applying the tools to these different patterns of cash flows requires us to take care in specifying the timing of the various cash flows.
- If the interest on alternative investments is stated in different terms, we can put these interest rates on a common basis so that we can determine the best alternative.
- Typically we specify an interest rate on an annual basis, using either the annual percentage rate or the effective annual rate. The latter method is preferred since it takes into consideration the compounding of interest within a year.
- Effective rates on loans are calculated like annual percentage yields on savings accounts, by calculating the periodic rate and translating it into an effective annual rate.

QUESTIONS

1. If you are offered two investments, one that pays 5% simple interest per year and one that pays 5% compound interest per year, which would you choose? Why?
2. Suppose you make a deposit today in a bank account that pays compounded interest annually. After one year, the balance in the account has grown.
   a. What has caused it to grow?
   b. After two years, the balance in the account has grown even more. What has caused the balance to increase during the second year?
3. The Florida lottery pays out winnings, after taxes, on the basis of 20 equal annual installments, providing the first installment at the time that the winning ticket is turned in.
   a. What type of cash flow pattern is the distribution of lottery winnings?
   b. How would you value such winnings?
4. Rent is typically paid at the first of each month. What pattern of cash flows, an ordinary annuity or an annuity due, does a rental agreement follow?
5. a. Under what conditions does the effective annual rate of interest (EAR) differ from the annual percentage rate (APR)?
   b. As the frequency of compounding increases within the annual period, what happens to the relation between the EAR and the APR?
6. Using the appropriate table, find the compound factor for each of the following combinations of interest rate per period and number of compounding periods:

Number of Periods	Interest Rate per Period	Compound Factor
2	2%	________
4	3%	________
3	4%	________
6	8%	________
8	6%	________

7. Using the appropriate table, find the discount factor for each of the following combinations of interest rate per period and number of discounting periods:

Number of Periods	Interest Rate Per Period	Discount Factor
2	2%	________
4	3%	________
3	4%	________
6	8%	________
8	6%	________

8. Using the appropriate table, find the future value annuity factor for each of the following combinations of interest rate per period and number of payments:

Number of Payments	Interest Rate Per Period	Future Value Annuity Factor
2	2%	________
4	3%	________
3	4%	________
6	8%	________
8	6%	________

9. Using the appropriate table, find the present value annuity factor for each of the following combinations of interest rate per period and number of payments:

Number of Payments	Interest Rate Per Period	Present Value Annuity Factor
2	2%	________
4	3%	________
3	4%	________
6	8%	________
8	6%	________

10. Using an 8% compounded interest rate per period, calculate the future value of a $100 investment:
    a. one period into the future.
    b. two periods into the future.
    c. three periods into the future.
    d. four periods into the future.
    e. five periods into the future.
    f. 40 periods into the future.
11. Suppose you deposit $1,000 into a savings account that earns interest at the rate of 4%, compounded annually. What would be the balance in the account:
    a. after two years?
    b. after four years?
    c. after six years?
    d. after 20 years?
12. If you deposit $10,000 in an account that pays 6% compounded interest per period, assuming no withdrawals:
    a. What will be the balance in the account after two periods?
    b. After the two periods, how much interest has been paid on the principal amount?
    c. After the two periods, how much interest has been paid on interest?

13. Using an 8% compounded interest rate, calculate the present value of $100 to be received:
    a. one period into the future.
    b. two periods into the future.
    c. three periods into the future.
    d. four periods into the future.
    e. five periods into the future.
    f. 40 periods into the future.
14. Ted wants to borrow from Fred. Ted is confident that he will have $1,000 available to pay off Fred in two years. How much will Fred be willing to lend to Ted in return for $1,000 two years from now if he uses a compounded interest rate per year of:
    a. 5%? b. 10%? c. 15%?
15. How much would you have to deposit into a savings account that earns 2% interest compounded quarterly, to have a balance of $2,000 at the end of four years, if you make no withdrawals?
16. What is the present value of $5,000 to be received five years from now, if the nominal annual interest rate (APR) is 12% and interest is compounded: a. annually? b. semiannually? c. quarterly? d. monthly?
17. Calculate the future value at the end of the second period of this series of end-of-period cash flows, using an interest rate of 10% compounded per period:

Period	End of Period Cash Flow
0	$100
1	200
2	400

18. An investor is considering the purchase of an investment at the end of Year 0 that will yield the following cash flows:

Year	End of Year Cash Flow
Year 1	$2,000
Year 2	$3,000
Year 3	$4,000
Year 4	$5,000

    If the appropriate discount rate for this investment is 10%, what will this investor be willing to pay for this investment?
19. Calculate the present value (that is, the value at the end of period 0) of the following series of end of period cash flows:

Period	End of Period Cash Flow
0	$100
1	200
2	400

20. Suppose that an investment promises to provide the following cash flows:

Year	End of Year Cash Flow
Year 1	$0
Year 2	$1,000
Year 3	$0
Year 4	–$1,000

If interest is compounded annually at 5%, what is the value of the investment at the end of: a. Year 1? b. Year 0?

21. Calculate the future value at the end of the third period an ordinary annuity consisting of three cash flows of $2,000 each. Use a 5% rate of interest per period.
22. What is the present value of $10 to be received each period, forever, if the interest rate is 6%?
23. If an investor is willing to pay $40 today to receive $2 every year forever, what is this investor's opportunity cost used to value this investment?
24. Calculate the present value of an annuity due consisting of three cash flows of $1,000 each, each one year apart. Use a 6% compounded interest rate per year.
25. Calculate the future value at the end of the third period of an annuity due consisting of three cash flows of $1,000 each, each one year apart. Use a 6% compounded interest rate per year.
26. Suppose that you have won the Florida Lotto worth $18 million. Further suppose that the State of Florida will pay you the winnings in 20 annual installments, starting immediately, of $900,000 each. If your opportunity cost is 10%, what is the value today of these 20 installments?
27. Calculate the required deposit to be made today so that a series of ten withdrawals of $1,000 each can be made beginning five years from today. Assume an interest rate of 5% per period on end of period balances.
28. How much would you need to deposit today so that you can withdraw $4,000 per year for ten years, starting three years from today?
29. Suppose you wish to invest $2,000 today so that you have $4,000 six years from now. What must the compounded annual interest rate be in order to achieve your goal?

30. The Bert and Ernie Bathtub Company is planning to finance a new truck with a loan of $20,000. This loan requires the company to pay five end-of-year-installments of $5,276 each. What is the effective annual interest rate that the company is paying for its new truck financing?
31. If interest is earned at the rate of 5%, compounded annually, how long will it take an investment of $10,000 to grow to:
    a. $15,000? b. $20,000? c. $30,000?
32. If interest is earned at the rate of 5%, compounded annually, how long would it take an investment to:
    a. double in value? b. triple in value?
33. Suppose you invest $2,500 today. How long would it take to grow to $5,000 if interest is compounded at the rate of 4% per quarter?
34. If interest is paid at a rate of 5% per quarter, what is the:
    a. annual percentage rate? b. effective annual rate?
35. L. Shark is willing to lend you $10,000 for three months. At the end of six months, L. Shark requires you to repay the $10,000, plus 50%.
    a. What is the length of the compounding period?
    b. What is the rate of interest per compounding period?
    c. What is the annual percentage rate associated with L. Shark's lending activities?
    d. What is the effective annual rate of interest associated with L. Shark's lending activities?
36. The Consistent Savings and Loan is designing a new account that pays interest quarterly. It wishes to pay, effectively, 16% per year on this account. Consistent desires to advertise the annual percentage rate on this new account, instead of the effective rate, since its competitors state their interest on an annualized basis. What is the APR that corresponds to an effective rate of 16% for this new account?
37. Which of the following financing arrangements offers the lowest cost of credit on an effective annual basis?
    a. Simple interest loan of 15% per year.
    b. Trade credit, on terms 1/10, net 40, paying on the net day.
    c. Pawn shop credit, on terms 25%, payable after 50 days.
    d. A bank loan with a nominal interest of 14%, with interest compounded monthly.
38. Ken invested $6,000 in an a savings account that pays interest at the rate of 1% per quarter. At the end of five years, Ken withdraws only the interest. How much does he withdraw?
39. The ABC Company wished to invest a sum of money today in an investment that grows at the rate of 12% per year, so that it may withdraw $1,000 at the end of every year for the next ten years. How much must be invested?

PART Two

The Fundamentals of Valuation

CHAPTER 8

Principles of Asset Valuation and Investment Returns

There are a number of factors that affect a stock's price and its value to investors. The financial manager regularly has to figure out whether a particular investment is good or bad. A good investment will enhance shareholder wealth. A bad one won't. To decide whether an investment is good or bad, the manager must determine whether the benefits from the investment—often expected in future periods—will outweigh its costs.

To make the best investment decisions, the financial manager must also consider the way the investment is financed. If the firm takes on more debt, is this harmful to shareholders? If the firm issues more shares of equity, how does this affect the value of equity? Furthermore, recognizing that the value of equity is the difference between the value of the firm's assets and its debt obligations, we must be aware of how debt securities are valued as well.

Valuation compares the benefits of a future investment decision with its cost. Another way of evaluating an investment is to answer the question: Given its cost and its expected future benefits, what return will a particular investment provide? In this chapter we focus on the principles of valuation and how to calculate the return on investments. In the next chapter we focus on the valuation of stocks and bonds.

PRINCIPLES OF ASSET VALUATION

Suppose you are offered the following investment opportunity by a company: Lend the company $90 today, and you will be paid $100 one year from today by the company. Whether or not this is a good investment depends on:

- what you could have done with your $90 instead of investing it with the company, and
- how certain you are that the company will pay the $100 in one year.

If your other opportunities with the same amount of uncertainty provide a return of 10%, is this loan a good investment? There are two ways to evaluate this.

First, you can figure out what you could have wound up with after one year, investing your $90 at 10%:

$$\begin{aligned}\text{Value at the end of one year} &= \$90 + 10\% \text{ of } \$90 \\ &= \$90(1 + 0.10) = \$99\end{aligned}$$

Since the $100 promised is more than $99, you are better off with the investment offered by the company.

Another way of looking at this is to figure out what the $100 promised in the future is worth today. To calculate its present value, we must discount the $100 at some rate. The rate we'll use is the opportunity cost of funds, which in this case is 10%:

$$\text{Value today of } \$100 \text{ in one year} = \frac{\$100}{(1 + 0.10)^1} = \$90.91$$

This means that you consider $90.91 today to be worth the same as $100 in one year. In other words, if you invested $90.91 today in an investment that yields 10%, you end up with $100 in one year. Since today's value of the receipt of $100 in the future is $90.91 and it only costs $90 to get into this deal, the investment is attractive: it costs *less* than what you have determined it is worth.

Since there are two ways to look at this—through its future value or through its present value—which way should you go? While both approaches get you to the same decision, the approach in terms of the present value of the investment is usually easier.

Let's look at another example. Suppose you have an opportunity to buy an asset expected to give you $500 in one year and $600 in two years. If your other investment opportunities with the same amount of risk give you a return of 5% a year, how much are you willing to pay today to get these two future receipts?

We can figure this out by discounting the \$500 in year 1 at 5% and \$600 in year 2 at 5%:

$$\text{Present value of an investment} = \frac{\$500}{(1+0.05)^1} + \frac{\$600}{(1+0.05)^2}$$
$$= \$476.19 + \$544.22 = \$1{,}020.41$$

This investment is worth \$1,020.41 today, so you will be willing to pay \$1,020.41 *or less* for this investment:

- if you pay more than \$1,020.41, you get a return of less than 5%;
- if you pay less than \$1,020.41 you get a return of more than 5%; and,
- if you pay \$1,020.41 you get a return of 5%.

Suppose you are evaluating an investment that promises \$10 every year forever. The value of this investment is the present value of the stream of \$10 to be received each year to infinity where each \$10 is discounted at the appropriate number of years at some annual rate i:

$$\text{Present value of an investment}$$
$$= \frac{\$10}{(1+i)^1} + \frac{\$10}{(1+i)^2} + \frac{\$10}{(1+i)^3} + \ldots + \frac{\$10}{(1+i)^\infty}$$

which we can write in shorthand notation using summation notation as:

$$\text{Present value of an investment} = \sum_{t=1}^{\infty} \frac{\$10}{(1+i)^t} = \$10 \sum_{t=1}^{\infty} \frac{1}{(1+i)^t}$$

Or, since the last term is equal to $1/i$, we can rewrite the present value of this perpetual stream as:

$$\text{Present value of investment} = \$10\ (1/i) = \$10/i$$

If the discount rate to translate this future stream into a present value is 10%, the value of the investment is \$100:

$$\text{Present value of investment} = \$10\ (1/0.10) = \$10/0.10 = \$100$$

The 10% is the discount rate, also referred to as the ***capitalization rate***, for the future cash flows comprising this stream. Let's look at this investment from another angle: If you consider the investment to be worth $100 today, you are capitalizing—translating future flows into a present value—the future cash flows at 10% per year.

As you see from these examples, the value of an investment depends on:

1. the amount and timing of the future cash flows, and
2. the discount rate used to translate these future cash flows into a value today.

This discount rate represents how much an investor is willing to pay today for the right to receive a future cash flow. Or, to put it another way, the discount rate is the rate of return the investor requires on an investment, given the price he or she is willing to pay for its expected future cash flow.

We can generalize this relationship a bit more. Let CF_t represent the cash flow from the investment in period t, so that CF_1 is the cash flow at the end of period 1, CF_2 is the cash flow at the end of period 2, and so on, until the last cash flow at the end of period N, CF_N. If the investment produces cash flows for N periods and the discount rate is i, the value of the investment—the present value—is:

$$\text{Present value of an investment} = \frac{CF_1}{(1+i)^1} + \frac{CF_2}{(1+i)^2} + \frac{CF_3}{(1+i)^3} + \ldots + \frac{CF_N}{(1+i)^N}$$

which we can write more compactly as:

$$\text{Present value of an investment} = \sum_{t=1}^{N} \frac{CF_t}{(1+i)^t}$$

In the special case where the cash flows are all equal, we can simplify this by letting CF represent each cash flow and use CF in place of CF_1, CF_2, and so on. The valuation relation becomes:

$$\text{Present value of an investment} = \sum_{t=1}^{N} \frac{CF}{(1+i)^t} = CF \sum_{t=1}^{N} \frac{1}{(1+i)^t}$$

which we can write in terms of the annuity factor,

$$\text{Present value of an investment} = CF(\text{Present value annuity factor})$$

If the cash flow stream is level and is promised each period forever, N is infinite. As the number of future periods approaches infinity, the present value annuity factor approaches $1/i$. Therefore, the present value of a perpetual stream of cash flows is equal to:

$$\text{Present value of an investment with a perpetual cash flow} = \frac{CF}{i}$$

Whether we are talking about a single future cash flow, a series of level cash flows, a series of cash flows having different amounts, or a perpetual series of cash flows, to determine its present value we need to know:

- the amount and timing of the future cash flows, and
- the discount rate that reflects the uncertainty of these cash flows.

The Role of the Marketplace in the Valuation of Assets

If you are faced with a decision whether to make a particular investment, you figure out what it is worth to you—its value—and compare it with what it will cost you. If the investment costs less than you think it is worth, you will buy it; if it costs more than you think it is worth, you will not buy it.

Now suppose several people are considering buying the same, one-of-a-kind asset. Each potential investor evaluates whether the asset is priced at more or less than what he or she thinks it is worth by making this comparison and either buying or selling the asset based on whether they think it is over- or underpriced, the buyers and sellers determine its price.

Let's see how this works. Three investors, A, B, and C, have an opportunity to buy an asset expected to generate \$100 each period forever. This is a perpetuity whose value is the ratio of the \$100 to the discount rate. If each investor thinks that this asset represents an investment that has a different amount of risk, they each will use a different discount rate to value it. If investors are ***risk averse***—they do not like risk—they will value an asset using a higher discount rate the more uncertain they are about the future cash flows.

Suppose:

Investor	uses the discount rate ...	and values the asset as...
A	8.0%	$1,250
B	10.0%	$1,000
C	12.5%	$800

And suppose the asset is owned by Investor C who has been looking at alternative investment opportunities with similar risk that offer a return of 12.5% and as a result figures that the asset is worth only $800. Both Investors A and B would be interested in buying it from C for more than $800 and C would be willing to sell it for more than $800. Since both A and B want this asset, they would bid for it.

So what is the market price of the asset? If its price is $1,000, Investor B would be indifferent between this asset and his other investments of similar risk. At $1,000, Investor A would still think it is underpriced and want to buy it. So the price is bid up to reflect the highest value investors are willing to pay: $1,250. If Investor A buys the asset for $1,250, he gets a return of 8%, which is what he thinks is appropriate given his assessment of the asset's risk.

What makes this process work is the desire of investors to exploit profitable opportunities: C to sell it for more than she thinks it is worth and A and B to buy it for less than they think it is worth. If we assume that investors are interested in maximizing their wealth, those investors thinking an asset is overpriced will want to sell it and those thinking it is underpriced will want to buy it.

Buyers and sellers will continue to buy and sell until they have exhausted what they believe are all the profitable opportunities. When that happens, the assets are neither over- or underpriced. This point where buying and selling is in balance is referred to as a ***market equilibrium***. The price of an asset is determined by the investor with the highest valuation of the asset. If the price of an asset is above or below its market equilibrium price, investors will buy and sell it until its price is the market equilibrium price.

As long as an asset can be traded without any restrictions in a market, buying and selling will determine its price. However, if there is a barrier to trading—such as a limit on the quantity that can be sold—this trading is inhibited and the asset's price will not reflect the valuation of the highest valuer.

In addition, if there are costs to trading—such as a fee each time a trade is made—investors will figure the costs into their bidding. For example, if there is a $100 fee to buy the asset, the most Investor A would be willing to pay is $1,150 and the most Investor B would be willing to pay is $900, considering there is a $100 fee to buy it.

RETURNS ON INVESTMENTS

As we have seen, whether investors are willing to make an investment, and the price they are willing to pay, depends on the return they expect. A ***return***, the benefit an investor receives from an investment, can be in the form of:

- a change the in the value of the asset—it's appreciation or depreciation,
- a cash flow from the investment, such as a dividend or an interest payment, or
- both a cash flow and a change in value.

The return on an investment is also referred to as the ***yield***. We saw the role of the yield in the value of an asset. Now let's see how to calculate returns on different investments.

Return on Investments with No Intermediate Cash Flows

Let's start by looking at an investment that involves no cash flows other than its purchase and then its sale. Suppose you bought a comic book in 1991 for the cover price of $1.00. In 2001, this comic book, in mint condition, was worth $3.00. If you sold at this time, your return on your investment would have been:

Return on investment = (Sales price – cost)/cost = ($3.00 – $1)/$1 = 200%

Before any commission by the comic book dealer, you have made a return of 200% over the ten years.

Because different investments have different lives, or are valued at different points in their lives, we will need to put their yields on some common basis to compare them. The most common way of reporting a return or yield is on an annual basis, expressed as the average annual return per year. We can translate the six-year return on our comic book investment into a return per year given the following:

Future value (sale price)	=	FV	=	$3.00
Present value (cost)	=	PV	=	$1.00
Number of periods	=	N	=	10 years

We can represent the return over the six years on a per year basis. Let i be the annual return on the investment. Using the basic valuation equation, where FV is the future value, PV is the present value, N is the number of compounding periods, and r is the interest rate per period:

$$FV = PV\,(1 + i)^N$$

$$\$3.00 = \$1.00\,(1 + i)^{10}$$

solving for i,

$$(1 + i)^{10} = \$3/\$1$$

Taking the tenth root of both sides:

$$1 + i = \sqrt[10]{\$3/\$1} = \sqrt[10]{3} = 3^{0.10} = 1.1161$$

and therefore i is equal to $1.1161 - 1 = 0.1161$.

The annual return is 11.61%. Holding onto the comic book for ten years provided an average return of 11.61% per year. Another name for this return is the ***internal rate of return*** (IRR).

The average annual return on an investment is the geometric average, not the arithmetic average. What's the difference? Compounding. Suppose today you invest \$100 that will earn 5% per year for two years. After two years your investment is worth $\$100\ (1 + 0.05)^2$ = \$110.25. You have earned \$10.25 or 10.25% over two years. But what have you earned per year? Five percent, which happens, not coincidentally, to be the geometric average return:

$$\begin{aligned}\text{Geometric average annual return} &= \sqrt[2]{\frac{FV}{PV}} - 1\\ &= \sqrt[2]{\frac{\$110.25}{\$100.00}} - 1 = \sqrt[2]{1.1025} - 1\\ &= 1.05 - 1 = 5\%\end{aligned}$$

The arithmetic average annual return is:

$$\text{Arithmetic average annual return} = \frac{\$10.25/\$100.00}{2} = 5.13\%$$

But the arithmetic average ignores any compounding! It says that we earn 5.13% of $100 in the first year, 5.13% in the second year, and do not earn interest on interest during the second period.

If we want to compare investments that have different time horizons and different frequencies of compounding, we need to place returns on a common basis. Since the geometric average gives us a return that considers compounding, this is the average we can compare meaningfully. So, when we refer to the average annual return or average annual yield, we are talking about the average that considers compounding: the geometric average.

Return on Investments with Even Cash Flows

For the comic book, we calculated a return that was derived solely from the appreciation in value. Now let's look at an example where the return is derived solely from a stream of cash inflows.

Suppose you buy an investment for $10,000 that promises to pay $4,000 per year for three years, beginning one year from the date you buy it. The return on this investment is the discount rate that equates its cost, $10,000, with the benefits it produces—the three cash inflows of $4,000 each—considering the time value of money. This is a present value problem. We use CF_t to indicate the cash flow at the end of period t:

Present value (cost of investment)	=	PV	=	$10,000
Cash flow at the end of the first year	=	CF_1	=	$4,000
Cash flow at the end of the second year	=	CF_2	=	$4,000
Cash flow at the end of the third year	=	CF_3	=	$4,000

The return is the value of i that solves:

$$PV = \frac{CF_1}{(1+i)^1} + \frac{CF_2}{(1+i)^2} + \frac{CF_3}{(1+i)^3}$$

Inserting the known values,

$$\$10{,}000 = \frac{\$4{,}000}{(1+i)^1} + \frac{\$4{,}000}{(1+i)^2} + \frac{\$4{,}000}{(1+i)^3}$$

Because the cash flows are level, we can represent this equation in summation form:

$$\$10{,}000 = \sum_{t=1}^{3} \frac{\$4{,}000}{(1+i)^t} = \$4{,}000 \sum_{t=1}^{3} \frac{1}{(1+i)^t}$$

Recognizing that

$$\sum_{t=1}^{3} \frac{1}{(1+i)^t}$$

is the present value annuity factor for three periods and some unknown i, the equation becomes:

$10,000 = $4,000(Present value annuity factor for $N = 3$ and $i = ?$)

We can calculate i either by:

1. trial and error,
2. using present value of annuity factors from a table of factors, or
3. using a financial calculator.

Using a financial calculator,

Hewlett-Packard 10B	Hewlett-Packard 12C	Hewlett-Packard 17B	Texas Instruments BA-II Plus
10000 ± PV	10000 CHS PV	FIN TVM	10000 ± PV
4000 PMT	4000 PMT	10000 ± PV	4000 PMT
3 N	3 n	4000 PMT	3 N
I/YR	i	3 N	I
		I%YR	

The value of i that solves this equation is $i = 0.09701$ or 9.701% per period. So, we say that this investment yields 9.701% per year.

Return on Investments with Uneven Cash Flows

If we are calculating a return on an investment from a change in the value of an investment, we can use the basic valuation equation to determine the return on the investment. If we are calculating a return on an investment that produces even cash flows throughout its life, we can use the annuity shortcut to figure out its return. But it is more difficult to calculate the return when the cash flows are neither the same amount each period nor a single lump sum. If an investment produces cash flows in different amounts, there are only two ways to solve for i:

1. trial and error, or
2. using a financial calculator.

Suppose you are offered an investment costing \$10,000 that promises cash flows of \$1,000 after one year, \$2,000 after two years, and returns the original \$10,000 at the end of the second year. The return on this investment is the rate i that solves:

$$\$10{,}000 = \frac{\$1{,}000}{(1+i)^1} + \frac{\$2{,}000}{(1+i)^2} + \frac{\$10{,}000}{(1+i)^2}$$

Combining the cash flows that occur at the end of the second year,

$$\$10{,}000 = \frac{\$1{,}000}{(1+i)^1} + \frac{\$12{,}000}{(1+i)^2}$$

The discount rate that solves this problem is 14.66% per year, the annual return on this investment.

Hewlett-Packard 10B	Hewlett-Packard 12C	Hewlett-Packard 17B	Texas Instruments BA-II Plus
10000 ± CFj	10000 CHS CF_0	FIN CFLO	CF
1000 CFj	1000 CFj	FLOW(0)=? 10000 ± INPUT	CF 10000 ± ENTER
12000 CFj	12000 CFj	FLOW(1)=? 1000 INPUT	↑ 1 ENTER
☐ IRR	f IRR	#TIMES(1)=1 INPUT	↑ 1000 ENTER
		FLOW(2)=? 12000 INPUT	↑ 1 ENTER
		#TIMES(1)=1 INPUT	↑ 12000 ENTER
		EXIT CALC IRR%	↑ 1 ENTER
			CPT IRR

The Reinvestment Assumption

The discount rate that equates an investment's initial cost with value of the future cash flows it produces is the internal rate of return. The internal rate of return is aptly named since we are assuming that the cash inflows are reinvested at the same return as the rest of the investment—its internal return.

How does this reinvestment work? Let's look again at the preceding problem. Suppose that instead of reinvesting the $1,000 you received after the first year, you place it under your mattress, where it earns nothing. The total value of the cash flows at the end of the second year is:

Year	Cash Flow	Value at the End of the Second Year
1	$1,000	$1,000
2	12,000	12,000
Total		$13,000

This reinvestment strategy provides you $13,000 at the end of the second year. The ***effective annual return*** on your investment—what you earn considering compounding—is calculated from the basic valuation equation:

$$FV = PV\,(1 + i)^N$$

Substituting the known values of FV, PV, and N,

$$\$13{,}000 = \$10{,}000\,(1 + i)^2$$

$$(1 + i)^2 = \frac{\$13{,}000}{\$10{,}000} = 1.3000$$

therefore,

$$i = \sqrt{1.3000} - 1 = 0.1402 \text{ or } 14.02\% \text{ per year}$$

By stuffing your first year's end-of-period cash flow into a mattress, where it earns no interest during the second year, your return is 14.02%.

If, instead of the mattress, you invested the $1,000 for the one year at 10%, what would be the return on your total investment?

Year	Cash Flow	Value at the End of the Second Year	
1	$1,000	$1,100	← $1,000 invested one period at 10%
2	$12,000	12,000	
Total		$13,100	

In this case, the return on your investment is:

$$i = \sqrt{\frac{\$13{,}100}{\$10{,}000}} - 1 = 14.46\% \text{ per year}$$

which is larger than mattress stuffing because the $1,000 earns $100 of interest during the second period.

But suppose you reinvest this $1,000 at a return = 14.66%?

Year	Cash Flow	Value at the End of the Second Year	
1	$1,000	$1,147	← $1,000 invested one period at 14.66%
2	$12,000	12,000	
Total		$13,147	

$$i = \sqrt{\frac{\$13{,}147}{\$10{,}000}} - 1 = 0.1466 \text{ or } 14.66\%$$

In this case, the return on your investment is 14.66%, the IRR!

When we solve for the internal rate of return, whether by trial and error, annuity tables, or the financial calculator, we are assuming that the cash flows are reinvested at the same rate as the rate of the investment that generated those cash flows. This is true whether we are calculating yields on stocks, bonds, comic books, or any other investment.

If we assume the cash flows are reinvested at a different return, the return on the investment is referred to as the ***modified internal rate of return*** (MIRR). For example, assuming reinvestment of the cash inflows at 10% provides us with a modified internal rate of return of 14.46%, which is less than the internal rate of return, 14.66%.

SUMMARY

- The value of any asset today depends on its expected future cash flows. These future cash flows may be a level, perpetual stream (as in the case of a preferred stock), a growing stream of cash flows (as in the case of many common stocks), or a series of uneven cash flows (as in the case of a bond).
- No matter the pattern of future cash flows, the basic valuation of these flows is the same: Each future cash flow is discounted to the present at

an interest rate that reflects both the time value of money and the uncertainty of the cash flow.

- Prices of assets are determined in a market. A market is in equilibrium if there is a balance between buying and selling.
- The price of an asset in the market is the most someone is willing to pay for it; in other words, prices are determined by the highest valuation of the asset.
- If there are transactions costs, these costs will affect the price of an asset in the market.
- The annual return on an investment is calculated as the geometric mean return. The geometric mean considers the compounding of returns. The geometric mean return also considers that any intermediate cash flows from the investment are reinvested at the average return.
- There is an inverse relation between the value of an asset and the discount rate applied to future cash flows: The higher this discount rate, the lower today's value, and the lower the discount rate, the higher today's value.
- The valuation of an asset requires discounting the expected future cash flows at some rate—the yield required by investors. Turning the problem around, we can figure out, for a given asset value, the yield on the asset. It's the same math as we performed with valuation, but instead of solving for the present value of future cash flows, we are solving for the discount rate—the yield.
- When we calculate the yield on a security, we are interested in translating that yield into some common basis—a year—so we can compare alternative investments.

QUESTIONS

1. What is the relation between the discount rate applied to future cash flows from an investment and the value of the investment today?
2. a. What is meant by the required rate of return?
   b. What is the relation between the required rate of return and the discount rate used to value future cash flows?
3. Consider the following three investments:

   Investment 1:
   Invest \$10,000 today and get \$15,000 three years from now.
   Investment 2:
   Invest \$10,000 today and get \$5,000 at the end of each of the next three years.

Investment 3:
Invest $10,000 today and get $2,500 at the end of every six months for three years.

If the cash flows from these investments have the same degree of uncertainty, which investment should you choose? Why?

4. Suppose an you have an opportunity to invest $1,000 today an get $1,200 one year from today. If your required rate of return on investments of similar risk is 10%, should you make this investment? Why?
5. Suppose you have the opportunity to invest in a project that provides you with $4,000 every year forever. If you require an 8% return on investments with similar risk, what is the most you would be willing to pay for this project?
6. Calculate the average annual return for the following investments that have no intermediate cash flows:

	Beginning Price	Ending Price	Number of Years
(a)	$1,000	$1,500	2
(b)	$10,000	$9,000	10
(c)	$978	$1,000	3

7. Island Corporation invested $1,000,000 in a new product on January 1, 2000. This product generated cash flows of $800,000 the first year, $400,000 the second year, and $200,000 the third year. At the end of the third year, Island abandoned the new product and disposed of the production equipment for $400,000 at the end of the third year. If these are the only cash flows from this new product, what was Island's return on their investment in this product?
8. Suppose you invest $10,000 in an investment that provides a return of 10% in the first year, 15% in the second and third years, and 12% in the fourth year. The investment has no cash flows, but rather the value of the investment grows each year.
   a. What is your investment worth at the end of the fourth year?
   b. What is the average annual return on this investment?
9. The Abel Company invested $100,000 in an investment that produced cash flows of $35,000 at the end of the first year, $45,000 at the end of the second year, and $50,000 at the end of the third year. If these are the only cash flows from this investment, what is Abel's annual return on its investment?
10. The Baker Company is considering an investment of $1 million. The investment is expected to produce the following cash flows:

Year	Cash Flow
Year 1	$400,000
Year 2	$300,000
Year 3	$300,000
Year 4	$400,000

a. What is the annual return on Baker Company's investment if it invests $1 million?
b. What is the most the Baker Company would invest so that the return on its investment is at least 10%?

CHAPTER 9

Valuation of Securities and Options

In Chapter 8, we explained and illustrated the principles of asset valuation and how to calculate the return on an investment. In this chapter we look at how to value three types of securities: common stock, preferred stock, and debt. We also discuss the key factors affecting the value of an option.

CASH FLOW CHARACTERISTICS

These securities have different types of cash flows and the uncertainty of each is different. We briefly describe the characteristics of the cash flows for these securities. By necessity, our discussion does not cover the finer points associated with the investment characteristics of these securities. These points will be covered in later chapters when we describe common stock (in Chapter 16), preferred stock (in Chapter 17), and various types of debt obligations (in Chapter 15).

If you invest in common stock, you buy shares that represent an ownership interest in a corporation. Shares of common stock are a perpetual security—there is no maturity. Owners of common stock have the right to receive a certain portion of any dividends—but dividends are *not* a sure thing. Typically we see some pattern in the dividends companies pay—dividends are either constant or grow at a somewhat constant rate.

There are three major differences between the dividends of preferred and common shares. First, the dividends on preferred stock usually are specified at a fixed contractual amount. This can be based on a fixed dividend rate or a variable dividend rate. The key point is that with the exception of certain types of preferred stock which are rarely issued, the amount that the investor can receive cannot exceed a contractually specified amount. Second, preferred shareholders' dividends must be paid before any dividends are paid on common stock. Third, if the preferred stock has a ***cumulative feature***, dividends not paid in one period accumulate and are carried over to the next period. Therefore, the dividends on preferred stock are more certain than those on common shares.

Notes and bonds are debt securities obligating the borrower to pay interest at regular intervals and to repay the principal amount borrowed, referred to as the ***face value***. The repayment of the principal can be at maturity or there can be scheduled principal repayments over the life of the debt obligation. Some notes and bonds can be paid off before the scheduled principal repayment date. The prepayment can be to retire the entire obligation at once by calling an issue before maturity or one or more scheduled payments before the maturity date. Debt securities are senior to equity securities. This means that the corporate borrower must satisfy its obligations to creditors before making payments to owners. Therefore, cash flows from debt securities are viewed as more certain than cash flows from either preferred stock or common stock. The features of preferred stock, common stock, and corporate debt are summarized in Exhibit 9.1.

EXHIBIT 9.1 **Summary of Features of Securities**

Security	Cash Flow	Certainty of Cash Flow	Maturity
Common stock	Dividend, no fixed rate or amount.	No obligation to pay, but paid at the discretion of the board of directors.	None
Preferred stock	Dividend; can be either a fixed rate or a contractually determined variable rate.	No obligation to pay but preferential to common stock if dividends are paid.	None
Debt	Includes interest and principal repayment. Many variations. Interest can be a fixed or a contractually determined variable rate. Principal repayment can be at maturity or repaid over the life of the debt. The principal repayment provision may allow the firm to prepay.	Legal obligation and given preference over common and preferred stocks. Possibility of early call or prepayments create uncertainty about the cash flow pattern.	Fixed

Valuation of Common Stock

When you buy a share of common stock, it is reasonable to assume that the price you pay reflects what you expect to receive from it in the form of a return on your investment. What you receive are cash dividends in the future. How can we relate that return to the value of a share of common stock? The value of a share of stock should be equal to the present value of all the future cash flows you expect to receive from that share:

$$\text{Price of a share of common stock} = \frac{\text{Dividends in first period}}{(1 + \text{Discount rate})^1} + \frac{\text{Dividends in second period}}{(1 + \text{Discount rate})^2} + \ldots$$

Because common stock never matures, today's value is the present value of an infinite stream of cash flows. And also, common stock dividends are not fixed, as in the case of preferred stock. Not knowing the amount of the dividends—or even if there will be future dividends—makes it difficult to determine the value of common stock.

So what are we to do? Well, we can attempt to determine the valuation of common stock by looking at its current dividend and making assumptions about any future dividends it may pay. We will describe the dividend valuation model and then focus on two related issues: determining the required rate of return and dealing with different assumptions regarding dividend growth.

Dividend Valuation Model

If dividends are constant forever, the value of a share of stock is the present value of the dividends per share per period, in perpetuity. Let D represent the constant dividend per share of common stock expected next period and each period thereafter, forever, P_0 represent the price of a share of stock today, and r_e the required rate of return on common stock. The ***required rate of return*** (RRR) is the return shareholders demand to compensate them for the time value of money tied up in their investment and the uncertainty of the future cash flows from these investments.

The current price of a share of common stock, P_0, is:

$$P_0 = \frac{D}{(1 + r_e)^1} + \frac{D}{(1 + r_e)^2} + \ldots + \frac{D}{(1 + r_e)^x}$$

which we can write using summation notation,

$$P_0 = \sum_{t=1}^{\infty} \frac{D}{(1+r_e)^t}$$

The summation of a constant amount discounted from perpetuity simplifies to:

$$P_0 = D/r_e$$

As an example, if the current dividend is $2 per share and the required rate of return is 10%, the value of a share of stock is:

$$P_0 = \$2/0.10 = \$20$$

Therefore, if you pay $20 per share and dividends remain constant at $2 per share, you will earn a 10% return per year on your investment every year. But dividends on common stock often change through time.

If dividends grow at a constant rate, the value of a share of stock is the present value of a *growing* cash flow. Let D_0 indicate this period's (i.e., end of period 0) dividend. If dividends grow at a constant rate, g, forever, the present value of the common stock is the present value of all future dividends:

$$P_0 = \frac{D_0(1+g)^1}{(1+r_e)^1} + \frac{D_0(1+g)^2}{(1+r_e)^2} + \ldots + \frac{D_0(1+g)^\infty}{(1+r_e)^\infty}$$

Pulling today's dividend D_0, from each term,

$$P_0 = D_0\left[\frac{(1+g)^1}{(1+r_e)^1} + \frac{(1+g)^2}{(1+r_e)^2} + \ldots + \frac{(1+g)^\infty}{(1+r_e)^\infty}\right]$$

Using summation notation:

$$P_0 = D_0 \sum_{t=1}^{\infty} \frac{(1+g)^t}{(1+r_e)^t}$$

which simplifies to:

$$P_0 = D_0 \frac{(1+g)}{(r_e - g)}$$

If we represent the next period's dividend, D_1, in terms of this period's dividend, D_0, compounded one period at the rate g (that is, $D_1 = D_0(1+g)$) and substitute for D_0:

$$P_0 = \frac{D_1}{r_e - g}$$

This equation is referred to as the ***Dividend Valuation Model*** (DVM).[1]

As an example, consider a firm expected to pay a constant dividend of \$2 per share, forever. If this dividend is capitalized at 10%, the value of a share is \$20. If, on the other hand, the current dividend is \$2 but these dividends are expected to grow at a rate of 6% per year, forever, the value of a share of stock is \$53:

$$P_0 = \frac{\$2(1+0.06)}{0.10 - 0.06} = \frac{\$2.12}{0.04} = \$53$$

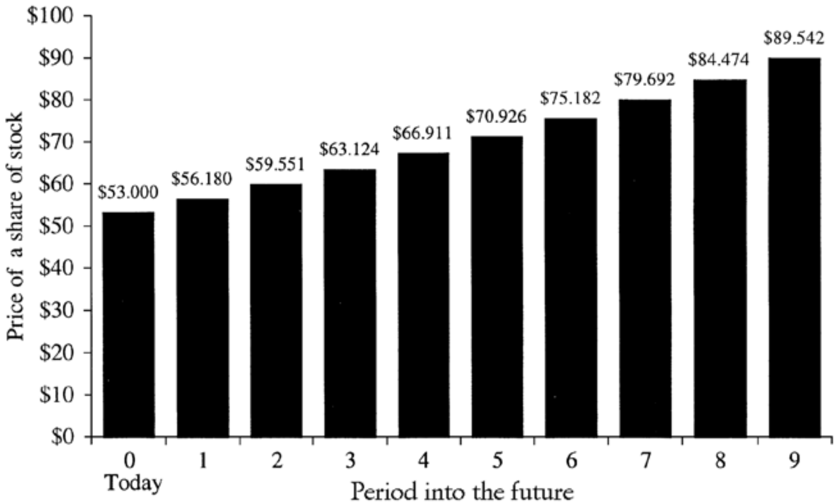
Does this make sense compared to the constant *amount* case where dividends are unchanged at \$2 per year? Yes: If dividends are expected to grow in the future, the stock is worth more than if the dividends are expected to remain the same.

If today's value of a share is \$53, what are we saying about the value of the stock next year? If we move everything up one year, D_1 is no longer \$2.12, but the current dividend of \$2 grows at 6% to $\$2(1 + 0.06)^2 = \$2.2472$. Therefore, we expect the price of the stock at the end of one year, P_1, to be \$5:

$$P_1 = \frac{\$2(1+0.06)^2}{0.10 - 0.06} = \frac{\$2.2472}{0.04} = \$56.18$$

[1] The Dividend Valuation Model is attributed to Myron Gordon, who popularized the constant growth model. A more formal presentation of this model can be found in published works by Gordon entitled "Dividends, Earnings and Stock Prices," (*Review of Economics and Statistics*, May 1959, pp. 99–105) and *The Investment Financing and Valuation of the Corporation* (Homewood, IL: R. D. Irwin, 1962). However, the foundation of common stock valuation is laid out—for both constant and growing dividends—by John Burr Williams in *The Theory of Investment Value* (Amsterdam: North-Holland Publishing Company, 1938), Chapters V, VI, and VII.

EXHIBIT 9.2 **The Price of a Share of Stock with a Current Dividend of \$2, a 6% Growth in Dividends, and a 10% Required Rate of Return**



At the end of two years, the price will be \$59.55. Since we expect dividends to grow each year, we also are expecting the price of the stock to grow through time as well. In fact, the price is expected to grow at the same rate as the dividends: 6% per period.

The relation between the growth rate of dividends, g, and the price of the stock expected in the future is illustrated in Exhibit 9.2. For a given required rate of return and dividend—in this case r_e = 10% and D_0 = \$2—we see that the price of a share of stock is expected to grow each period at the rate g.

What if the dividends are expected to decline each year? That is, what if g is negative? We can still use the Dividend Valuation Model, but each dividend in the future is expected to be *less* than the one before it. For example, suppose a stock has a current dividend of \$2 per share and the required rate of return is 10%. If dividends are expected to *decline* 6% each year, what is the value of a share of stock today? We know that D_0 = \$2, r_e = 10%, and g = –6%. Therefore,

$$P_0 = \frac{\$2(1 - 0.06)}{0.10 + 0.06} = \frac{\$1.88}{0.16} = \$11.75$$

Two periods from now, the expected price is even lower:

$$P_1 = \frac{\$2(1-0.06)^2}{0.10+0.06} = \frac{\$1.7672}{0.16} = \$11.045$$

Let's look at another situation, one in which growth is expected to *change* but at different growth rates as time goes on. Consider a share of common stock whose dividend is currently \$3.00 per share and is expected to grow at a rate of 8% per year for five years and afterward at a rate of 4% per year after five years. To tackle this problem, let's break it into two manageable parts: the first five years and after five years, or:

P_0 = Present value of dividends in the first five years
+ Present value of dividends received after the first five years to infinity

Assuming a required rate of return of 10%,

$$P_0 = \underbrace{\frac{D_1}{(1+0.10)^1} + \frac{D_2}{(1+0.10)^2} + \frac{D_3}{(1+0.10)^3} + \frac{D_4}{(1+0.10)^4} + \frac{D_5}{(1+0.10)^5}}_{\text{Dividends growing at a rate of 8\% per year}}$$

$$+ \underbrace{\frac{D_6}{(1+0.10)^6} + \frac{D_7}{(1+0.10)^7} + \ldots + \frac{D_\infty}{(1+0.10)^\infty}}_{\text{Dividends growing at a rate of 4\% per year}}$$

The present value of the dividends in the first five years is:

Present value of dividends received during the first five years

$$= \frac{\$3.24}{1.1000} + \frac{\$3.4992}{1.2100} + \frac{\$3.7791}{1.3310} + \frac{\$4.0815}{1.4641} + \frac{\$4.4080}{1.6105}$$

$$= \$2.9455 + \$2.8919 + \$2.8393 + \$2.7877 + \$2.7370 = \$14.2014$$

The present value of dividends received after the fifth year—evaluated five years from today—is the expected price of the stock in five years, P_5:

$$P_5 = \frac{D_5(1+0.04)}{0.10-0.04} = \frac{\$4.5843}{0.06} = \$76.4053$$

The price expected at the end of five years is \$76.4053, which we translate into a value today by discounting it five years at 10%:

Present value of dividends to be received after the first five years

$$= \frac{\$76.4053}{(1+0.10)^5} = \frac{\$76.4053}{1.6105} = \$47.4420$$

Putting together the two pieces,

$$P_0 = \$14.2014 + \$47.4420 = \$61.6434$$

The value of a share of this stock is \$61.6434.

We can represent the Dividend Valuation Model in terms of a share's price to earnings ratio (P/E ratio). Let's start with the Dividend Valuation Model with constant growth in dividends:

$$P_0 = \frac{D_1}{r_e - g}$$

If we divide both sides of this equation by earnings per share, we can represent the dividend valuation model in terms of the price-earnings (P/E) ratio:

$$\frac{P_0}{\text{EPS}_1} = \frac{\dfrac{D_1}{\text{EPS}_1}}{r_e - g}$$

$$\text{P/E} = \frac{\text{Dividend payout ratio}}{r_e - g}$$

This tells us the P/E ratio is influenced by the dividend payout ratio, the required rate of return on equity, and the expected growth rate of dividends.

The Dividend Valuation Model makes some sense regarding the relation between the value of a share of stock, the growth in dividends, and the discount rate:

- The greater the current dividend, the greater the value of a share of stock.

- The greater the expected growth in dividends, the greater the value of a share of stock.
- The more uncertainty regarding future dividends, the greater the discount rate and the lower the value of a share of stock.

However, the DVM has some drawbacks. How do you deal with dividends that do not grow at a constant rate? As you can see in the last example, this model does not accommodate nonconstant growth easily. What if the firm does not pay dividends now? In that case, D_0 would be zero and the expected price would be zero. But the price of a share of stock cannot be zero. Therefore, the DVM may be appropriate to use to value the stock of companies with stable dividend policies, but it is not applicable for all firms.

Despite its drawbacks, the DVM captures the valuation for many companies' securities. We can use the DVM to take a closer look at investors' required rate of return and the expected rate of growth in future dividends. Moreover, the DVM has been modified to allow for different types of dividend patterns.[2]

Required Rate of Return

The DVM is based on the idea that future cash flows—dividends in the case of common stock—are discounted to the present at some rate that reflects the share owners' opportunity cost. This opportunity cost is what they could have earned on alternative investments with similar risk. This minimum return is the required rate of return—it's the discount rate compensating the share owners for the time value of money and risk:

$$\begin{aligned}&\text{Required rate of return}\\&= \text{Time value of money} + \text{Compensation for bearing risk}\end{aligned}$$

To calculate the required rate of return, we start with the DVM with a constant growth rate, and solve for r_e:

$$r_e = (D_1/P_0) + g$$

we see that the required rate of return is made up of the ***dividend yield***, D_1/P_0, plus the rate we expect share price to grow, the ***capital yield***, g:

$$r_e = \text{Dividend yield} + \text{Capital yield}$$

[2] For a discussion of these models, see William J. Hurley and Frank J. Fabozzi, "Dividend Discount Models," Chapter 9 in *Handbook of Portfolio Management* (New Hope, PA: Frank J. Fabozzi Associates, 1998).

In other words, if we know next period's dividend, the current price, and the expected growth rate, we can determine the required rate of return. For example, suppose a share of stock is currently selling for \$40 per share. If next period's dividend is expected to be \$2 and dividends are expected to grow at a rate of 4% per year, the required rate of return is:

$$r_e = (\$2/\$40) + 4\% = 5\% + 4\% = 9\%$$

In other words, a share of stock is valued so that it yields 9%.

Growth Rate of Future Dividends

If we assume that a constant proportion of earnings are paid in dividends—a constant dividend payout—we can tie the growth rate of dividends to the return on equity. Let's start with the DVM with constant growth. If we divide both sides by earnings per share for the next period, EPS_1, we get:

$$\frac{P_0}{\text{EPS}_1} = \frac{\dfrac{D_1}{\text{EPS}_1}}{r_e - g}$$

Inverting both sides,

$$\frac{\text{EPS}_1}{P_0} = \frac{r_e - g}{\dfrac{D_1}{\text{EPS}_1}}$$

Recognizing that the return on equity, r_e, is equal to EPS_1/P_0 and simplifying to solve for the growth rate,

$$g = r_e\left(1 - \frac{D_1}{\text{EPS}_1}\right)$$

or,

$$\begin{aligned}&\text{Expected growth rate of dividends}\\ &= \text{Return on equity}(1 - \text{Dividend payout ratio})\end{aligned}$$

From this we see that:

- the greater the return on equity, the greater the expected growth rate of dividends; and
- the greater the dividend payout, the lower the growth rate of dividends.

Does this make sense? Yes. The more the firm can earn, the greater the expected future growth in dividends. Also, the more the firm pays out in dividends, the less it has to reinvest into the firm for the future and the lower the expected growth rate of dividends in the future.

Returns on Common Stock

As we saw in the preceding section, the value of a stock is the present value of future cash flows, discounted at the required rate of return. If we know the future cash flows and the required rate of return, we can determine today's value. Suppose that instead of determining today's value, we wish to determine the return on a stock. For example, we may want to determine whether a particular stock provides a return over the next five years that is appropriate for its risk. In this case, we know the value of the stock today, we estimate its value in five years, and estimate any intermediate cash flows (e.g., dividends). The missing piece is the return.

We can calculate the return on an investment in common stocks just as we did the internal rate of return in the preceding example. The return on stock is comprised of two components: (1) the appreciation (or depreciation) in the market price of the stock—the capital yield—and (2) the return in the form of dividends—the dividend yield:

$$\text{Return on stock} = \text{Capital yield} + \text{Dividend yield}$$

Let's first ignore dividends. The return on common stock over a period of time where there are no dividends is the change in the stock's price divided by the beginning share price:

$$\text{Return on stock} = \frac{(\text{End-of-period price} - \text{Beginning-of-period price})}{\text{Beginning-of-period price}}$$

Suppose that at the beginning of 2000, Hype.com stock was \$10 per share, and at the end of 2001 Hype.com stock was \$15 a share. The return on Hype.com during 2000–2001 was:

$$\text{Return on Hype.com stock} = \frac{\$15 - \$10}{\$10} = 50\%$$

Hype.com stock appreciated \$5.00 per share, providing a return of 50% for the two years. To make the return comparable to returns on other investments, we usually restate the return as a return per year. The return on Hype.com per year is calculated using the time value of money relationship:

$$PV = \$10 \quad FV = \$15 \quad N = 2$$

Solving for r, the return is 22.47% per year.

Let's work through another illustration. Suppose you bought one share of Berkshire Hathaway stock at the end of 1986 for \$2,430. And suppose you sold this share of stock at the end of 1998 for \$70,000. Over the twelve years, you earned over 2,700%! But in order to compare this return with other stocks' returns, we need to place it on a common basis, a year. Given the following, we can translate the twelve-year return on Berkshire Hathaway stock into a return per year:

Future value (sales price) = FV = \$70,000
Present value (cost) = PV = \$2,430
Number of periods = N = 12 years

We can represent the return over the nine years on a per-year basis. Let r be the annual return on the investment. Using the basic valuation equation, $FV = PV\,(1 + r_e)^N$, we substitute the known elements into the basic valuation equation,

$$\$70{,}000 = \$2{,}430\,(1 + r_e)^{12}$$

Next, we rearrange in terms of r_e:

$$r_e = \sqrt[23]{\frac{\$70{,}000}{\$2{,}430}} - 1 = 32.32\%$$

The return on your investment (not considering any commissions paid) is 32.32% per year.

If a stock pays dividends, we need to consider them as cash inflows, as well as the change in the share's price, in determining the return. The simplest way to calculate the return is to assume that dividends are received at the end of the period:

$$\text{Return on a stock} = \frac{\text{End-of-period price} - \text{Beginning-of-period price} + \text{Dividends at end of period}}{\text{Beginning-of-period price}}$$

Or, if we let:

P_0 = beginning-of-period price
P_1 = end-of-period price
D_1 = dividends received at the end of period

we can write:

$$\text{Return on a stock} = \frac{P_1 - P_0 + D_1}{P_0}$$

We can break this return into one part representing the return due to the change in price and another part representing the return due to dividends:

$$\text{Return on a stock} = \underbrace{\frac{P_1 - P_0}{P_0}}_{\text{capital yield}} + \underbrace{\frac{D_1}{P_0}}_{\text{dividend yield}}$$

The first part is the capital yield and the second part is the dividend yield. If a company doesn't pay dividends, the dividend yield is zero and the return on the stock is its capital yield.

When using this equation, be careful to specify the timing of the prices at the beginning and the end of the period and the timing of the dividends. Because we're dealing with the time value of money, we have to be very careful to be exact about the timing of all cash flows.

To simplify our analysis, let's ignore our stockbroker's commission, though we will discuss these costs later in this chapter. Suppose we bought 100 shares of Internet.com common stock at the end of 1997 at 35¼. We have invested 100 × \$35.25 = \$3,525 in Internet.com stock. During 1997, Internet.com paid \$0.43 per share in dividends, so we earned \$43.00 in dividends. If we sold the Internet.com shares at the end of 1997 for 43 (\$43.00 per share, or \$4,300.00 for all 100 shares), what was the return on our investment? It depends on when the dividends were received. If we assume that the dividends were received at the end of 1997, our return was:

$$\text{Return on Internet.com for 1997} = \frac{\$4{,}300.00 - \$3{,}525.00 + \$43.00}{\$3{,}525.00}$$

$$= \frac{\$818.00}{\$3{,}525.00} = 0.2321 \text{ or } \$23.21\%$$

We can break this return into its capital yield and dividend yield components:

$$\begin{aligned}\text{Return on Internet.com for 1997} &= \frac{\$4{,}300.00 - \$3{,}525.00}{\$3{,}525.00} + \frac{\$43.00}{\$3{,}525.00}\\ &= \text{Capital yield} + \text{Dividend yield}\\ &= 0.2199 + 0.0122\\ &= 21.99\% + 1.22\%\\ &= 23.21\%\end{aligned}$$

Most of the return on Internet.com stock was from the capital yield—the appreciation in the stock's price.

Now suppose instead that Internet.com is not sold at the end of 1997, but rather sold at the end of 1999 at $50 per share. This is a more complicated problem to solve because we not only have to consider each cash flow—the purchase price, any dividends paid during the 1997–1999, and the sale price—and the time value of money.

$$P_0 = \frac{D_{1997}}{(1+r_e)^1} + \frac{D_{1998}}{(1+r_e)^2} + \frac{D_{1999}}{(1+r_e)^3} + \frac{P_{1999}}{(1+r_e)^3}$$

If dividends in 1998 and 1999 are the same as those in 1997,

$$\$3{,}525 = \frac{\$43}{(1+r_e)^1} + \frac{\$43}{(1+r_e)^2} + \frac{\$43}{(1+r_e)^3} + \frac{\$5{,}000}{(1+r_e)^3}$$

or

$$\$3{,}525 = \frac{\$43}{(1+r_e)^1} + \frac{\$43}{(1+r_e)^2} + \frac{\$5{,}043}{(1+r_e)^3}$$

Where do we begin? We can solve this using a financial calculator or trial and error. Using trial and error, we want to find the return that equates the present value of the investment—the $3,525—with the present value of the future cash flows. For example, if we try an r_e of 10%, the present value of the future cash flows is $3,863.51. Since this value is not equal to $3,525, 10% is not the rate we seek.

To reduce the present value, we must use a larger value of r_e. If we try 15%, the present value is $3,385.76 and therefore 15% is not the value we seek. So we know that the return is between 10% and 15%. Using a financial calculator we would find that the answer is 13.45%.

You can see that we can compute returns on investments whether or not we have sold them. In the cases where we do not sell the asset represented in the investment, we compute the capital yield (gain or loss) based on the market value of the asset at the point of time we are evaluating the investment. It becomes important to consider whether or not we actually realize the capital yield only when we are dealing with taxes. We must pay taxes on the capital gain only when we realize it. As long we don't sell the asset, we are not taxed on its capital appreciation.

VALUATION OF PREFERRED STOCK

The value of preferred stock is the present value of all future dividends. If a share of preferred stock has a 5% dividend (based on a \$100 par value), paid at the end of each year, today's price is the present value of the stream of \$5's forever, discounted at the rate r_p:

$$\text{Present value of preferred stock} = \frac{\$5}{(1+r_p)^1} + \frac{\$5}{(1+r_p)^2} + \frac{\$5}{(1+r_p)^3} + \ldots + \frac{\$5}{(1+r_p)^\infty} = \frac{\$5}{r_p}$$

If the discount rate is 10%, the present value of the preferred stock is \$50. That is, investors are willing to pay \$50 today for the promised stream of \$5 per year since they consider 10% to be sufficient compensation for both the time value of money and the risk associated with the perpetual stream of \$5s.

Let's rephrase this relation, letting P_p indicate today's price, D_p indicate the perpetual dividend per share per period, and r_p indicate the discount rate, (i.e., the required rate of return on the preferred stock). Then:

$$P_p = D_p/r_p$$

We can make some generalizations about the value of preferred stock:

- The greater the dividend rate, the greater the value of a share of preferred stock.
- The greater the required rate of return—the discount rate—the lower value of a share of preferred stock.

Here is another example of valuing a share of preferred stock. Consider a share of preferred stock with a par value of \$100 and a dividend rate of 12%. If the required rate of return is 15%, the value of the preferred stock is less than \$100:

$$P_p = \$12/0.15 = \$80$$

If the required rate of return declines to 10%, the price would rise to \$120.

Let's look at a feature of preferred stock that may affect its value: the call feature. If preferred stock has a ***call feature***, the issuer has the right to call it—buy it back—at a specified price per share, referred to as the ***call price***.

Suppose the dividend rate on preferred stock is \$6 per share and the preferred stock is callable after three years at par value, \$100. If the preferred stock has a required rate of return of 5%, the value of a share of preferred stock without the call is:

$$P_p = \$6/0.05 = \$120$$

Considering the call feature and assuming the issue is called in three years, we need to alter our valuation equation so that we find the present value of the first three dividends and the present value of the call price:

$$\begin{aligned} P_p &= \underbrace{\frac{\$6.00}{(1+0.05)^1} + \frac{\$6.00}{(1+0.05)^2} + \frac{\$6.00}{(1+0.05)^3}} + \underbrace{\frac{\$100.00}{(1+0.05)^3}} \\ &= \$5.71 + \$5.44 + \$5.18 + \$86.38 \\ &= \$102.71 \end{aligned}$$

If the preferred shares did not have a call feature, they would be worth more—the call feature reduces the value of the shares. What is the likelihood that the firm will call in the preferred shares? If the required rate of return is 5%—that is, investors demand a 5% return—and the stock pays \$6 on the par of \$100, or 6%, the firm can call in the 6% preferred shares and issue 5% shares. Since calling in the preferred shares makes sense—the firm can lower its costs of raising capital—it is very likely the firm will call in the preferred shares when they can.

VALUATION OF LONG-TERM DEBT SECURITIES

Long-term debt securities, such as notes and bonds, are promises by the borrower to repay the principal amount. Notes and bonds typically require the borrower to pay interest periodically, typically semiannually in the United States, and are generally stated as a percentage of the face value of the bond or note. We refer to the interest payments as coupon payments or ***coupons*** and the percentage rate as the ***coupon rate***. If these coupons are a constant amount, paid at regular intervals, we refer to the security paying them as having a ***straight coupon***. A debt security that does not have a promise to pay interest periodically but only at the maturity date is referred to as a ***zero-coupon*** note or bond.

The value of a debt security today is the present value of the promised future cash flows—the interest and the maturity value. Therefore, the present value of a debt is the sum of the present value of the interest payments and the present value of the maturity value:

Value of debt security = Present value of future interest payments
+ Present value of maturity value

To figure out the value of a debt security, we have to discount the future cash flows—the interest and maturity value—at some rate that reflects both the time value of money and the uncertainty of receiving these future cash flows. We refer to this discount rate as the ***yield***. The more uncertain the future cash flows, the greater the yield. It follows that the greater the yield, the lower the present value of the future cash flows—hence, the lower the value of the debt security.

In the case of a straight coupon security, the present value of the interest payments is the present value of an annuity. In the case of a zero-coupon security, the present value of the interest payments is zero, so the present value of the debt is the present value of the maturity value.

We can rewrite the formula for the present value of a debt security using some new notation and some familiar notation. Since there are two different cash flows—interest and maturity value—let C represent the coupon payment promised each period and M represent the maturity value. Also, let N indicate the number of periods until maturity, t indicate a specific period, and r_d indicate the yield. The present value of a debt security, V, is:

$$V = \underbrace{\sum_{t=1}^{N} \frac{C}{(1+r_d)^t}}_{\text{Present value of future interest payments}} + \underbrace{\frac{M}{(1+r_d)^t}}_{\text{Present value of maturity value}}$$

To see how the valuation of future cash flows from debt securities works, let's look at the valuation of a straight coupon bond and a zero-coupon bond.

Straight Coupon Bond

Suppose you are considering investing in a straight coupon bond that:

- Promises interest of $100, paid at the end of each year.
- Promises to pay the principal amount of $1,000 at the end of 12 years.
- Investors require an annual yield of 5%.

What is this bond worth today? We are given the following:

Interest, C	=	$100 every year
Number of periods, N	=	12 years
Maturity value, M	=	$1,000
Yield, r_d	=	5% per year

$$V = \sum_{t=1}^{12} \frac{\$100}{(1+0.05)^t} + \frac{\$1{,}000}{(1+0.05)^t} = \$886.32 + \$556.84 = \$1{,}443.16$$

Using a financial calculator,

Hewlett-Packard 10B	Hewlett-Packard 12C	Hewlett-Packard 17B	Texas Instruments BA-II Plus
100 PMT	100 PMT	100 PMT	100 PMT
12 N	12 n	12 N	12 N
1000 FV	1000 FV	1000 FV	1000 FV
5 I/YR	5 i	5 I%YR	5 I/Y
PV	PV	PV	CPT PV

This bond has a present value greater than its maturity value, so we say that the bond is selling at a ***premium*** from its maturity value. Does this make sense? Yes: The bond pays interest of 10% of its face value every year. But what investors require on their investment—the capitalization rate considering the time value of money and the uncertainty of the future cash flows—is 5%. So what happens? The bond paying 10% is attractive—*so* attractive that its price is bid upward to a price that gives investors the going rate, 5%. In other words, an investor who buys the bond for $1,443.16 will get a 5% return on it if it is held until maturity. We say that at $1,443.16, the bond is priced to yield 5% per year.

Suppose, instead, the interest on the bond is \$50 every year—a 5% coupon rate—instead of \$100 every year. Then,

Interest, C	=	\$50 every year
Number of periods, N	=	12 years
Maturity value, M	=	\$1,000
Yield, r_d	=	5% per year

$$V = \sum_{t=1}^{12} \frac{\$50}{(1+0.05)^t} + \frac{\$1{,}000}{(1+0.05)^t} = \$443.16 + \$556.84 = \$1{,}000.00$$

The bond's present value is equal to its maturity value and we say that the bond is selling "at par." Investors will pay the maturity value for a bond that pays the going rate for bonds of similar risk. In other words, if an investor buys the 5% coupon bond for \$1,000.00, the investor will earn a 5% annual return on the investment if the bond is held until maturity.[3]

Suppose, instead, the interest on the bond is \$20 every year—a 2% coupon rate. Then,

Interest, C	=	\$20 every year
Number of periods, N	=	12 years
Maturity value, M	=	\$1,000
Yield, r_d	=	5% per year

$$V = \sum_{t=1}^{12} \frac{\$20}{(1+0.05)^t} + \frac{\$1{,}000}{(1+0.05)^t} = \$177.26 + \$556.84 = \$734.10$$

The bond sells below its maturity value and is said to be trading at a *discount* from its maturity value. Why? Because investors are not going to pay the maturity value for a bond that pays less than the going rate for bonds of similar risk. If an investor can buy other bonds that yield 5%, why pay the maturity value—\$1,000 in this case—for a bond that pays only 2%? They wouldn't. Instead, the price of this bond would fall to a price that provides an investor a yield of 5%. In other words, if an investor buys the 2% coupon bond for \$734.10, the investor will earn a 5% annual return on the investment if the bond is held until maturity.

[3] This statement will be qualified later when we discuss assumptions inherent in a yield-to-maturity calculation.

So when we look at the value of a bond, we see that its present value is dependent on the relation between the coupon rate and the yield. We can see this relation in our example:

If a bond has a yield of 5% and a coupon rate of ...	it will sell for ...	so we say it is selling at ...
10%	\$1,443.16	a premium
5%	\$1,000.00	par
2%	\$734.10	a discount

As another example for valuing a straight coupon bond, suppose we have a \$1,000 face value bond with a 10% coupon rate, that pays interest at the end of each year and matures in five years. If the required yield is 5%, the value of the bond is:

$$V = \sum_{t=1}^{5} \frac{\$100}{(1+0.05)^t} + \frac{\$1{,}000}{(1+0.05)^5} = \$432.95 + \$783.53 = \$1{,}216.48$$

If the yield is 10%, the same as the coupon rate, the bond sells at maturity value:

$$V = \sum_{t=1}^{5} \frac{\$100}{(1+0.10)^t} + \frac{\$1{,}000}{(1+0.10)^5} = \$379.08 + \$620.92 = \$1{,}000.00$$

If the yield is 15%, the bond's value is less than its maturity value:

$$V = \sum_{t=1}^{5} \frac{\$100}{(1+0.15)^t} + \frac{\$1{,}000}{(1+0.15)^5} = \$335.21 + \$497.18 = \$832.39$$

When we hold the coupon rate constant and vary the required yield, we see that:

If a bond has a coupon rate of 10% and a yield of ...	it will sell for ...	so we say it is selling at ...
5%	\$1,216.48	a premium
10%	\$1,000.00	par
15%	\$832.39	a discount

We see a relation developing between the coupon rate, the yield, and the value of a debt security:

- If the coupon rate is more than the yield, the security is worth more than its maturity value—it sells at a premium.
- If the coupon rate is less than the yield, the security is less than its maturity value—it sells at a discount.
- If the coupon rate is equal to the yield, the security is valued at its maturity value.

We can extend the valuation of debt to securities that pay interest every six months. But before we do this, we must grapple with a bit of semantics. In Wall Street parlance, the term ***yield-to-maturity*** is used to describe an annualized yield on a security if the security is held to maturity. For example, if a bond has a return of 5% over a six-month period, the annualized yield-to-maturity for a year is 2 times 5% or 10%.

$$\text{Yield-to-maturity} = r_d \times 2$$

If a debt security promises interest every six months, there are a couple of things to watch out for in calculating the security's value. First, the r_d we use to discount cash flows is the *six-month yield*, not an annual yield. Second, the number of periods is the number of *six-month periods* until maturity, not the number of years to maturity.

Suppose we are interested in valuing a bond with a maturity value of \$1,000 that matures in five years and promises a coupon of 4% per year, with interest paid semiannually. This 4% coupon rate tells us that 2%, or \$20, is paid every six months. What is the bond's value if the yield-to-maturity is 6%? From the bond's description we know that:

Interest, C	=	\$20 every six months
Number of periods, N	=	$5 \times 2 = 10$ six-month periods
Maturity value, M	=	\$1,000
Yield, r_d	=	$6\%/2 = 3\%$ for six-month period

The value of the bond is:

$$V = \sum_{t=1}^{10} \frac{\$20}{(1+0.03)^t} + \frac{\$1{,}000}{(1+0.03)^{10}} = \$170.60 + \$744.09 = \$914.70$$

If the yield-to-maturity is 8%, then:

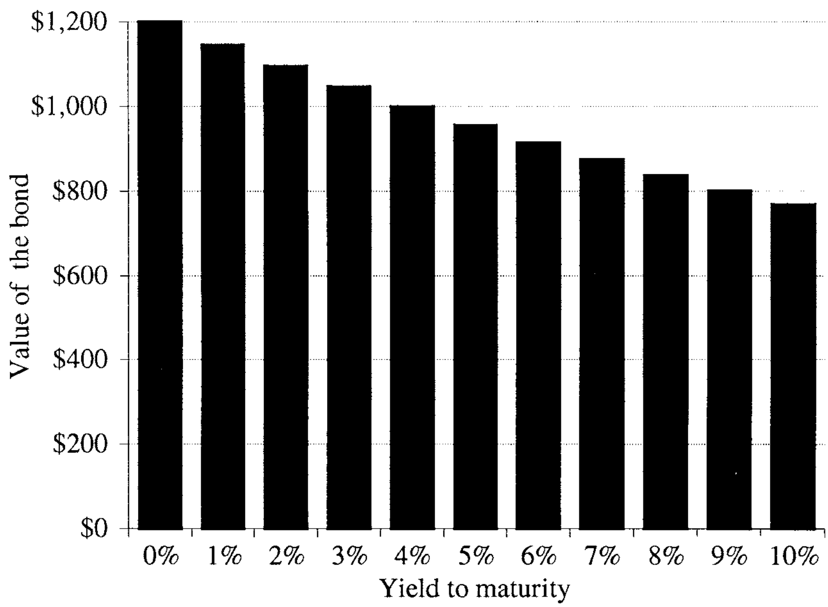
Interest, C	=	\$20 every six months
Number of periods, N	=	$5 \times 2 = 10$ six-month periods
Maturity value, M	=	\$1,000
Yield, r_d	=	8%/2 = 4% for six-month period

and the value of the bond is:

$$V = \sum_{t=1}^{10} \frac{\$20}{(1+0.04)^t} + \frac{\$1{,}000}{(1+0.04)^{10}} = \$162.22 + \$675.56 = \$837.78$$

We can see the relation between the yield-to-maturity and the value of the 4% coupon bond in Exhibit 9.3. The greater the required yield, the lower the present value of the bond. This makes sense since a higher yield-to-maturity required by the market means that the future cash flows are discounted at higher rates.

EXHIBIT 9.3 **Value of a 4% Coupon Bond with Five Years to Maturity and Semiannual Interest**



Bond-Equivalent Yield

Notice in dealing with straight-coupon bonds that pay interest semiannually, we moved from the six-month yield-to-maturity to the annual yield-to-maturity by simply doubling the six-month yield-to-maturity. The shortcoming of this approach follows from our discussion of the time value of money in Chapter 7. What the investor is interested in is the effective annual yield computed as follows for bonds that pay interest semiannually:

$$\text{Effective annual yield} = (1 + r_d)^2 - 1$$

For now, it is important to understand that the ***convention*** in the bond market for annualizing a six-month yield is to double it. The yield computed following this convention is called the ***bond-equivalent yield***. Or equivalently, a yield computed by doubling a semiannual yield is said to be computed on a ***bond-equivalent basis***.

This convention for annualizing yields must be kept in mind. We'll see why next when we show how to compute the yield on a zero-coupon bond. In addition, there are bonds that are issued in other countries that pay interest annually rather than semiannually. Consequently, the yield on either the bond that pays interest semiannually or the bond that pays interest annually must be adjusted to compare yields. While we have looked at this from the perspective of an investor, from an issuer's perspective this difference in the frequency of payments must be recognized when comparing bonds that may be issued in the U.S. bond market as well as countries such as Japan which pay interest semiannually, and bond markets in many European countries which pay interest annually.

In fact, in Chapter 26 we will discuss a funding source for corporations called asset-backed securities. Such securities typically pay interest monthly. A yield-to-maturity for such securities can be computed. (Actually, the yield computed for such securities is called a "cash flow yield.") However, the yield-to-maturity calculated is a monthly yield. How is that monthly yield annualized? The convention is to compute the effective six-month yield (we discuss the effective yield later in this chapter) and then to double that yield.

If this makes little sense to you—doubling a six-month yield—it should. All that is important is that you recognize that there is a convention used in the bond market for annualizing a yield and that an investor who is deciding among different bond alternatives or a manager considering issuing bonds in different countries should take this into consideration.

Zero-Coupon Bond

The value of a zero-coupon bond is easier to figure out than the value of a coupon bond. Let's see why. Suppose we are considering investing in a zero-coupon bond that matures in five years and has a maturity value of $1,000. If this bond does not pay interest every period no one will buy it at its maturity value. Instead, investors pay some amount *less* than the maturity value, with its return based on the difference between its maturity value and what they pay for it—assuming they hold it to maturity.

The yield to maturity on bonds with semiannual coupons is determined by taking the six-month yield and multiplying it by two—following the bond-equivalent basis convention. Though this yield is not the effective or true yield on the bond because we have not considered compounding within the year. Now what happens when we are dealing with yields on bonds that have no coupon? Convention still has us dealing with a yield-to-maturity calculated by taking the six-month yield and multiplying it by two.

If a zero-coupon bonds is priced to yield 10% its price is the present value of $1,000, discounted five years at 10%. We are given:

Maturity value, M	=	\$1,000
Number of periods, N	=	5 years × 2 = 10 six-month periods
Yield, r_d	=	10%/2 = 5% per year

The value of the debt security is:

$$V = \frac{\$1{,}000}{(1 + 0.05)^{10}} = \$613.91$$

If, instead, these bonds are priced to yield 5%,

Maturity value, M	=	\$1,000
Number of periods, N	=	5 years × 2 = 10
Yield, r_d	=	5%/2 = 2.5% per year

and the value of a bond is:

$$V = \frac{\$1{,}000}{(1 + 0.025)^{10}} = \$781.20$$

Returns on Bonds

If you invest in a bond, you realize a return from the interest it pays (if it is a coupon bond) and from either the sale, the maturity, or call of the bond. We calculate the return on a bond in the same way we calculate the return for a stock, except in the case of stock the cash flow is dividend income, rather than interest income.

There is another dimension to consider with bonds that we needn't consider with common stocks: Bonds have a finite life since they either mature or are called. Therefore, we are interested in:

- the realized return, which is the return over a specific period of time,
- the yield if the bond is held to maturity, which is the return assuming the bond is held to maturity, and
- the yield to call, which is the return on the bond assuming the bond is called.

Realized Return

A bond's return comprises the return from the appreciation or depreciation in the value of the bond over the period—the capital yield—and the return from the interest received during the period—the ***coupon yield***.

$$\text{Return} = \text{Capital yield} + \text{Coupon yield}$$

Let's look at an investment in 100 Olympic Power bonds that mature in the year 2007 with a coupon rate of 8⅞ and a par value of 1000. At the beginning of 1997, these bonds were selling at 96½ (that is, 96.5% of face value, or $965.00 per bond); at the end of 1997, they were selling for 97½. The coupon rate of 8⅞% means that they pay 8.875% on the par value of $1,000, or $88.75 for the year. If interest were paid at the end of the year, the return on 100 bonds for 1997 is:

$$\text{Return on Olympic Power } 8\tfrac{7}{8} \text{ bonds maturing in 2007}$$
$$= \frac{\$97{,}500 - \$96{,}500 + \$8{,}875}{\$96{,}500} = \frac{\$9{,}875}{\$96{,}500} = 10.2332\%$$

Breaking down this return into its capital yield and coupon yield:

$$\begin{array}{c}\text{Return on Olympic}\\ \text{Power } 8\tfrac{7}{8} \text{ bonds}\\ \text{maturing in 2007}\end{array} = \underbrace{\frac{\$97{,}500 - \$96{,}500}{\$96{,}500}}_{\text{Capital yield}} + \underbrace{\frac{\$8{,}875}{\$96{,}500}}_{\text{Coupon yield}} = 10.2332\%$$
$$= 1.0363\% + 9.1969\%$$

Because the interest is paid semiannually (each bond pays $44.375 on June 30th and December 31st), what return could you have earned if you bought 100 of these bonds on January 1, 1997 and held them through December 31, 1997? The semiannual interest payments make our computations a bit more complicated. But we can make our job easier if we lay out the cash flows in an orderly fashion:

	Beginning of January 1997	End of June 1997	End of December 1997
Bond value	$96,500.00		$97,500.00
Interest		$4,437.50	4,437.50
Total	$96,500.00	$4,437.50	$101,937.50

The yield on these bonds is such that an investment of $96,500.00 will produce cash flows of $4,437.50 after six months and $101,947.50 after 12 months. Stated in the form of a present value equation, with r_d representing the six-month yield,

$$\$96{,}500.00 = \frac{\$4{,}437.50}{(1+r_d)^1} + \frac{\$101{,}947.50}{(1+r_d)^2}$$

Where do we start to solve for r_d? We can begin at either of two places.

For one, we know these bonds are selling at a discount from their par value of $1,000. This tells us the yield is greater than the coupon rate because investors are not willing to pay the maturity value, $1,000, to get interest of 8.875% per year. Therefore, the market rate must be something greater than 8.875%. So, we know the effective annual yield must be greater than 8.875%, which means that the six-month yield must be greater than 8.875%/2 = 4.4375%.

What we know, then, is that the semiannual yield is above 4.992% Using a financial calculator, r_d is 5.1087%. If the yield over six months is 5.1087%, the effective annual yield for a year is 5.1087% compounded for two six-month periods:

$$\text{Effective annual yield} = (1 + 0.051087)^2 - 1 = 10.48\%$$

Now let's look at an example of the return on a zero-coupon bond. Suppose on January 1, 2001, you bought 10 Dot.com zero-coupon bonds maturing on December 31, 2011 for 47¾ or $477.50 per bond. On December 31, 2000, these bonds sold for 50¼, or $502.50 per bond. What is your effective annual return on these bonds during 2000?

Return on Dot.com bonds during 2000

$$= \frac{\$5{,}020 - \$4{,}775}{\$4{,}775} = \frac{\$245}{\$4{,}775} = 5.13\%$$

Yield-to-Maturity

The annual return on a bond is a measure of the yield or benefit (realized or unrealized) over a year. But for some bonds, we may be interested in knowing what yield we would earn over the longer term, such as holding them until maturity. Yield-to-maturity is the annual yield on an investment assuming the investor *holds the bond until maturity*. It considers all an investment's expected cash flows—in the case of a bond, the interest and principal. When we look at yield-to-maturity, we once again see a relation between a bond's yield and its value today.

Zero-Coupon Bonds

Looking again at the Dot.com bonds, let's figure out the return if they were held to maturity. If you hold these bonds to maturity, you will receive the $1,000 par value on each of your bonds, or $10,000.[4] To make the calculations simpler, let's assume they mature on December 31st of 2011. If you buy the bonds and hold them to maturity, you would hold them for 12 years.

The return on these bonds over the 12-year period is:

Return on Dot.com bonds 2000–2001

$$= \frac{\$10{,}000 - \$4{,}775}{\$4{,}775} = \frac{\$5{,}225}{\$4{,}775} = 109.42\%$$

Looks impressive! But this return is over 12 years. Let's see what this return is on an annual basis so that we can compare it with the annual return of other investments. And to make the yield comparable with that on a coupon bond, let's calculate the yield-to-maturity on a bond equivalent basis.

For this example:

PV = $4,775.00
FV = $10,000.00
N = 12 years × 2 = 24 six-month periods

---

[4] Of course we are assuming that the issuer of the bond, in this case Dot.com, will be able to pay the principal at maturity.

Using the basic valuation equation and inserting the known values for *FV*, *PV*, and *N*, and solving for the annual return:

$$
\begin{aligned}
FV &= PV(1 + r_d)^N \\
\$10{,}000 &= \$4{,}775.00(1 + r_d)^{24} \\
(1 + r_d)^{24} &= \$10{,}000/\$4{,}775.00 = 2.0942 \\
1 + r_d &= \sqrt[24]{2.0942} = 1.0312788 \\
r_d &= 1.0312788 - 1 = 3.12788\%
\end{aligned}
$$

The yield to maturity is therefore 3.12788% × 2 = 6.25577%. Buying these bonds at the beginning of 1990 and holding them to maturity provides an average annual return of 6.26%—the yield-to-maturity. Using a financial calculator,

Hewlett-Packard 10B	**Hewlett-Packard 12C**	**Hewlett-Packard 17B**	**Texas Instruments BA-II Plus**
24 N	24 n	24 N	24 N
10000 FV	10000 FV	10000 FV	10000 FV
4775 ± PV	4775 ± PV	4775 ± PV	4775 ± PV
I/YR	i	I%YR	CPT I/Y
× 2 =	2×	× 2 =	× 2 =

Coupon Bonds

The present value of a bond is its current market price, which is the discounted value of all future cash flows of the bond—the interest and principal. The yield to maturity on a coupon bond is the discount rate, put on an annual basis, that equates the present value of the interest and principal payments to the present value of the bond. So, in the case of a bond that pays interest semiannually, we first solve for the six-month yield, and then translate it to its equivalent annual yield-to-maturity.

Now let's look at a the yield-to-maturity on a coupon bond. Going back to the Olympic Power 8⅞% coupon bonds maturing in 2007 and with interest paid semiannually, what is the yield to maturity on these bonds if you bought them on January 1, 1997 for \$96,500.00? Or, put another way, what annual yield equates the investment of \$96,500.000 with the present value of the 22 interest cash flows and maturity value?

In this example, we know the following:

$$
\begin{aligned}
V &= \$1{,}000 \times 96.5\% \times 100 &&= \$96{,}500.00 \\
C &= (0.08875/2) \times \$1{,}000 \times 100 \text{ bonds} &&= \$4{,}437.50
\end{aligned}
$$

$$
\begin{aligned}
M &= \$1{,}000 \times 100 \text{ bonds} &&= \$100{,}000.00\\
N &= 11 \text{ years} \times 2 &&= 22 \text{ six-month periods}
\end{aligned}
$$

and t identifies the six-month period we're evaluating. Therefore,

$$\$96{,}500 = \sum_{t=1}^{22} \frac{\$4{,}437.50}{(1+r_d)^t} + \frac{\$100{,}000.00}{(1+r_d)^{22}}$$

Where do we start looking for a solution to r_d? Before we revert to our financial calculators, let's think about the value of r_d. If the bonds yielded 8⅞%, they would be selling close to par ($100,000 for our 100 bonds). This would be equivalent to a six-month value of $r_d = 4.4375\%$ for six months.

But these bonds are priced *below* par. That is, investors are not willing to pay the maturity value for these bonds because they can get a better return on similar bonds elsewhere. As a result, the price of the bonds is driven downward until these bonds provide a return or yield-to-maturity equal to that of bonds with similar risk.

Given this reasoning, the yield on these bonds must be greater than the coupon rate, so the six-month yield must be greater than 4.4375%. Using the trial and error approach, we would start with 5% as the six-month yield and look at the relation between the present value of the cash inflows (interest and principal) discounted at 5% and the price of the bonds (the $96,500.00):

$$\text{Present value of bonds using a 5\% discount rate}$$
$$= \sum_{t=1}^{22} \frac{\$4{,}437.50}{(1+0.05)^t} + \frac{\$100{,}000.00}{(1+0.05)^{22}} = \$92{,}595.81$$

or,

$$\text{Present value of bonds using a 5\% discount rate} \neq \text{Present value of bonds}$$

$$\$92{,}595.81 \neq \$96{,}500.00$$

In fact, using 5%, we have discounted too much, since the present value of the bonds using 5% is less than the present value of the bonds. Therefore, we know that r_d should be less than 5%. We now have an idea of where the yield lies: between 4.4375% and 5%. Using a financial calculator, we find the value of $r_d = 4.70\%$, a six-month yield:

Hewlett-Packard 10B	Hewlett-Packard 12C	Hewlett-Packard 17B	Texas Instruments BA-II Plus
4437.5 PMT	4437.5 PMT	4437.5 PMT	4437.5 PMT
22 N	22 n	22 N	22 N
100000 FV	100000 FV	100000 FV	100000 FV
96500 ± PV	96500 CHS PV	96500 ± PV	96500 ± PV
I/YR	i	I%YR	CPT I/Y

Translating the six-month yield into an annual yield, we find that these bonds are valued such that the yield-to-maturity is 9.4%:

$$\text{Yield to maturity} = 4.7\% \times 2 = 9.4\%$$

Another way of saying this is that the bonds are priced to yield 9.4% per year.

Why is the yield to maturity different from the annual yield of 10.48% that we calculated earlier? The annual yield was calculated using the beginning and end-of-year values of the bonds ($96,500.00 and $97,500.00), as well as the two interest payments. But the yield-to-maturity assumes that we buy the bonds for $96,500 and *hold them until 2007*, getting 22 interest payments and the $100,000 principal. So, we know that if we buy and hold these bonds for one year, we would have gotten a 10.48% annual return on our investment. But if we held onto these bonds, we would have gotten a 9.4% annual return. Remember: When we bought the bonds at the beginning of 1997, we didn't know if the price of the bonds was going to go up, down, or stay the same since we didn't know what was going to happen to interest rates during the year. But when we buy the bonds at the beginning of 1997 we do know what we will get at maturity—assuming the bond issuer is able to pay the principal at that time.

The bond's price changes from January 1, 1997 to December 31, 1997 for two reasons:

- As time progresses, the value of a bond tends toward its maturity value (we'll show why and how next).
- The value of the bonds change as yields change.

We now take a brief look at both of these considerations.

The Value of Bonds as They Approach Maturity

Let's focus on maturity, holding the yield constant at the January 1, 1997 yield. What is value of the bond if the yield-to-maturity is 9.4% per year and there are now 20 interest payments left, instead of 22? This is the same as asking: What is the value of the bonds as of December 1997—two six-month periods later—if the yield-to-maturity does not change?

$$\text{Present value of bonds on December 31, 1997}$$
$$= \sum_{t=1}^{20} \frac{\$4{,}437.50}{(1+0.047)^t} + \frac{\$100{,}000.00}{(1+0.047)^{22}} = \$96{,}643.83$$

Moving ahead one more year, to December of 1992:

$$\text{Present value of bonds on December 31, 1998}$$
$$= \sum_{t=1}^{18} \frac{\$4{,}437.50}{(1+0.047)^t} + \frac{\$100{,}000.00}{(1+0.047)^{22}} = \$96{,}858.27$$

In Exhibit 9.4, we continue this calculation for each year to maturity. We see that the value of the bond increases until it approaches the maturity value. The interest payments contribute less to the bond's present value as time goes on since there are fewer interest payments through time, yet the maturity value contributes more as the bond approaches maturity—and hence more valuable—as we get closer to maturity. The change in the value of the bond as it approaches maturity is referred to as the *time path* of the bond.

EXHIBIT 9.4 **Value of Olympic Power 8⅞% Bonds, Interest Paid Semiannually, Maturing December 31, 2000 as Maturity Approaches**

Date	Number of Periods Remaining to Maturity	Present Value of Interest Payments	Present Value of Maturity Value	Present Value of Bonds
December 31, 1997	20	$56,735.24	$39,908.59	$96,643.83
December 31, 1998	18	53,110.12	43,748.15	96,858.27
December 31, 1999	16	49,136.23	47,957.12	97,093.35
December 31, 2000	14	44,780.01	52,571.03	97,351.04
December 31, 2001	12	40,004.69	57,628.83	97,633.53
December 31, 2002	10	34,769.94	63,173.24	97,943.19
December 31, 2003	8	29,031.56	69,251.08	98,282.64
December 31, 2004	6	22,741.10	75,913.66	98,654.75
December 31, 2005	4	15,845.43	83,217.23	99,062.66
December 31, 2006	2	8,286.34	91,223.48	99,506.82
December 31, 2007	0	0	100,000.00	100,000.00

The Value of Bonds as Yields Change

If the yield-to-maturity had remained constant at 9.4% per year, what would these bonds be worth at the end of 1997? According to Exhibit 9.4, \$96,643.83. What is their value at the end of 1997 in our previous example? \$97,500. Why isn't the actual value of the bonds equal to the value predicted according to the time path of the bond? Because yields have changed. At the beginning of 1997, the bonds were priced to yield 9.4% per year to maturity. At the end of 1997, however, the value of the bonds is greater than what we would expect, given simply the passage of time.

The yield to maturity as of December 31, 1997, given a value of \$97,500, is calculated by solving for the six-month yield that equates the new market value to the present value of the interest and maturity value:

$$\text{Present value of bonds on December 31, 1997}$$
$$= \sum_{t=1}^{20} \frac{\$4{,}437.50}{(1+r_d)^t} + \frac{\$100{,}000.00}{(1+r_d)^{20}} = \$97{,}500$$

which gives us:

$$r_d = 4.632\% \text{ semiannually,}$$

or a $4.632\% \times 2 = 9.264\%$ yield-to-maturity.

If yields did not change during the year and the bonds were valued to yield 9.4% per year to maturity, the value of the bonds would have crept up to \$96,643.83 by the end of 1997. But instead, the value of the bonds increased from \$96,500 to \$97,500. Since the cash flows have not changed, the only thing that could cause the value of the bonds to deviate from \$96,643.83 is the discount rate—the yield.

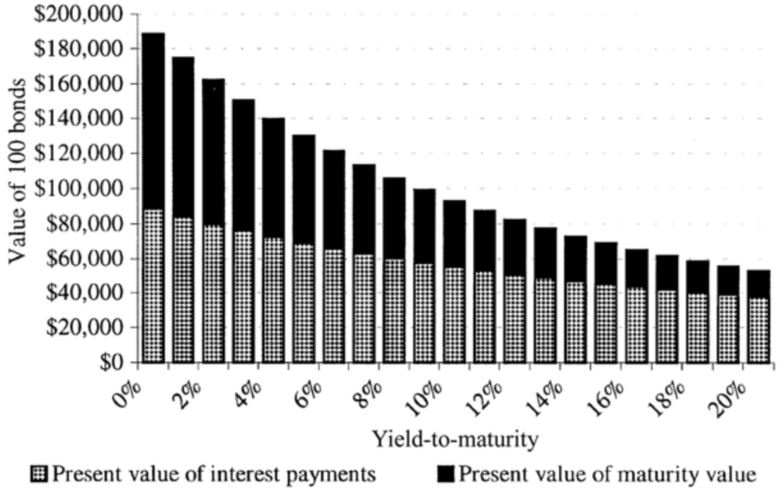
As we saw from the calculations, the yield-to-maturity decreased from 9.4% to 9.264% per year. As the yield decreased, the value of the bond increased.

Let's look once again at the value of a bond:

$$V = \sum_{t=1}^{N} \frac{C}{(1+r_d)^t} + \frac{M}{(1+r_d)^t}$$

If we hold C, T, and M constant, we see that an increase in r_d—the six-month yield—decreases the present value of the bond. Likewise, a decrease in r_d increases the present value of the bond. The value of bonds is therefore sensitive to the yield.

EXHIBIT 9.5 Value of Olympic Power Bonds for Different Yields-to-Maturity



How sensitive is the value to different yields? Let's look at an example using the Olympic Power bonds as of the end of 1997. We saw that if the bonds are valued at $97,500, this is equivalent to saying their yield to maturity is 9.264% per year. We also saw that if the bonds are valued at $96,643.83, their yield-to-maturity is 9.4% per year.

We can see the different values of the bonds as of December 1997 for different yields in Exhibit 9.5. If the bonds are priced to yield 2%, they would be worth $162,032. If the bonds are price to yield 20%, they would be worth $52,643.

Callable Bonds

Some bonds have a feature, referred to as a ***call feature***, that allows the bond issuer to buy back the bonds from the investor at a specified price—the call price—during a specified period prior the bond's maturity date. A bond with this feature is referred to as a ***callable bond***. If a bond is callable, investors are concerned with not just its yield-to-maturity, but also its return if the bond is called away. ***Yield-to-call*** is a concept similar to the yield-to-maturity. It is the yield to the date when the bond is expected to be called, instead of a yield to a bond's maturity. The yield-to-call is calculated like the yield-to-maturity, except:

- instead of the number of periods to the maturity date, N is the number of periods to some date when the bonds are expected to be called, and

- the call price of the bond is used as the maturity value, M.

The ***call price*** is specified in the bond indenture. When the bond may be called is also specified in the indenture, but it is usually a range of dates, so the precise date the firm will actually call the bond is not specified. Therefore, some assumption has to be made regarding when the bond will be called away.

Let's look at a callable bond to see how this works. Illinois Bell Telephone 8¼ debentures due in 2016 are callable in any year before maturity until the year 2011. But the call price depends on the year called. For example, if Illinois Bell calls in the bonds in 1991, the company must pay 104.95, or $1,049.50 per bond; if called in 2011, it's 100.24 or $1,002.40 per bond.

On January 1, 1991, the price was 88, or $880 per bond. This bond pays $82.50 interest each year, or $41.25 every six months. As of January 1, 1991, there were 52 interest payments remaining to maturity. Therefore, the yield to maturity as of January 1, 1991 will be the annual yield equivalent to the six-month rate that solves:

$$\$880 = \sum_{t=1}^{52} \frac{\$41.25}{(1+r_d)^t} + \frac{\$1{,}000}{(1+r_d)^{52}}$$

We know that since this is a discount bond (the value is less than its par value), the yield-to-maturity is greater than the 8¼% coupon rate. Using a financial calculator, the six-month rate is 0.0475 or 4.75%. The yield-to-maturity is therefore 4.75% × 2 = 9.5%.

The yield-to-call is calculated is a similar manner. For example, if the bonds are called at the end of 1991, Illinois Bell must pay $1,049.50 per bond at the call date and prior to the call has paid two interest payments since January 1, 1991: June and December of 1991.

Using this information (M = $1,049.50 and N = 2), the yield-to-call is calculated by determining the six-month yield and translating it into an annual yield:

$$\$880 = \sum_{t=1}^{2} \frac{\$41.25}{(1+r_d)^t} + \frac{\$1{,}049.50}{(1+r_d)^{2}}$$

The six-month rate is 13.7% and the yield-to-call is 27.4% per year.

Though the yield-to-call is usually calculated using the first available call date, we can calculate the yield-to-call for any possible call date. For bonds called at the end of the year 2000, the yield-to-call is the annual yield equivalent to the six-month yield solving using the call price for 2000 of 102.83:

$$\$880 = \sum_{t=1}^{20} \frac{\$41.25}{(1+r_d)^t} + \frac{\$1{,}028.30}{(1+r_d)^{20}}$$

The six-month yield is 5.19% and the yield-to-call is 10.38% per year.

VALUATION OF OPTIONS

In Chapter 4, we discussed options and how the price of an option can be decomposed into intrinsic value and time premium. The factors that affect the time value of an option are:

1. the value of the underlying asset;
2. the exercise price;
3. the time value of money;
4. the expected volatility in the value of the underlying asset; and
5. the time to maturity.

To see how these factors influence the value of an option, let's look at a simple option—a stock option. A ***stock option*** is the right to buy or sell a particular common stock at a specified price within a specified period. These options are not created by the company that issued the underlying stock; rather, they are created by the exchange on which the option is to be traded.

To illustrate the influence of these factors an option's value, consider the following stock option:

> *The right to buy a share of ABC stock at $40 a share before December 15th.*

Since this is a right to buy an asset, we refer to this as a ***call option***. This option gives the investor the right to buy a share of ABC stock at $40 per share—the ***exercise price***, also called the ***strike price***—before December 15th—the ***expiration date***.

If ABC stock is currently trading for $35 a share, this option is referred to as ***out-of-the-money***; that is, the current stock price is less that the exercise price of $40. Is this option worthless? The answer is no. The option to buy ABC stock at $40 a share is valuable (that is, the option is worth more than $0) since there is *some* chance that the price of ABC stock will rise above $40 a share prior to December 15th.[5]

If ABC stock is currently trading for $40 a share, this option is referred to as ***at-the-money***; that is, the current stock price is equal to the strike price. Again, the option would be worth something since ABC stock may rise above the exercise price prior to December 15th.

If the ABC stock is currently trading for $45 a share, this option is referred to as ***in-the-money***; that is, the current stock price is greater than the strike price. The option will be worth more than $5. Why? Because an investor today can buy the stock at $40 (exercising the option) and then sell it for $45 in the market, making a $5 profit. Therefore, the option is at least worth $5. Again, since the stock price has a chance of rising further prior to December 15th, the option will be worth more than $5.

From this analysis, we can see that the greater the price of the underlying asset (the stock, in this case), the greater the value of the call option. That is, there is a direct relation between the price of the underlying asset and the value of the call option.

The option value is also affected by the exercise price. For a given price of ABC stock, the lower the exercise price, the greater the value of the option. For example, assume that the price of ABC stock is $45. The option with an exercise price of $40 will have a value greater than $5. Compare this with an option on ABC with an exercise price of $35. In this latter case, the ABC option will trade for some value greater than $10. Therefore, there is an inverse relation between the exercise price and the value of the call option.

The value of the option is also affected by the time value of money. The call option is the right to buy an asset sometime in the future. Since the option represents buying in the future, the greater the opportunity cost of funds, the greater the value of the option. By delaying the purchase of the asset, you can invest your funds in other assets—the greater the return available, the greater is the value of deferring the purchase of the asset. In other words, the greater the opportunity cost, the more valuable it is to have the option, which allows you to purchase the asset in the future (instead of today).

The value of the option is also influenced by the volatility of the value of the underlying asset. If ABC stock is currently trading for $35 a

[5] Of course, the price of the stock may fall below $40. If the price of the stock is below $40, the option owner will not choose to exercise the option (that is, the option owner will not purchase the stock). Since a call option is an option to buy and not an option to sell the stock, the option's value depends on the probability that the stock's price will be *above* the strike price prior to expiration.

share, the value of the option to buy ABC stock at $40 a share is influenced by the probability that ABC stock will rise above $40 prior to the expiration date. What affects this probability? The more volatile the value of the underlying asset is expected to be, the more likely that it may increase in value prior to the expiration date. Therefore, there is a direct relation between the expected volatility of the underlying asset's value and the value of the call option.

The time remaining to expiration also affects the value of the option. For example, if today is October 15th, and the ABC stock is trading for $35 a share, there would be two months prior to the option's expiration. If, instead, today is November 15th and the ABC stock is trading for $35 a share, there is one month prior to expiration. In which case is it more likely that the option will become in-the-money before expiration? October 15th, because there is more time for the stock price to move upward. Therefore, there is a direct relation between the time to maturity of an option and the option's value.

If we alter our example to make the option a *put option* in ABC stock—that is, an option to *sell* ABC stock—we would have a different set of relations between these factors and the value of the option. Consider a put option on ABC stock:

> *The right to sell a share of ABC stock at $40 a share before December 15th.*

The put becomes more valuable in the following circumstances:

- the lower the value of the asset, since the investor in the put option gains when the exercise price is more than the asset's actual value,
- the higher the exercise price, since this means the investor can sell the asset for a higher price,
- the lower the time value of money, since the investor is delaying selling the asset and getting the proceeds from that sale,
- the more the expected volatility of the underlying asset's value, since there is no profit if the price of the underlying asset does not move, and
- the longer the time to maturity, since there is more time for the underlying asset's price to move below the exercise price.

The factors that affect the value of call and put options and their relation to the value of an option are summarized in Exhibit 9.6. Incorporating these factors mathematically into the valuation of an option or option-like security is quite complex. That's part of your advanced studies in finance.

EXHIBIT 9.6 **Relation between Call and Put Option Features and the Value of an Option**

Feature	Relation to a Call Option Value	Relation to a Put Option Value
Value of the underlying asset	*Direct relation* The greater the value of the underlying asset, the greater the value of the option.	*Inverse relation* The greater the value of the underlying asset, the lower the value of the option.
Exercise price	*Inverse relation* The lower the exercise price, the greater the value of the option.	*Direct relation* The greater the exercise price, the greater the value of the option.
Time value of money	*Direct relation* The greater the time value of money, the greater the value of the option.	*Inverse relation* The greater the time value of money, the lower the value of the option.
Volatility of the underlying asset's value	*Direct relation* The greater the volatility of the value of the underlying asset, the greater the value of the option.	*Direct relation* The greater the volatility of the value of the underlying asset, the greater the value of the option.
Time to maturity	*Direct relation* The greater the time remaining to maturity, the greater the value of the option.	*Direct relation* The greater the time remaining to maturity, the greater the value of the option.

In addition to options on stocks, as we discussed in our ABC example, there are other types of option securities.

- A ***warrant*** is the right to buy a specified stock at a specified price in a specified time period, generally attached to a corporate bond as a "sweetener" to make the bond more attractive. A warrant is therefore a call option.
- A ***detachable warrant*** is a warrant that can be sold separately from the bond and traded as a security.
- A ***right*** is a call option given to shareholders to buy additional stock in the issuing corporation (usually at a discount from the current market price) for a limited period of time. Rights can be sold by shareholders or exercised. If they are sold to another investor, they are traded as securities.

In addition to these option securities, there are also securities with option-like features. A ***convertible bond*** is a bond that can be converted into common stock at the option of the investor. This bond is therefore a combination of a straight bond (a bond without such a conversion feature) and an option to convert the bond to shares of stock. Another example is the putable bond. A ***putable bond*** is a bond that gives the investor the right to put or sell the bonds back to the issuer at a specified price, under certain specified conditions.

There are many option-like features that may affect the value of the security. These features include callability and convertibility. A bond with a call feature gives the bond issuer the right to buy back the bond from the investor for a specified price during a specified period. This feature provides the issuer with flexibility—for example, if interest rates decline, the issuer can call, or buy back, the bonds and then sell new bonds with a lower interest rate. Since the issuer is likely to call the bond when interest rates have declined below the bond's coupon rate, the investor must reinvest the proceeds received when the bond is called at a lower interest rate. Consequently, a call feature increases the risk to the investor because the investor is exposed to the risk that the proceeds received will have to be reinvested at a lower rate. As a result of this risk, investors demand a higher yield to invest in a bond that has a call feature relative to an otherwise comparable bond that does not have this feature. Looked at from the issuer's perspective, the issuer must pay a higher cost (in the form of a higher coupon rate) by issuing a bond with a call feature than one without a call feature.

A bond with a convertible feature gives the investor the right to exchange the bond for common stock of the issuer at a specified rate of exchange. This feature gives the investor flexibility. For example, if the common stock's price increases sufficiently, the investor could exchange the bond for common stock. A convertible feature therefore increases the potential return on the bond since it could be turned into stock when it is attractive to do so.

We already know the value of a debt security is affected by its return (in the form of interest and principal payments) and the uncertainty associated with these interest and principal payments. Now we know features such as callability and convertibility also affect the value of debt securities.

In addition to the options found in securities, the financial manager faces investment decisions that have options. In deciding whether or not to invest in a new product, the financial manager has the option to postpone or defer investment. This is a call option—the option to invest in the product at some future point in time.

Another example is the abandonment option. In evaluating an investment that was made in the past, the financial manager has the option to abandon the investment—stop production and sell off the assets. The option to abandon is a put option, since it is an option to *sell* the investment.

Looking at options in a broader perspective, we see that the owners of a firm have the option to not pay the creditors, halting operations, selling off assets, and distributing the proceeds. This is a put option held by the owners since they control whether or not to pay off creditors or to default.

Whether we are talking about securities that are options, securities with option-like features, or financial decisions that contain options, the same five factors listed in Exhibit 9.6 apply in valuing them. Though the precise calculation of the value of options is beyond the scope of this text, you should be able to recognize the factors affecting the value of an option and how they could influence the financial decisions you will have to make.

SUMMARY

- The value of any asset depends on the expected cash flows and the uncertainty associated with those cash flows.
- The value of a share of stock is the present value of all future dividends on the stock. These dividends may be fairly predictable and constant, as in the case of a preferred stock, or fairly unpredictable in amount and timing, as in the case of some common stock.
- The value of a debt security is the present value of the promised interest and principal payments, discounted at a rate that reflects the uncertainty associated with these cash flows.
- The Dividend Valuation Model (also known as the Gordon Model) is a formula that can be used to value a share of stock if the dividend is either constant or grows at a constant rate. The model states that the value of a share of stock is equal to the ratio of next period's dividend to the difference between the required rate of return and the growth rate of dividends.
- If dividends on a stock are expected to grow at one rate for a finite number of years and to grow at another rate after that time, the Dividend Valuation Model can be modified to accommodate these two growth rates.
- Using the Dividend Valuation Model, we can see that the required rate of return on a stock is a function of the stock's dividend yield and its capital yield. Using the same model, we also can see that the growth rate is a function of the dividend payout such that the lower the payout, the greater the growth of future dividends.

- Calculating returns on an asset uses the same tools we used to value the asset, but this time we solve for the return instead of the present value.
- The return on an asset over a specified time interval involves determining the return based on the given present value, future value, any intermediate cash flows (such as interest or dividends), and the number of periods.
- The return on a bond if it is held to maturity is referred to as the yield-to-maturity. The yield-to-maturity is the annualized return assuming that any interest earned is reinvested at the yield-to-maturity and that the bond is held until maturity. For the case of bonds of U.S. corporations or governmental entities, interest is paid semiannually; therefore, the calculation of the yield-to-maturity requires first calculating the six-month yield then multiplying this yield by two.
- To be consistent with conventions, the value and returns of zero-coupon bonds are calculated using an annualized six-month yield.
- The value of a bond may change when either the bond's yield-to-maturity changes or time passes. As a bond approaches maturity, the value of the bond converges upon the maturity value.
- If a bond is callable, an additional yield is also calculated: the yield-to-call. This is the annualized yield on the bond assuming the bond is called at a specified time and at the specified call price.
- The value of an option—whether an actual security, a security with option-like features, or embedded in an investment or financing decision—is influenced by the exercise price, the time remaining to the expiration of the option, the value of the underlying asset, the expected volatility of the value of the underlying asset, and the time value of money.

QUESTIONS

1. What are the major differences between the dividends expected to be paid to preferred stockholders and common stockholders?
2. a. Why do investors view the cash flows from debt securities of a corporation as more certain than the common stock of the corporation?
   b. What is the uncertainty regarding the cash flows for a bond that is callable?
3. Using the dividend valuation model with dividends growing at a constant rate, what is the relation between dividend growth, share price growth, and earnings growth?
4. Which of the following situations does not work with the dividend valuation model? a. no growth in dividends. b. growth in dividends

that is greater than the required rate of return. c. negative growth in dividends. d. no current dividends.

5. What is the relation between the price-earnings ratio and the growth rate of dividends?
6. If the dividend rate on preferred stock is reset every year to the going market yield on preferred stocks of similar risk, at what price would a share of preferred stock trade?
7. a. What is the relation between the expected growth rate of common stock dividends and the dividend payout?
   b. What is the rationale behind this relation?
8. The Goofy Gadget Company currently pays a dividend of $2.50 per common share. If dividends are expected to grow at a rate of 5% per year and the required rate of return on Goofy common stock is 8%, what is the value of a share of Goofy stock?
9. The Common Company has paid the following dividends during the past four years of:

Year	Dividend per Share
1997	$2.00
1998	$2.10
1999	$2.30
2000	$2.52

   If dividends are expected to grow at the same rate as the past four years and the required rate of return on Common common is 10%, what is the expected price of a share of Common common at the end of 2000?

10. The Grow-all Company has 1,000,000 shares of common stock outstanding. The company paid dividends of $6,000,000 on common stock this year. Dividends are expected to grow at a rate of 4% per year and the required rate of return on common stock is 7%. Using the dividend valuation model, what is the value of a share of Grow-all common stock?
11. The Change-all Company currently pays $2.00 of dividends on each share of common stock. The required rate of return on Change-all stock is 10%.
    a. If the expected dividend growth rate is 5% each year, forever, what is the value of a share of Change-all common stock?
    b. If the expected dividend growth rate is 2% each year, forever, what is the value of a share of Change-all common stock?
    c. If the dividend growth is expected to be 5% for the next five years and 2% thereafter, what is the value of a share of Change-all common stock?

12. The AlterG Corporation currently pays $3.00 of dividends per share of common stock. The required rate of return on AlterG stock is 5%.
    a. If the expected dividend growth rate is 2% per year, forever, what is the value of a share of AlterG common stock?
    b. If the expected dividend growth rate is 4% per year, forever, what is the value of a share of AlterG common stock?
    c. If the dividend growth rate is expected to be 4% per year of the next four years and 2% thereafter, what is the value of a share of AlterG common stock?
13. Ross purchased 100 shares of stock for $30 a share on January 1, 1996. On December 31, 2001, he sold these shares for $25 per share. What was the yield on his investment?
14. The Babson Software Company common stock currently pays dividends of $2.00 per share. Babson's stock earnings are expected to grow at a rate of 10% for the next three years and then grow at a rate of 5% thereafter. Investors demand a return of 12% on Babson's stock. Using the dividend valuation model, what is the value of Babson common stock?
15. Burlington Northern Santa Fe, Inc., paid the following dividends per share (DPS) on its common stock:

Year	DPS	Year	DPS	Year	DPS
1984	$1.10	1988	$2.20	1992	$1.20
1985	$1.45	1989	$1.20	1993	$1.20
1986	$1.55	1990	$1.20	1994	$1.20
1987	$2.05	1991	$1.20	1995	$1.20

Source: Value Line Investment Survey, Edition 2 (March 22, 1996) p. 285.

Calculate the average annual growth rate in dividends from:
    a. 1984 through 1987
    b. 1984 through 1991
    c. 1984 through 1995
16. The Perpetual Corporation issued shares of preferred at a price of $90 per share. If the dividend is fixed at $9 per share, what is the yield on the preferred shares?
17. Suppose the Everlasting Company has shares of preferred stock outstanding that pay $5 per share and are priced to yield 10%. If the yield on this stock were to change to 8%, what would be the expected effect on the shares' price?
18. For each of the following pairs of coupon rates and yields, assuming interest is paid at the end of each year, determine whether the bond will sell for more than, at, or less than its par value:

Bond	Coupon Rate	Yield-to-Maturity
A	5%	7%
B	2%	3%
C	6%	6%
D	3%	6%
E	8%	4%

19. Consider two bonds, Bond X and Bond Y, each with a maturity value of $1,000 and maturing in five years. Bond X has a coupon rate of 5% and Bond Y has a no coupon. If Bond X and Bond Y are considered to be of equal risk, which bond will have a higher value today?
20. If you determine that the yield-to-maturity on a bond with annual coupons is 10%, what rate of return are we assuming that these coupons earn when they are reinvested?
21. Consider a bond with a face value of $1,000, a coupon rate of 8% (paid annually), and a maturity in three years. What is the value of the bond if it is priced to yield 6%?
22. The IM Company issued a bond with a maturity value of $1,000, a coupon rate of 5% (paid annually), and it reaches maturity in five years. What is the value of the IM bond today if the yield-to-maturity is 4%?
23. Suppose three years ago you bought an ABC Company bond that pays 6% per year (paid semiannually) and it has three years to maturity at its par value of $1,000.
    a. If you sell the bond when it is priced to yield 8%, what is your gain or loss on this investment?
    b. If you sell the bond when it is priced to yield 4%, what is your gain or loss on this investment?
    c. If you sell the bond when it is priced to yield 10%, what is your gain or loss on this investment?
24. Arthur purchased a zero-coupon bond on January 1, 1990 for $500. On December 31, 2001, Arthur sold this bond for $750. What was the yield on this investment?
25. Consider a bond that has a current value of $1,081.11, a face value of $1,000.00, a coupon rate of 10% (paid semiannually) and five years remaining to maturity.
    a. What is the bond's yield-to-maturity today?
    b. If the bond's yield does not change, what is its value one year from today?
    c. If the bond's yield does not change, what is its value two years from today?

26. What is the value today of a zero-coupon bond with a maturity value of $1,000 and five years remaining to maturity if it is priced to yield: a. 5%? b. 8%? c. 10%? d. 12%? e. 14%?
27. The R. T. Ely Corporation issued bonds at par on January 1, 1995, with a face value of $1,000, an original maturity of five years and a coupon rate of 5% (paid annually). What is the price of one Ely bond on January 1, 1998 if Ely bonds are priced to yield 6%?
28. On January 1, 1981, the Huntington Railroad Company issued $100 million of 9⅝ bonds due 2020. Interest is paid semiannually in January and June of each year. These bonds are callable according to the following schedule:

1990–2000	at 103.0
2001–2005	at 102.0
2006–2010	at 101.0
2011–2015	at 100.5
2016–2020	at 100.0

These bonds are also convertible into shares of stock, with each $1,000 face value bond convertible into 15 shares of Huntington common stock. Huntington common stock paid a dividend of $2 per share in 1997. Its dividends are expected to grow at a rate of 10% per year for the years 1998–2002 and then slow to a rate of 5% per year thereafter. The current required rate of return on Huntington common stock is 14%.

The current yield (i.e., annual interest/market price) on the Huntington bonds is 7.5%. Interest rate forecasts for the next six years are as follows:

1998	8.00%	2001	8.75%
1999	8.50%	2002	9.00%
2000	8.50%	2003	9.00%

All indications are that yields will remain at 9% through 2020.

a. Calculate the yield-to-call for the Huntington bonds for each year from today, the end of 1997, to maturity. Plot the yield-to-call against time.

b. Forecast the stock price of Huntington common stock for each year from 1998 through 2020. Plot the predicted stock price against time.

c. Based on the yield and dividend growth forecasts, at what point in the future would it be profitable to convert the Huntington bonds into stock? Explain the basis of your decision. What other factors enter into this decision?

29. Suppose you are offered an option on an asset. This option gives you the right to buy the asset for $1,000 any time before December 31st of this year. Currently, this asset is worth $800.
    a. Is this a call option or a put option?
    b. What determines the value of this option?
    c. Currently, is this option "at-the-money," in-the-money," or "out-of-the money"?
    d. Since the exercise price is more than the asset's value today, does this mean the option is worthless? Explain.
30. a. Why is a warrant an option?
    b. Why is a right an option?
31. Explain why each of the following bonds can be viewed as a bond with an embedded option?
    a. a callable bond
    b. a putable bond
    c. a convertible bond.
32. For which type of option (put or call) does the price of the option vary inversely with the:
    a. exercise price?
    b. value of the underlying asset?
33. If the expected volatility of the underlying asset's value increases, what would happen to the price of
    a. a call option?
    b. a put option?
34. If interest rates in the market decline below the coupon rate on a callable bond that is currently callable, why would an investor say that the embedded call option is "in the money"?

CHAPTER 10

Risk and Expected Return

By now it should be clear that to make any investment or financing decision you must make your best determination of the costs involved and the benefits, or return, that will result from it. What may not yet be as clear is that there is always risk that returns may not turn out to be what you though they would be. What we're getting at, of course, is risk. Specifying a return by itself doesn't mean very much unless you also specify its risk.

After we have explained the concept of risk, we will look at how to quantify the risk of an expected return and how to incorporate risk in financial decision-making. By becoming familiar with modern portfolio theory and the role of risk in valuing assets, you will understand how a financial manager can manage risk and its relation to expected return.

RISK

Whenever you make a financing or investment decision, there is some uncertainty about the outcome. ***Uncertainty*** means not knowing exactly what will happen in the future. There is uncertainty in most everything we do as financial managers, because no one knows precisely what changes will occur in such things as tax laws, consumer demand, the economy, or interest rates.

Though the terms "risk" and "uncertainty" are often used to mean the same thing, there is a distinction between them. Uncertainty is not knowing what's going to happen. ***Risk*** is how we characterize *how much* uncertainty exists: The greater the uncertainty, the greater the risk. Risk is the degree of uncertainty.

In financing and investment decisions there are many types of risk we must consider. These include:

- Cash flow risk
    - Business risk
        - Sales risk
        - Operating risk
    - Financial risk
    - Default risk
- Reinvestment risk
    - Prepayment risk
    - Call risk
- Interest rate risk
- Purchasing power risk
- Currency risk
- Portfolio risk
    - Diversifiable risk
    - Nondiversifiable risk

Let's take a look at each of these types of risk.

Cash Flow Risk

Cash flow risk is the risk that the cash flows of an investment will not materialize as expected. For any investment, the risk that cash flows may not be as expected—in timing, amount, or both—is related to the investment's business risk.

Business Risk

Business risk is the risk associated with operating cash flows. Operating cash flows are not certain because neither are the revenues nor the expenditures comprising the cash flows.

> ***Revenues:*** depending on economic conditions and the actions of competitors, prices or quantity of sales (or both) may be different from what is expected. This is ***sales risk***.
> ***Expenditures:*** operating costs are comprised of fixed costs and variable costs. The greater the fixed component of operating costs, the less easily a company can adjust its operating costs to changes in sales.

The mixture of fixed and variable costs depends largely on the type of business. For example, fixed operating costs make up a large portion of an airline's operating costs: No matter how many passengers are flying, the airline still needs to pay gate fees, pay a pilot, and buy fuel. The variable costs for an airline—the costs that change depending on the number of passengers—amount to a little bit of fuel and the cost of the meal.

Even within the same line of business, companies can vary their fixed and variable costs. For example, an airline could develop a system that allows it to vary the number of cabin stewards and baggage handlers according to passenger traffic, varying more of its operating costs as demand changes.

We refer to the risk that comes about from the mix of fixed and variable costs as ***operating risk***. The greater the fixed operating costs relative to variable operating costs, the greater the operating risk.

Let's take a look at how operating risk affects cash flow risk. Remember back in economics when you learned about elasticity? That's a measure of the sensitivity of changes in one item to changes in another. We can look at how sensitive a firm's operating cash flows are to changes in demand, as measured by unit sales. We'll calculate the operating cash flow elasticity, which we call the ***degree of operating leverage (DOL)***.

The degree of operating leverage is the ratio of the percentage change in operating cash flows to the percentage change in units sold. Let's simplify things and assume that we sell all that we produce in the same period. Then,

$$\text{DOL} = \frac{\text{Percentage change in operating cash flows}}{\text{Percentage change in units sold}}$$

Suppose the price per unit is \$30, the variable cost per unit is \$20, and the total fixed costs are \$5,000. If we go from selling 1,000 units to selling 1,500 units, an increase of 50% of the units sold, operating cash flows change from:

	1,000 Units Sold	**1,500 Units Sold**
Sales	\$30,000	\$45,000
Less variable costs	20,000	30,000
Less fixed costs	5,000	5,000
Operating cash flow	\$5,000	\$10,000

Operating cash flows doubled when units sold increased by 50%. What if the number of units decreases by 25%, from 1,000 to 750?

	1,000 Units Sold	750 Units Sold
Sales	\$30,000	\$22,500
Less variable costs	20,000	15,000
Less fixed costs	5,000	5,000
Operating cash flow	\$5,000	\$2,500

Operating cash flows decline by 50%. For any 1% change in units sold, the operating cash flow changes by 2%, in the same direction. So if units sold increased by 10%, operating cash flows would increase by 20%; if units sold decreased by 10%, operating cash flows would decrease by 20%.

We can represent the degree of operating leverage in terms of the basic elements of the price per unit, variable cost per unit, number of units sold, and fixed operating costs. Operating cash flows are:

$$\begin{aligned}\text{Operating cash flow} = {} & (\text{Price per unit})(\text{Number of units sold}) \\ & - (\text{Variable cost per unit})(\text{Number of units sold}) \\ & - (\text{Fixed operating costs})\end{aligned}$$

How much do operating cash flows change when the number of units sold changes? It changes by the difference between the price per unit and the variable cost per unit—called the ***contribution margin***—times the change in units sold. The percentage change in operating cash flows for a given change in units sold is:

$$\text{DOL} = \frac{\begin{pmatrix}\text{Number}\\ \text{of units}\\ \text{sold}\end{pmatrix}\begin{pmatrix}\text{Price} & & \text{Variable}\\ \text{per} & - & \text{cost}\\ \text{unit} & & \text{per unit}\end{pmatrix}}{\begin{pmatrix}\text{Number}\\ \text{of units}\\ \text{sold}\end{pmatrix}\begin{pmatrix}\text{Price} & & \text{Variable}\\ \text{per} & - & \text{cost}\\ \text{unit} & & \text{per unit}\end{pmatrix} - \begin{pmatrix}\text{Fixed}\\ \text{operating}\\ \text{costs}\end{pmatrix}} \tag{10-1}$$

Applying the formula for DOL using the data in the example, we can figure out the sensitivity to change in units sold from 1,000 units:

$$\text{DOL for 1,000 units} = \frac{1{,}000(\$30 - \$20)}{1{,}000(\$30 - \$20) - \$5{,}000} = 2$$

A DOL of 2.0 means that a 1% change in units sold results in a 1% × 2.0 = 2% change in operating cash flow.

Why do we specify that the DOL is at a particular quantity sold (in this case 1,000 units)? Because the DOL will be different at different numbers of units sold. For example, at 10,000 units,

$$\text{DOL for 10,000 units} = \frac{10{,}000(\$30 - \$20)}{10{,}000(\$30 - \$20) - \$5{,}000} = 1.05$$

Let's look at situation in which the firm has shifted some of the operating costs away from fixed costs and into variable costs. Suppose the firm has a unit sales price of \$30, a variable cost of \$24 a unit, and \$1,000 in fixed costs. A change in units sold from 1,000 to 1,500—a 50% change—changes operating cash flows from \$5,000 to \$8,000, or 60%:

	1,000 Units Sold	1,500 Units Sold
Sales	\$30,000	\$45,000
Less variable costs	24,000	36,000
Less fixed costs	1,000	1,000
Operating cash flow	\$5,000	\$8,000

Then:

$$\text{DOL at 1,000 units} = \frac{1{,}000(\$30 - \$24)}{1{,}000(\$30 - \$24) - \$1{,}000} = 1.2$$

and

$$\begin{aligned} &\text{Percentage change inoperating cash flows} \\ &= \text{DOL(Percentage change in units sold)} \\ &= 1.2(50\%) = 60\% \end{aligned}$$

What we see in our calculations here is what we saw a bit earlier in our reasoning of fixed and variable costs: The greater use of fixed, relative to variable operating costs, the more sensitive operating cash flows are to changes in units sold and, therefore, more operating risk.

At 1,000 units produced and sold, we see that the DOL is 2.0; at 10,000 units, the DOL is 1.2. The degree of operating leverage is sensitive to the number of units produced and sold.

We can gain additional insight into the firm's profitability and its uncertainty by looking at the relation between profitability and the number of units produced and sold. What number of units must be produced and sold to just break even (that is, to just cover the fixed operating costs)? The answer to this question is found by rearranging the operating cash flow equation:

$$\begin{aligned}\$0 = {} & (\text{Price per unit})(\text{Number of units sold}) \\ & - (\text{Variable cost per unit})(\text{Number of units sold}) \\ & - (\text{Fixed operating costs})\end{aligned}$$

The break-even number of units, Q_{BE}, is:

$$Q_{BE} = \frac{(\text{Fixed operating costs})}{(\text{Price per unit} - \text{Variable cost per unit})} \tag{10-2}$$

Consider the example in which fixed operating costs are \$5,000, price per unit is \$30, and variable cost per unit is \$20. The break-even quantity is:

$$Q_{BE} = \frac{\$5{,}000}{(\$30 - \$20)} = 500 \text{ units}$$

If the firm produces and sells 500 units, there are no operating profits and the DOL is undefined.

Both sales risk and operating risk influence a firm's operating cash flow risk. And both sales risk and operating risk are determined in large part by the type of business the firm is in. But management has more opportunity to manage and control operating risk than they do sales risk.

Suppose a firm is deciding on which equipment to buy to produce a particular product. The sales risk is the same no matter what equipment is chosen to produce the product. But the available equipment may differ in terms of fixed and variable operating costs of producing the product. Financial managers need to consider the operating risk associated with their investment decisions.

Financial Risk

When we refer to the cash flow risk of a security, we expand our concept of cash flow risk. Since a security represents a claim on the income and assets of a business, the risk of the security is not just the risk of the cash flows of the business, but also the risk related to how these cash flows are distributed among the claimants—the creditors and owners of the business. Therefore, cash flow risk of a security includes both its business risk *and* its financial risk.

Financial risk is the risk associated with how a company finances its operations. If a company finances with debt, it is a legally obligated to pay the amounts comprising its debts when due. By taking on fixed obligations, such as debt and long-term leases, the company increases its financial risk. If a company finances its business with equity, either generated from operations (retained earnings) or from issuing new equity, it does not incur fixed obligations.

The more fixed-cost obligations (i.e., debt) incurred by the firm, the greater its financial risk. We can quantify this risk somewhat in the same way we did for operating risk, looking at the sensitivity of the cash flows available to owners when operating cash flows change. This sensitivity, which we refer to as the ***degree of financial leverage (DFL)***, is:

$$\text{DFL} = \frac{\text{Percentage change in cash flows to owners}}{\text{Percentage change in operating cash flows}}$$

The cash flows to owners are equal to operating cash flows, less interest and taxes. If operating cash flows change, how do cash flows to owners change? Suppose operating cash flows change from $5,000 to $6,000 and suppose the interest payments are $1,000 and, for simplicity and wishful thinking, the tax rate is 0%:

	Operating Cash Flow of $5,000	**Operating Cash Flow of $6,000**
Operating cash flow	$5,000	$6,000
Less interest	1,000	1,000
Cash flows to owners	$4,000	$5,000

A change in operating cash flow from $5,000 to $6,000—a 20% increase—increased cash flows to owners by $1,000—a 25% increase.

What if, instead, our fixed financial costs are $3,000? A 20% change in operating cash flows results in a 50% change in the cash flows available to owners:

	Operating Cash Flow of $5,000	**Operating Cash Flow of $6,000**
Operating cash flow	$5,000	$6,000
Interest	3,000	3,000
Cash flows to owners	$2,000	$3,000

Using more debt financing increases the sensitivity of owners' cash flows.

We can write the sensitivity of owners' cash flows to a change in operating cash flows as:

$$\text{DFL} = \frac{\begin{pmatrix}\text{Number}\\\text{of units}\\\text{sold}\end{pmatrix}\begin{pmatrix}\text{Price} & & \text{Variable}\\\text{per} & - & \text{cost}\\\text{unit} & & \text{per unit}\end{pmatrix} - \begin{pmatrix}\text{Fixed}\\\text{operating}\\\text{costs}\end{pmatrix}}{\begin{pmatrix}\text{Number}\\\text{of units}\\\text{sold}\end{pmatrix}\begin{pmatrix}\text{Price} & & \text{Variable}\\\text{per} & - & \text{cost}\\\text{unit} & & \text{per unit}\end{pmatrix} - \begin{pmatrix}\text{Fixed}\\\text{operating}\\\text{costs}\end{pmatrix} - \begin{pmatrix}\text{Fixed}\\\text{financing}\\\text{costs}\end{pmatrix}} \tag{10-3}$$

If

Number of units sold	=	1,000
Price per unit	=	\$30
Variable cost per unit	=	\$20
Fixed operating costs	=	\$5,000
Fixed financing costs	=	\$1,000

$$\text{DFL for 1,000 units} = \frac{1{,}000(\$30 - \$20) - \$5{,}000}{1{,}000(\$30 - \$20) - \$5{,}000 - \$1{,}000} = 1.25$$

Again, we need to qualify our degree of leverage by the level of production since DFL is different at different levels operating cash flows.

The firm must produce and sell a sufficient number of units to make a profit for owners. How many units are necessary? The answer is similar to what we did for the break-even in terms of operating profits, but this time we have to also cover the fixed financial costs (that is, interest). The break-even number of units considering both operating and financial costs, indicated as Q^*_{BE}, is:

$$Q^*_{BE} = \frac{(\text{Fixed operating costs}) + (\text{Fixed financing costs})}{(\text{Price per unit} - \text{Variable cost per unit})} \tag{10-4}$$

In other words, the firm must produce and sell more than Q^*_{BE} units to make a profit. In our example, the break-even number of units, with total fixed costs of \$6,000, is:

$$Q^*_{BE} = \frac{\$6{,}000}{(\$30 - \$20)} = 600 \text{ units}$$

If the firm sells 600 units, profits to owners will be zero. If the firm sells less than the 600 units, the firm has a loss and if the firm sells more than the 600 units, the firm has a profit.[1]

[1] If the firm produces and sells exactly 600 units, the DFL is undefined.

The greater the use of financing sources that require fixed obligations, such as interest, the greater the sensitivity of cash flows to owners to changes in operating cash flows.

Operating and Financial Risk

The degree of operating leverage gives us an idea of the sensitivity of operating cash flows to changes in sales. And the degree of financial leverage gives us an idea of the sensitivity of owners' cash flows to changes in operating cash flows. But often we are concerned about the *combined* effect of both operating leverage and financial leverage. Owners are concerned about the combined effect because both contribute to the risk associated with their future cash flows. And financial managers, making decisions to maximize owners' wealth, need to be concerned with how investment decisions (which affect the operating cost structure) and financing decisions (which affect the capital structure) affect owners' risk.

Let's look back on the example using fixed operating costs of \$5,000 and fixed financial costs of \$1,000. The sensitivity of owners' cash flow to a given change in units sold is affected by both operating and financial leverage.

Consider increasing the units sold up 50%. If there was no interest (and therefore no financial leverage), the owners' cash flow would equal operating cash flow. Then a 50% increase in units sold would result in a 100% increase in cash flows to owners. Now consider decreasing units sold by 50%. This would result in a 100% decrease in cash flows to owners.

But if there is financial leverage, this leverage exaggerates the effect of operating leverage. Consider again the case where there is \$1,000 of interest:

	1,000 Units Sold	1,500 Units Sold	500 Units Sold
Sales	\$30,000	\$45,000	\$15,000
Less variable costs	20,000	30,000	10,000
Less fixed costs	5,000	5,000	5,000
Operating cash flow	\$5,000	\$10,000	\$0
Less interest	1,000	1,000	1,000
Cash flows to owners	\$4,000	\$9,000	–\$1,000

If the number of units sold increases by 50%, from 1,000 to 1,500 units,

- Operating cash flows increase by 100%.

- Cash flows to owners increase by 125%.

If units sold decrease by 50%, from 1,000 to 500 units,

- Operating cash flows decrease by 100%.
- Cash flows to owners decrease by 125%.

Combining a firm's degree of operating leverage with its degree of financial leverage results in the ***degree of total leverage (DTL)***, a measure of the sensitivity of the cash flows to owners to changes in unit sales:

$$\text{DTL} = \frac{\text{Percentage change in cash flows to owners}}{\text{Percentage change in units sold}}$$

which is the same as:

$$\text{DFL} = \frac{\begin{pmatrix}\text{Number}\\ \text{of units}\\ \text{sold}\end{pmatrix}\begin{pmatrix}\text{Price} & & \text{Variable}\\ \text{per} & - & \text{cost}\\ \text{unit} & & \text{per unit}\end{pmatrix}}{\begin{pmatrix}\text{Number}\\ \text{of units}\\ \text{sold}\end{pmatrix}\begin{pmatrix}\text{Price} & & \text{Variable}\\ \text{per} & - & \text{cost}\\ \text{unit} & & \text{per unit}\end{pmatrix} - \begin{pmatrix}\text{Fixed}\\ \text{operating}\\ \text{costs}\end{pmatrix} - \begin{pmatrix}\text{Fixed}\\ \text{financing}\\ \text{costs}\end{pmatrix}} \quad (10\text{-}5)$$

and which simplifies to:

$$\text{DTL} = \text{DOL} \times \text{DFL}$$

Suppose:

Number of unit sold	=	1,000
Price per unit	=	\$30
Variable cost per unit	=	\$20
Fixed operating cost	=	\$5,000
Fixed financing cost	=	\$1,000

Then,

$$\text{DTL for 1,000 units} = \frac{1{,}000(\$30 - \$20)}{1{,}000(\$30 - \$20) - \$5{,}000 - \$1{,}000} = \frac{\$10{,}000}{\$4{,}000} = 2.5$$

which we could also have gotten from multiplying the DOL, 2, by the DFL, 1.25. This means that a 1% increase in units sold will result in a 2.5% increase in cash flows to owners; a 50% increase in units sold results in a 125% increase in cash flows to owners; a 5% decline in units sold results in a 12.5% decline in cash flows to owners; and so on.

In the case of operating leverage, the fixed operating costs act as a fulcrum: The greater the proportion of operating costs that are fixed, the more sensitive are operating cash flows to changes in sales. In the case of financial leverage, the fixed financial costs, such as interest, act as a fulcrum: The greater the proportion of financing with fixed cost sources, such as debt, the more sensitive cash flows available to owners are to changes in operating cash flows. Combining the effects of both types of leverage, we see that fixed operating and financial costs together act as a fulcrum that increases the sensitivity of cash flows available to owners to changes in the umber of units sold.

Default Risk

When you invest in a bond, you expect interest to be paid (usually semi-annually) and the principal to be paid at the maturity date. However, the more burdened a firm is with debt—required interest and principal payments—the more likely it is that payments promised to bondholders will not be made and that there will be nothing left for the owners. We refer to the cash flow risk of a debt security as ***default risk*** or ***credit risk***.

Technically, default risk on a debt security depends on the specific obligations comprising the debt. Default may result from:

- Failure to make an interest payment when promised (or within a specified period).
- Failure to make the principal payment as promised.
- Failure to make sinking fund payments (that is, amounts set aside to pay off the obligation), if these payments are required.
- Failure to meet any other condition of the loan.
- Bankruptcy.

Why do financial managers need to worry about default risk? Because they invest their firm's funds in the debt securities of other firms; because they are concerned about how investors perceive the risk of their own debt securities; and because the greater the perceived default risk of a firm's securities, the greater the firm's cost of financing.

Default risk is affected by both business risk—which includes sales risk and operating risk—and financial risk. We need to consider the effects operating and financing decisions have on the default risk of the securities a firm issues, since the risk accepted through the financing decisions affects the firm's cost of financing.

Reinvestment Rate Risk

Another type of risk is the uncertainty associated with reinvesting cash flows, not surprisingly called ***reinvestment rate risk***.

Suppose you buy a U.S. Treasury Bond that matures in five years. There is no default risk, since the U.S. government could simply print more money to pay the interest and principal. Does this mean there is no risk when you own a Treasury bond? No. You need to do something with the interest payments as you receive them and the principal amount when it matures. You could stuff them under your mattress, reinvest in another Treasury bond, or invest them otherwise. If yields have been falling, however, you cannot reinvest the interest payments from the bond and get the same return you are getting on the bond. When your Treasury bond matures, you face reinvestment risk.

If we look at an investment that produces cash flows before maturity or sale, such as a stock (with dividends) or a bond (with interest), we face a more complicated reinvestment problem. In this case we're concerned with the reinvestment of the final proceeds (at maturity or sale), but also with the reinvestment of the intermediate dividend or interest cash flows (between purchase and maturity or sale).

Let's look at the case of a five-year bond issued by Company Y, that pays 10% interest (at the end of each year, to keep things simple), and has a par value of $1,000. This bond is a ***coupon bond***; that is, interest is paid at the coupon rate of 10% per year, or $100 per bond. If you buy the bond when it is issued at the beginning of Year 1 and hold it to maturity, you will have the following cash flows:

Company Y Bond

Date	Cash Flow	
January 1, Year 1	–$1,000.00	← Purchase of bond
December 31, Year 1	100.00	
December 31, Year 2	100.00	
December 31, Year 3	100.00	
December 31, Year 4	100.00	
December 31, Year 5	1,100.00	← Proceeds of maturity and last interest payment

You face five reinvestment decisions along the life of this bond: the four intermediate flows at the end of each year, and the last and largest cash flow that consists of the last interest payment and the par value.

Suppose we wish to compare the investment in the Company Y bond with another five-year bond, issued by Company Z, that has a different cash flow stream, but a yield that is nearly the same. Company Z's bond is a zero-coupon bond; that is, it has no interest payments, so the only cash flow to the investor is the face value at maturity:

Company Z bond

Date	Cash Flow		
January 1, Year 1	–$1,000.00	←	Purchase of bond
December 31, Year 5	+$1,610.51	←	Proceeds at maturity

Both bonds have the same annual yield-to-maturity of 10%. If the yield is the same for both bonds, does this mean that they have the same reinvestment rate risk? No. Just from looking at the cash flows from these bonds we see there are intermediate cash flows to reinvest from Company Y's bond, but not from Company Z's bond.

Let's see just how sensitive the yield on the investment is to changes in the assumptions on the reinvestment of intermediate cash flows. Suppose we can reinvest the interest payments at 5%, not 10%. We calculate the yield on the bonds assuming reinvestment at 5%—a ***modified internal rate of return***—by calculating the future value of the reinvested cash flows and determining the discount rate that equates the original investment of $1,000 to this future value:

	Company Y Bond		Company Z Bond	
Date	Cash Flow	Values as of December 31, Year 5	Cash Flow	Values as of December 31, Year 5
December 31, Year 1	$100.00	$121.55		
December 31, Year 2	100.00	115.76		
December 31, Year 3	100.00	110.25		
December 31, Year 4	100.00	105.00		
December 31, Year 5	1,100.00	1,100.00	$1,610.50	$1,610.51
Future value, with cash flows reinvested at 5%		$1,552.56		$1,610.51

Using the value of the cash flow as of December 31, Year 5 as the future value and the $1,000 investment as the present value, the modified internal rates of return are 9.2% for Company Y's bond and 10% for Company Z's bond. You'll notice that the modified internal rate of return for Company Z's bond is the same as its yield-to-maturity—because there are no intermediate cash flows.

If we compare two bonds with the same yield-to-maturity and the same coupon rate, the bond with the *longer* maturity has *more* reinvestment risk. That's because it has more cash flows to reinvest throughout its life.

If we compare two bonds with the same yield-to-maturity and the same time to maturity, the bond with the *greater* coupon rate has *more* reinvestment rate risk. That's because it has more of its value coming sooner in the form of cash flows.

Two types of risk closely related to reinvestment risk of debt securities are prepayment risk and call risk. ***Prepayment risk*** is associated with certain asset-backed securities. These securities, which are discussed in Chapter 26, are created by pooling loans and using the pool as collateral for the securities. Examples of asset-backed securities issued by corporations are those backed by residential mortgage loans, automobile loans, and equipment leases. The loans have a schedule for the repayment of principal. Typically the borrower has the right to prepay a loan without a penalty at any time prior to the scheduled principal prepayment date. A payment made in excess of the schedule principal repayment is referred to as a ***prepayment***. A borrower may benefit from exercising the option to prepay if interest rates decline below the loan's interest rate. A prepayment that occurs when interest rates decline below the loan's interest rate is a disadvantage to the investor in an asset-backed security because it forces the investor to reinvest the proceeds received at a lower interest rate. This risk is referred to as prepayment risk.

Call risk is the risk that a callable security will be called by the issuer. If you invest in a callable security, there is a possibility that the issuer may call it in (buy it back). While you may receive a call premium (a specified amount above the par value), you have to reinvest the funds you receive.

There is reinvestment risk for assets other than stocks and bonds, as well. if you are investing in a new product—investing in assets to manufacture and distribute it—you expect to generate cash flows in future periods. You face a reinvestment problem with these cash flows: What can you earn by investing these cash flows? What are your future investment opportunities?

If we assume that investors do not like risk—a safe assumption—then they will want to be compensated if they take on more reinvestment rate risk. The greater the reinvestment rate risk, the greater the expected return demanded by investors.

Reinvestment rate risk is relevant to investment decisions no matter the asset and you must consider this risk in assessing the attractiveness of investments. The greater the cash flows during the life of an investment, the greater the reinvestment rate risk of the investment. And if an investment has a greater reinvestment rate risk, this must be factored into decisions.

Interest Rate Risk

Interest rate risk is the sensitivity of the change in an asset's value to changes in market interest rates. And, you should remember that market interest rates determine the rate we must use to discount a future value to a present value. The value of any investment depends on the rate used to discount its cash flows to the present. If the discount rate changes, the investment's value changes.

Suppose you invest in a project that you expect to have in operation for ten years. Two years into the project, you find that returns on alternative investments have increased. Does this affect the value of this two-year-old project? Sure. You now have a higher opportunity cost—the return on your best investment opportunity. Therefore the value of the two-year-old project is now less, and you need to determine whether to continue or terminate it. Reassessment is necessary, also, if the opportunity cost declines as well. If the return on your next best investment opportunity declines, the existing project will look even better.

Interest rate risk also is present in debt securities. If you buy a bond and intend to hold it until its maturity, you don't need to worry about its value changing as interest rates change: your return is the bond's yield-to-maturity. But if you do not intend to hold the bond to maturity, you need to worry about how changes in interest rates affect the value of your investment. As interest rates go up, the value of your bond goes down. As interest rates go down, the value of your bond goes up. This may seem wrong to you. But it's not, it's correct. Here's why.

Let's compare the change in the value of the Company Y bond to the change in the value of the Company Z bond as the market interest rate changes. (We presented these bonds in the previous section.) Suppose that it is now January 1, Year 2. If yields remain at 10%, the value of the bonds are:

$$\text{Value of Company Y bond} = \frac{\$100.00}{(1+0.10)^1} + \frac{\$100.00}{(1+0.10)^2} + \frac{\$100.00}{(1+0.10)^3} + \frac{\$1{,}100.00}{(1+0.10)^4} = \$1{,}000.00$$

and

$$\text{Value of Company Z bond} = \frac{\$1{,}610.51}{(1+0.10)^4} = \$1{,}100.00$$

If market interest rates change causing the bonds to yield 12%, the value of the Company Y and Company Z bonds are less:

$$\text{Value of Company Y bond} = \frac{\$100.00}{(1+0.12)^1} + \frac{\$100.00}{(1+0.12)^2} + \frac{\$100.00}{(1+0.12)^3} + \frac{\$1{,}100.00}{(1+0.12)^4} = \$939.25$$

and

$$\text{Value of Company Z bond} = \frac{\$1{,}610.51}{(1+0.12)^4} = \$1{,}023.51$$

If market interest rates change causing the bonds to yield 8%, the value of the Company Y and Company Z bonds is more than $1,000:

$$\text{Value of Company Y bond} = \frac{\$100.00}{(1+0.08)^1} + \frac{\$100.00}{(1+0.08)^2} + \frac{\$100.00}{(1+0.08)^3} + \frac{\$1{,}100.00}{(1+0.08)^4} = \$1{,}066.24$$

and

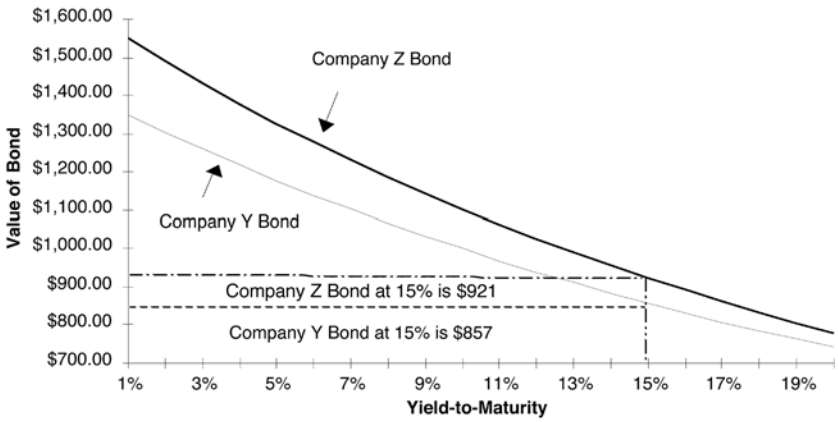
$$\text{Value of Company Z bond} = \frac{\$1{,}610.51}{(1+0.08)^4} = \$1{,}183.77$$

But how sensitive are the values of the bond to changes in market interest rates? If the bonds' yield changed on January 1, Year 2 from 10% to 12%, the value of Company Y bond would drop from $1,000.00 to $938.25—a drop of $61.75, or 6.18% of the bond's value. The drop would be greater for Company Z's bond—a drop of $76.50 or 6.95% of its value. Looking at changes in the value of the bonds for different yield changes, we see that the Company Z bond's value is more sensitive to changes in yields than is Company Y's.

The values of the two bonds for different yields as of January 1, Year 2 are shown in Exhibit 10.1. As you can see, the Company Z bond's value is more sensitive to the yield changes than is Company Y's bond.

- For a given maturity, the *greater the coupon rate*, the *less sensitive* the bond's value to a change in the yield. Why? The greater the coupon rate, the more of the bond's present value is derived from cash flows that are affected less by discounting.

EXHIBIT 10.1 **The Value of Company Y and Company Z bonds on January 1, Year 2, for Different Yields**



As another example, compare two bonds that have the same time remaining to maturity, five years, the same face value, $1,000, and both are priced to yield 10%. If Bond HC has a 10% coupon and Bond LC has a 5% coupon, a change in the yield has a greater effect on the value of Bond LC than on Bond HC. When yields change from 10% to 12%,

Bond HC's value changes from $1,000 to $928, down 7.20%
Bond LC's value changes from $810 to $748, down 7.65%

We can make an additional generalization about the sensitivity of a bond's value to changes in yields.

- For a given coupon rate, the *longer the maturity* of the bond, the *more sensitive* the bond's value to changes in market interest rates.

Compare the change in the value of two bonds that have the same coupon rate, 10% and the same face value, $1,000, with interest paid annually. If Bond SM has five years remaining to maturity and Bond LM has ten years remaining to maturity, a change in the yield on the bonds from 10% to 12% results in a greater change in Bond LM's value. When interest changes from 10% to 12%,

Bond SM's value changes from $1,000 to $928, down 7.2%
Bond LM's value changes from $1,000 to $887, down 11.3%

Purchasing Power Risk

Purchasing power risk is the risk that the price level may increase unexpectedly. If a firm locks in a price on your supply of raw materials through a long-term contract and the price level increases, it benefits from the change in the price level and your supplier loses—the firm pays the supplier in cheaper currency. If a firm borrows funds by issuing a long-term bond with a fixed coupon rate and the price level increases, the firm benefits from an increase in the price level and its creditor is harmed since interest and the principal are repaid in a cheaper currency.

Consider the 11.0% and 9.1% inflation rates for the years Year 1 and Year 2, respectively. If you borrowed \$1,000 at the beginning of Year 1 and paid it back two years later, you are paying back \$1,000 in end-of-Year 2 dollars. But how much is a Year 2 dollar worth relative to beginning-of-Year 1 dollars? We can use the compounding relation to work this out. We know that the future value is \$1,000. We also know that the rate of inflation over the two-year period is determined from compounding the two inflation rates:

$$\begin{aligned} r &= (1 + \text{Inflation rate for Year 1})(1 + \text{Inflation rate for Year 2}) - 1 \\ &= (1 + 0.110)(1 + 0.091) - 1 \\ &= 0.2110 \text{ or } 21.10\% \text{ over the two years} \end{aligned}$$

We can solve the basic valuation relation for today's value, PV, considering r to be a *two-year rate* (that is, a period is defined as the two-year stretch from the beginning of Year 1 through the end of Year 2):

$$\begin{aligned} FV &= PV(1 + r) \\ \$1{,}000 &= PV(1 + 0.2110) \end{aligned}$$

and rearranging to solve for *PV*,

$$PV = \frac{\$1{,}000}{(1 + 0.2110)} = \$825.76$$

Therefore, the \$1,000 you paid back at the end of Year 2 was really only worth \$825.80 at the beginning of Year 1. As a borrower, you have benefitted from inflation and your lender has lost.

Purchasing power risk is the risk that future cash flows may be worth less or more in the future because of inflation or deflation, respectively, *and* that the return on the investment will not compensate for the unanticipated inflation. If there is risk that the purchasing power of a currency will change, investors—who do not like risk—will demand a higher return.

Financial managers need to assess purchasing power risk in terms of both their investment decisions—making sure to figure in the risk from a change in purchasing power of cash flows—and their financing decisions—understanding how purchasing power risk affects the costs of financing.

Currency Risk

In assessing the attractiveness of an investment, we estimated future cash flows from the investment to see whether their value today—the benefits—out-weigh the cost of the investment. If we are considering making an investment that generates cash flows in another currency (some other nation's currency), there is some risk that the value of that currency will change relative to the value of our domestic currency. We refer to the risk of the change in the value of the currency as ***currency risk***. It is discussed further in Chapter 25.

Currency risk is the risk that the *relative values* of the domestic and foreign currencies will change in the future, changing the value of the future cash flows. As financial managers, we need to consider currency risk in our investment decisions that involve other currencies and make sure that the returns on these investments are sufficient compensation for the risk of changing values of currencies.

RETURN AND RISK

We refer to both future benefits and future costs as expected returns. ***Expected returns*** are a measure of the tendency of returns on an investment. This doesn't mean that these are the only returns possible, just our best measure of what we expect.

Expected Return

Suppose you are evaluating the investment in a new product. You do not know and cannot know precisely what the future cash flows will be. But from past experience, you can at least get an idea of possible flows and the likelihood—the probability—they will occur. After consulting with colleagues in marketing and production management, you determine that there are two possible cash flow outcomes, success or failure, and the probability of each outcome. Next, consulting with colleagues in production and marketing for sales prices, sales volume, and production costs, we develop the following possible cash flows in the first year:

Scenario	Cash Flow	Probability of Cash Flow
Product success	\$4,000,000	40%
Product flop	–2,000,000	60%

But what is the expected cash flow in the first year? The expected cash flow is the average of the possible cash flows, weighted by their probabilities of occurring:

$$\begin{aligned}\text{Expected cash flow} &= 0.40(\$4{,}000{,}000) + 0.60(-\$2{,}000{,}000)\\ &= \$400{,}000\end{aligned}$$

The expected cash flow is \$400,000.

The expected value is a guess about the future outcome. It is not necessarily the *most likely* outcome. The most likely outcome is the one with the highest probability. In the case of our example, the most likely outcome is –\$2,000,000.

A general formula for any expected value is:

$$\text{Expected value} = E(x) = p_1x_1 + p_2x_2 + p_3x_3 + \ldots + p_nx_n + \ldots + p_Nx_N$$

where

$E(x)$ = the expected value
n = possible outcome
N = number of possible outcomes
p_n = probability of the nth outcome
x_n = value of the nth outcome

We can abbreviate this formula by using summation notation:

$$\text{Expected value} = E(x) = \sum_{n=1}^{N} p_nx_n \qquad (10\text{-}6)$$

The calculation of the expected value requires that all possible outcomes be included. Therefore, the probabilities (the p_i's) must sum to 1.00 or 100%—if not, you have left out a possible outcome.

Applying the general formula to our example,

N = 2 (there are two possible outcomes)
p_1 = 0.40
p_2 = 0.60

$$x_1 = \$4,000,000$$
$$x_2 = -\$2,000,000$$

$$\begin{aligned} E(\text{cash flow}) &= \sum_{n=1}^{2} p_n x_n \\ &= p_1 x_1 + p_2 x_2 \\ &= (0.40(\$4,000,000) + 0.60(-\$2,000,000) = \$400,000) \end{aligned}$$

Considering the possible outcomes and their likelihoods, we expect a \$400,000 cash flow.

Standard Deviation of the Possible Outcomes

The expected return gives us an idea of the tendency of the future outcomes—what we expect to happen, considering all the possibilities. But the expected return is a single value and does not tell us anything about the diversity of the possible outcomes. Are the possible outcomes close to the expected value? Are the possible outcomes much different than the expected value? Just how much uncertainty *is* there about the future?

Since we are concerned about the degree of uncertainty (risk), as well as the expected return, we need some way of quantifying the risk associated with decisions.

Suppose we are considering two products, Product A and Product B, with estimated returns under different scenarios and their associated probabilities:

Scenario	Probability of Outcome	Possible Return on Investment
Product A		
Success	25%	24%
Moderate success	50	10
Failure	25	–4
Product B		
Success	10%	40%
Moderate success	30	30
Failure	60	–5

We refer to a product's set of the possible outcomes and their respective probabilities as the ***probability distribution*** for those outcomes.

We can calculate the expected cash flow for each product as follows:

Scenario	p_n	x_n	$p_n x_n$
Product A			
Success	0.25	0.24	0.0600
Moderate success	0.50	0.10	0.0500
Failure	0.25	–0.04	–0.0100
Expected return			0.1000 or 10%
Product B			
Success	0.10	0.40	0.0400
Moderate success	0.30	0.30	0.0900
Failure	0.60	–0.05	–0.0300
Expected return			0.1000 or 10%

Both Product A and Product B have the same expected return. Let's now see if there is any difference in the possible outcomes for the two products.

The possible returns for Product A range from –4% to 24%, where the possible returns for Product B range from –5% to 40%. The ***range*** is the span of possible outcomes. For Product A the span is 28%; for Product B the it is 45%. A wider span indicates more risk, so Product B has more risk than Product A.

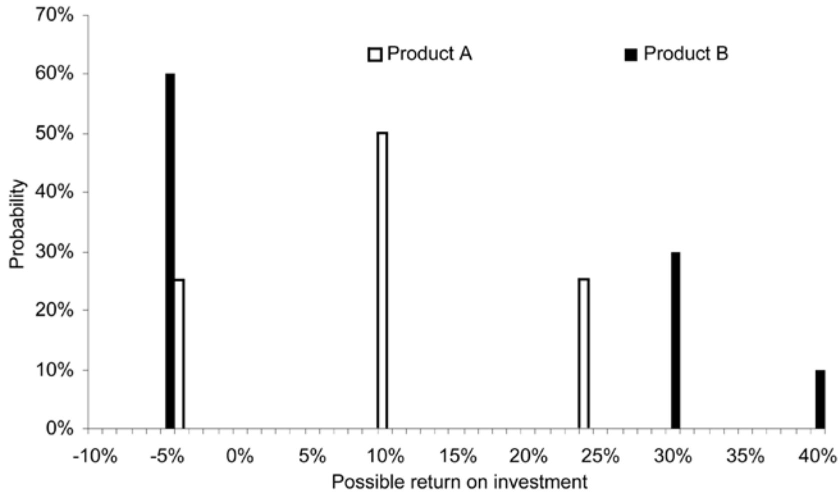
If we represent graphically the possible cash flow outcomes for Products A and B, with their corresponding probabilities, as in Exhibit 10.2, we see there is more dispersion of possible outcomes with Product B—they are more spread out—than those of Product A.

But the range by itself doesn't tell us much about the possible cash flows at these extremes nor within the extremes. Nor does the range tell us anything about the probabilities at or within the extremes.

A measure of risk that does tell us something about how much to expect and the probability that it will happen is the standard deviation. The ***standard deviation*** is a measure of dispersion that considers the values and probabilities for each possible outcome. The larger the standard deviation, the greater the dispersion of possible outcomes from the expected value. The standard deviation considers the distance (deviation) of each possible outcome from the expected value and the probability associated with that distance:

$$\text{Standard deviation of possible outcomes} = \sigma(x) = \sqrt{\sum_{n=1}^{N} p_n[x_n - E(x)]^2} \tag{10-7}$$

EXHIBIT 10.2 Probability Distribution for Product A



THE VARIANCE AND THE STANDARD DEVIATION

The variance and the standard deviation are both measures of dispersion. In fact, they are related: The standard deviation is the square root of the variance. So why do we go beyond the calculation of the variance to get the standard deviation? For two reasons.

First, the variance is in terms of squared units of measure (say, squared dollars or squared returns), whereas the standard deviation is in terms of the original unit of measure. It gets tough trying to interpret squared dollars or squared returns.

Second, if the probability distribution is approximately normally distributed (that is, bell-shaped, with certain other characteristics), we can use the standard deviation to compactly describe the probability distribution; not so with the variance. There are uses for the variance in statistical analysis, but for purposes of describing and comparing probability distributions, we focus on the expected value and the standard deviation.

The calculation of the standard deviation can be made manageable with a worksheet such as Exhibit 10.3, used to calculate the standard deviations of possible outcomes for Products A and B.

EXHIBIT 10.3 Calculation of the Standard Deviation of Possible Outcomes for Product A and Product B

Product A

Outcome	p	x	px	$x - E(x)$	$(x - E(x))^2$	$p(x - E(x))^2$
1	0.25	0.24	0.0600	0.1400	0.0196	0.0049
2	0.50	0.10	0.0500	0.0000	0.0000	0.0000
3	0.25	–0.04	–0.0100	–0.1400	0.0196	0.0049
			0.1000			σ^2 = 0.0098
						σ = 0.0990 = 9.90%

Product B

Outcome	p	x	px	$x - E(x)$	$(x - E(x))^2$	$p(x - E(x))^2$
1	0.10	0.40	0.0400	0.3000	0.0900	0.0090
2	0.30	0.30	0.0900	0.2000	0.0400	0.0120
3	0.60	–0.05	–0.0300	–0.1500	0.0225	0.0135
			0.1000			σ^2 = 0.0345
						σ = 0.1857 = 18.57%

Summarizing, we have calculated the following:

	Expected Return	Standard Deviation of Possible Outcomes
Product A	10%	9.90%
Product B	10%	18.57%

While both products have the same expected value, they differ in the distribution of possible outcomes. When we calculate the standard deviation around the expected value, we see that Product B has a larger standard deviation. The larger standard deviation for Product B tells us that Product B has more risk than Product A since its possible outcomes are more distant more from its expected value.

Return and the Tolerance for Bearing Risk

Which product investment do you prefer, A or B? Most people would choose A since it provides the same expected return with less risk. Most people do not like risk—they are ***risk averse***. Does this mean a risk averse person will not take on risk? No—they will take on risk if they feel they are compensated for it.

A ***risk neutral*** person is indifferent toward risk. Risk neutral persons do not need compensation for bearing risk. A ***risk preference*** person likes risk—someone even willing to pay to take on risk. Are there such people? Yes. Consider people who play the state lotteries, where the expected value is always negative: The expected value of the winnings is less than the cost of the lottery ticket.

When we consider financing and investment decisions, we assume that most people are risk averse. Managers, as agents for the owners, make decisions that consider risk "bad" and that if risk must be borne, they make sure there is sufficient compensation for bearing it. As agents for the owners, managers cannot have the "fun" of taking on risk for the pleasure of doing so.

Risk aversion is the link between return and risk. To evaluate a return you must consider its risk: Is there sufficient compensation (in the form of an expected return) for the investment's risk?

EXPECTED RETURN, RISK, AND DIVERSIFICATION

As managers, we rarely are consider investing in only one project at time. Small businesses and large corporations alike can be viewed as collections of different investments, made at different points in time. We refer to a collection of investments as a ***portfolio***.

While we usually think of a portfolio as a collection of securities (stocks and bonds), we can think of a business in much the same way—a portfolio of assets such as buildings, inventories, trademarks, patents, and so forth. As managers, we are concerned about the overall risk of the business's portfolio of assets.

Suppose you invested in two assets, Thing One and Thing Two, having 20% and 8% returns over the next year.

Suppose you invest equal amounts, say \$10,000, in each asset for one year. At the end of the year you expect to have \$10,000(1 + 0.20) = \$12,000 from Thing One and \$10,000(1 + 0.08) = \$10,800 from Thing Two, or a total value of \$22,800 from our original \$20,000 investment. The return on our portfolio is therefore:

$$\text{Return} = \frac{\$22{,}800 - \$20{,}000}{\$20{,}000} = 14\%$$

If instead, we invested \$5,000 in Thing One and \$15,000 in Thing Two, the value of our investment at the end of the year would be:

$$\text{Value of investment} = \$5{,}000(1 + 0.20) + \$15{,}000(1 + 0.08)$$
$$= \$6{,}000 + \$16{,}200 = \$22{,}200$$

and the return on our portfolio would be:

$$\text{Return} = \frac{\$5{,}000(1 + 0.20) + \$15{,}000(1 + 0.08) - \$20{,}000}{\$20{,}000} = 11\%$$

which we can also write as:

$$\text{Return} = \frac{\$5{,}000}{\$20{,}000}(0.20) + \frac{\$15{,}000}{\$20{,}000}(0.08) = 11\%$$

As you can see more immediately by the second calculation, the return on our portfolio is the weighted average of the returns on the assets in the portfolio, where the weights are the proportion invested in each asset.

We can generalize the formula for a portfolio return, r_p, as the weighted average of the returns of *all* assets in the portfolio, letting:

i = a particular asset in the portfolio
w_i = proportion invested in asset i
r_i = return on asset i
S = number of assets in the portfolio

Thus, $r_p = w_1 r_1 + w_2 r_2 + \ldots + w_S r_S$.

We can write more compactly as:

$$r_p = \sum_{i=1}^{S} w_i r_i \tag{10-8}$$

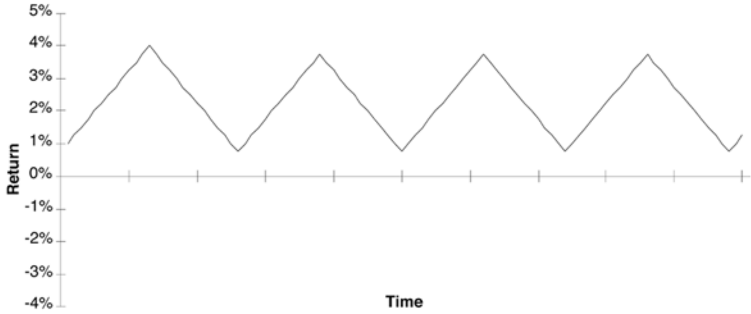
Diversification and Risk

In any portfolio, one investment may do well while another does poorly. The projects' cash flows may be "out of sync" with one another. Let's see how this might happen.

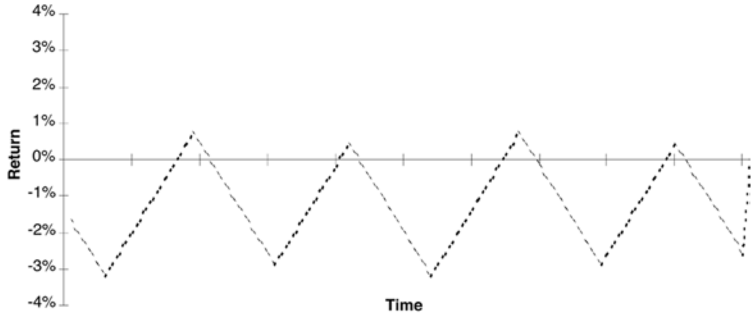
Suppose you own Asset P that produces the returns over time shown in Exhibit 10.4(a). These returns vary up and down within a wide range. Suppose you also invested in Asset Q whose returns over time are shown in Exhibit 10.4(b). These returns also vary over time within a wide band. But since the returns on Asset P and Asset Q are out of sync, each tends to provide returns when the other doesn't. The result is that your portfolio's returns vary within a narrower range as shown in Exhibit 10.4(c).

EXHIBIT 10.4 Returns on Asset P, Asset Q, and a Portfolio over Time

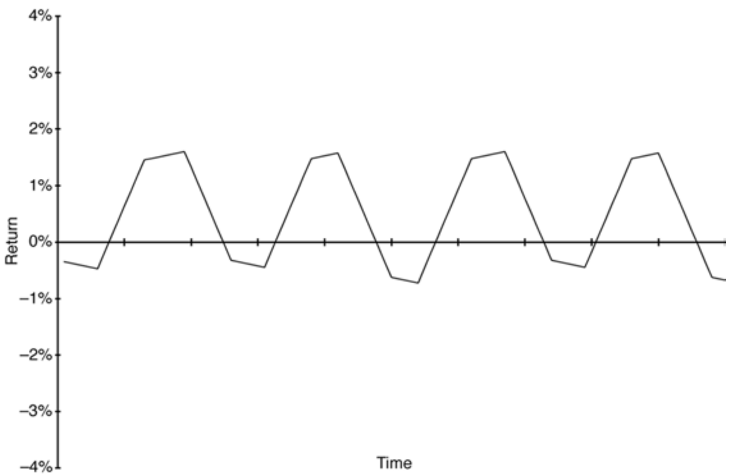
Panel A: Returns on Asset P over Time



Panel B: Returns on Asset Q over Time



Panel C: Returns on a Portfolio Comprised of Asset P and Asset Q over Time



Let's look at the idea of "out-of-syncness" in terms of expected returns, since this is what we face when we make financial decisions. Consider Investment C and Investment D and their probability distributions:

Scenario	Probability of Scenario	Return on Investment C	Return on Investment D
Boom	30%	20%	–10%
Normal	50	0	0
Recession	20	–20	45

We see that when Investment C does well, in the boom scenario, Investment D does poorly. Also, when Investment C does poorly, as in the recession scenario, Investment D does well. In other words, these investments are out of sync with one another.

Now let's look at how their "out-of-syncness" affects the risk of the portfolio of C and D. Suppose we invest an equal amount in C and D. The calculation of the expected return and standard deviation for Investment C, Investment D, and the portfolio consisting of C and D is shown in Exhibit 10.5. The expected return on Investment C is 2% and the expected return on Investment D is 6%. The return on a portfolio comprised of equal investments of C and D is expected to be 4%. The standard deviation of Investment C's return is 14% and of Investment D's return is 19.97%, but the *portfolio's* standard deviation, calculated using the weighted average of the returns on Investment C and D in each scenario, is 4.77%. This is *less* than the standard deviations of each of the individual investments because the returns of the two investments do not move in the same direction at the same time, but rather tend to move in opposite directions.

The portfolio comprised of Investments C and D has less risk than the individual investments because each moves in different directions with respect to the other. A statistical measure of how two variables—in this case, the returns on two different investments—move together is the ***covariance***. Covariance is a statistical measure of how one variable changes in relation to changes in another variable. Covariance in this example is calculated in four steps:

Step 1: For each scenario and investment, subtract the investment's expected value from its possible outcome.
Step 2: For each scenario, multiply the deviations for the two investments.
Step 3: Weight this product by the scenario's probability.
Step 4: Sum these weighted products to arrive at the covariance.

EXHIBIT 10.5 Calculation of the Standard Deviations for Investment C, Investment D, and the Portfolio Comprised of Investment C and Investment D

Investment C

Scenario	Probability p_n	Return x_n	Probability Times Return $p_n x_n$	Deviation from Expected Value $x_n - E(x)$	Squared Deviation from Expected Value $(x_n - E(x))^2$	Weighted Squared Deviations $p_n(x_n - E(x))^2$
Boom	30%	20%	0.0600	0.1800	0.0324	0.00972
Normal	50	0	0.0000	–0.0200	0.0004	0.00020
Recession	20	–20	–0.0400	–0.2200	0.0484	0.00968
	100%		$E(x) = 0.0200$			$\sigma^2(x) = 0.01960$
						$\sigma(x) = 0.1400$ or 14%

Investment D

Scenario	Probability p_n	Return x_n	Probability Times Return $p_n x_n$	Deviation from Expected Value $x_n - E(x)$	Squared Deviation from Expected Value $(x_n - E(x))^2$	Weighted Squared Deviations $p_n(x_n - E(x))^2$
Boom	30%	–10%	–0.0300	–0.1600	0.0256	0.00768
Normal	50	0	0.0000	–0.0600	0.0036	0.00018
Recession	20	45	0.0900	0.3900	0.1521	0.03042
	100%		$E(x) = 0.0600$			$\sigma^2(x) = 0.03990$
						$\sigma(x) = 0.1997$ or 19.97%

EXHIBIT 10.5 (Continued)

Portfolio of Investment C and Investment D

Scenario	Probability p_n	Return x_n	Probability Times Return $p_n x_n$	Deviation from Expected Value $x_n - E(x)$	Squared Deviation from Expected Value $(x_n - E(x))^2$	Weighted Squared Deviations $p_n(x_n - E(x))^2$
Boom	30%	5.0%	0.0150	0.0100	0.0001	0.00003
Normal	50	0.0	0.0000	–0.0400	0.0016	0.00080
Recession	20	12.5	0.0250	0.0850	0.0072	0.00145
	100%		$E(x) = 0.0400$			$\sigma^2(x) = 0.002275$
						$\sigma(x) = 0.477$ or 4.77%

Notes:
p_n = probability of outcome n occurring
x_n = outcome n
$E(x)$ = expected value
$\sigma(x)$ = standard deviation
$\sigma^2(x)$ = variance

	Step 1: Deviation of Return on Investment from its Expected Return			Step 2: Multiply Deviations Together	Step 3: Weight the Product by the Probability
Scenario	Probability	Investment C	Investment D		
Boom	0.30	0.1800	–0.1600	–0.0288	–0.00864
Normal	0.50	–0.0200	–0.0600	0.0012	0.00060
Recession	0.20	–0.2200	0.3900	–0.0858	–0.01716
				Step 4: Covariance =	–0.02520

As you can see in these calculations, in a boom economic environment, when Investment C is above its expected return (deviation is positive), Investment D is below its expected return (deviation is negative). In a recession, Investment C's return is below its expected value and Investment D's return is above its expected value. The tendency is for the returns on these portfolios to co-vary in *opposite* directions—producing a *negative* covariance of –0.0252.

Let's see the effect of this negative covariance on the risk of the portfolio. The portfolio's variance depends on:

- The weight of each asset in the portfolio.
- The standard deviation of each asset in the portfolio.
- The covariance of the assets' returns.

Let $\text{cov}_{1,2}$ represent the covariance of two assets' returns. We can write the portfolio variance as:

$$\text{Portfolio variance} = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2\text{cov}_{1,2}w_1w_2 \qquad (10\text{-}9)$$

The portfolio standard deviation is:

$$\text{Portfolio standard deviation} = \sqrt{\text{Portfolio variance}} \qquad (10\text{-}10)$$

We can apply this general formula to our example, with Investment C's characteristics indicated with a 1 and Investment D's with a 2,

w_1 = 0.50 or 50%
w_2 = 0.50 or 50%
σ_1 = 0.1400 or 14.00%
σ_2 = 0.1997 or 19.97%
$\text{cov}_{1,2}$ = -0.0252

Then:

$$\begin{aligned}&\text{Portfolio variance}\\&= 0.50^2(0.1400^2) + 0.50^2(0.1997^2) + 2(-0.0252)(0.50)(0.50)\\&= 0.002275\end{aligned}$$

and:

$$\text{Portfolio standard deviation} = \sqrt{0.002275} = 0.0477 \text{ or } 4.77\%$$

which, not coincidentally, is what we got when we calculated the standard deviation directly from the portfolio returns under the three scenarios.[2]

As we saw above, the standard deviation of the portfolio is lower than the standard deviations of each of the investments because the returns on Investments C and D are negatively related: When one is doing well the other may be doing poorly, and vice-versa. That is, the covariance is negative. The investment in assets whose returns are out of step with one another is the whole idea behind diversification. ***Diversification*** is the combination of assets whose returns do not vary with one another in the same direction at the same time.

If the returns on investments move together, we say that they are *correlated* with one another. ***Correlation*** is the tendency for two or more sets of data—in our case returns—to vary together. The returns on two investments are:

- *Positively correlated* if one tends to vary in the same direction at the same time as the other.
- *Negatively correlated* if one tends to vary in the opposite direction with respect to the other.
- *Uncorrelated* if there is no relation between the changes in one with changes in the other.

Statistically, we can measure correlation with a ***correlation coefficient.*** The correlation coefficient reflects how the returns of two securities vary together and is measured by the covariance of the two securities' returns, divided by the product of their standard deviations:

$$\begin{aligned}&\text{Correlation coefficient}\\&= \frac{\text{Covariance of two assets' returns}}{\begin{pmatrix}\text{Standard deviation of}\\ \text{returns on first asset}\end{pmatrix}\begin{pmatrix}\text{Standard deviation of}\\ \text{returns on second asset}\end{pmatrix}}\end{aligned} \tag{10-11}$$

[2] If we can calculate the standard deviation directly from the portfolio's returns, why calculate it using the individual assets' standard deviations and the covariance? We did it to illustrate the role of the assets' covariance in the portfolio's risk.

By construction, the correlation coefficient is bounded between –1 and +1.[3] We can interpret the correlation coefficient as follows:

- A **correlation coefficient of +1** indicates a perfect, positive correlation between the two assets' returns.
- A **correlation coefficient of –1** indicates a perfect, negative correlation between the two assets' returns.
- A **correlation coefficient of 0** indicates no correlation between the two assets' returns.
- A **correlation coefficients falling between 0 and +1** indicates positive, but not perfect positive correlation between the two assets' returns.
- A **correlation coefficient falling between –1 and 0** indicates negative, but not perfect negative correlation between the two assets' returns.

In the case of Investments C and D, the covariance of their returns is:

$$\begin{aligned}&\text{Correlation of returns on Investments C and D}\\&= \frac{\text{Covariance of returns Investments C and D}}{\begin{pmatrix}\text{Standard deviation of}\\ \text{returns on Investment C}\end{pmatrix}\begin{pmatrix}\text{Standard deviation of}\\ \text{returns on Investment D}\end{pmatrix}}\\&= \frac{-0.0252}{(0.1400)(0.1997)} = -0.9014\end{aligned}$$

Therefore, the returns on Investment C and Investment D are negatively correlated with one another.

By investing in assets with less than perfectly correlated cash flows, you can get rid of—diversify away—some risk. The less correlated the cash flows, the more risk you can diversify away—to a point.

Let's think about what this means for a company. Consider Proctor & Gamble whose products include Tide detergent, Prell shampoo, Pampers diapers, Jif peanut butter, and Old Spice cologne. Are the cash flows from these products positively correlated? To a degree, yes. The cash flows from these products depend on consumer spending for consumption goods. But are they *perfectly* correlated? No. For example, diaper sales depend on the diaper wearing population, whereas cologne products depend on the male cologne-wearing population. The cash flows of these different products also depend on the actions of competitors—the degree of competition may be different for the diaper market than for the peanut butter market. Further,

[3] Dividing the covariance by the product of the standard deviations insures (mathematically) that this statistic is bounded by –1 and +1, allowing a cleaner interpretation of the relation between assets' returns.

the cash flows of the products are affected by different input pricing—the costs of the raw inputs to make these products. If there is a bad year for the peanut crop, the price of peanuts may increase substantially, reducing cash flows from Jif—but this increase in peanut prices is not likely to affect the costs of, say, producing laundry detergent.

Portfolio Size and Risk

What we have seen for a portfolio with two assets can be extended to include any number of assets. The calculations become very complicated, because we have to consider the covariance between *every possible pair of assets*! But the basic idea is the same. The risk of a portfolio declines as it includes more assets whose returns are not perfectly correlated with the returns of the assets already in the portfolio.

The idea of diversification is based on beliefs about what will happen in the future: expected returns, standard deviation of all possible returns, and expected covariance between returns. How valid are our beliefs about anything in the future? We can get an idea by looking at the past. So we look at historical returns on assets—returns over time—to get an idea of how some asset's returns increase while at the same time others do not or decline.

Let's look at the effects of diversification with common stocks. As we add common stocks to a portfolio, the standard deviation of returns on the portfolio declines—to a point. We can see this in Exhibit 10.6, where the portfolio standard deviation is plotted against the number of different stocks in the portfolio. After around twenty different stocks, the portfolio's standard deviation is about as low as it is going to get.

Why does the risk seem to reach some point and not decline any farther? Because common stocks' returns, in general, are positively correlated with one another. There just aren't enough negatively correlated stocks' returns to reduce portfolio risk beyond a certain point.

We refer to the risk that goes away as we add assets as ***diversifiable risk***. We refer to the risk that *cannot* be reduced by adding more assets as ***nondiversifiable risk***. Diversifiable and nondiversifiable components of a portfolio's risk are shown in Exhibit 10.6.

The idea that we can reduce the risk of a portfolio by introducing assets whose returns are not highly correlated with one another is the basis of ***modern portfolio theory (MPT)***. MPT tells us that by combining assets whose returns are not correlated with one another, we can determine combinations of assets that provide the least risk for each possible expected portfolio return.

Though the mathematics involved in determining the optimal combinations of assets are beyond this text, the basic idea is provided in

Exhibit 10.7. In panel (a), the expected return and standard deviation for all possible portfolios is shown. Each point in the graph represents a *possible* portfolio that can be put together comprising different assets and different weights. The points in this graph represent every possible portfolio. As you can see in this diagram:

- Some portfolios have a higher expected return than other portfolios with the same level of risk.
- Some portfolios have a lower standard deviation than other portfolios with the same expected return.

Because investors like high returns and low risk, some portfolios are preferable to others. Portfolio that deliver the highest return for the level of risk make up what is called the ***efficient frontier***. If investors are rational, they will go for the portfolios that fall on this efficient frontier. All the possible portfolios, as well as the efficient frontier, are diagrammed in Exhibit 10.7, panel (b).

So what is the relevance of MPT to financial managers? MPT tells us that:

- We can manage risk by judicious combinations of assets in our portfolios.
- There are some combinations of assets that are preferred over others.

In the next section, we will see what MPT can teach us about valuation.

EXHIBIT 10.6 **The Average Standard Deviation of a Portfolio for Different Portfolio Sizes**

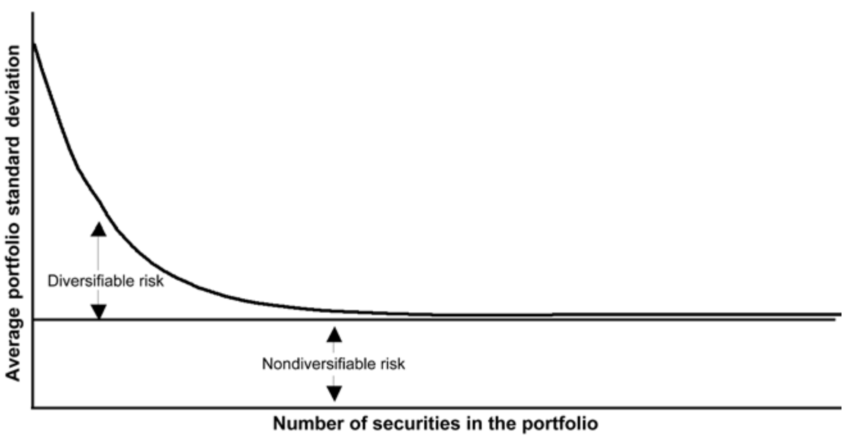
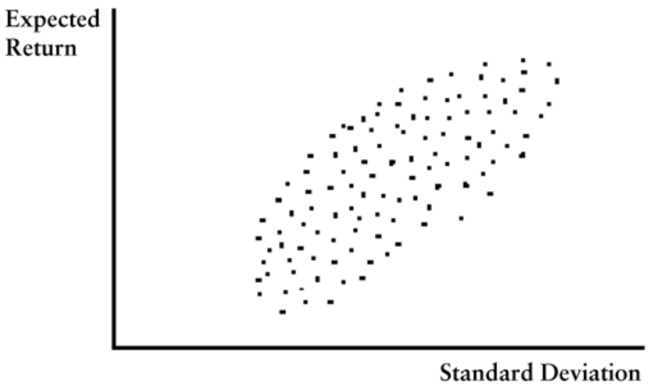
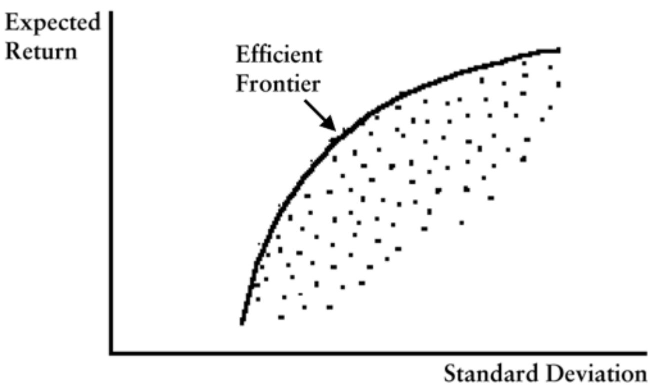


EXHIBIT 10.7 **Possible Portfolios and the Efficient Frontier**
Panel A: The Expected Return and Risk for Different Portfolios



Panel B: The Efficient Frontier



MODERN PORTFOLIO THEORY AND ASSET PRICING

The relation between portfolio returns and portfolio risk was recognized by two Nobel Laureates in Economics, Harry Markowitz and William Sharpe. Harry Markowitz tuned us into the idea that investors hold portfolios of assets and therefore their concern is focused upon the portfolio return and the portfolio risk, not on the return and risk of individual assets.[4]

[4] Harry M. Markowitz, "Portfolio Selection," *Journal of Finance* (March 1952) pp. 77–91.

The ***relevant risk*** to an investor is the portfolio's risk, not the risk of an individual asset. If an investor considers buying an additional asset or selling an asset from the portfolio, what must be considered is how this change will affect the *risk* of the portfolio. This concept applies whether we are talking about an investor who holds 30 different stocks or a business that has invested in 30 different projects. The important thing in valuing an asset is its contribution to the portfolio's return and risk.

The Capital Asset Pricing Model

William Sharpe took the idea that portfolio return and risk are the only elements to consider and developed a model that deals with how assets are priced.[5] This model is referred to as the ***capital asset pricing model (CAPM)***.

We just saw in Exhibit 10.7 that there is a set of portfolios that make up the efficient frontier—the best combinations of expected return and standard deviation. All the assets in each portfolio, even on the frontier, have some risk. Now let's see what happens when we add an asset with no risk—referred to as the risk-free asset. Suppose we have a portfolio along the efficient frontier that has a return of 4% and a standard deviation of 3%. Suppose we introduce into this portfolio the risk-free asset, which has an expected return of 2% and, by definition, a standard deviation of zero. If the risk-free asset's expected return is certain, there is *no* covariance between the risky portfolio's returns and the returns of the risk-free asset.

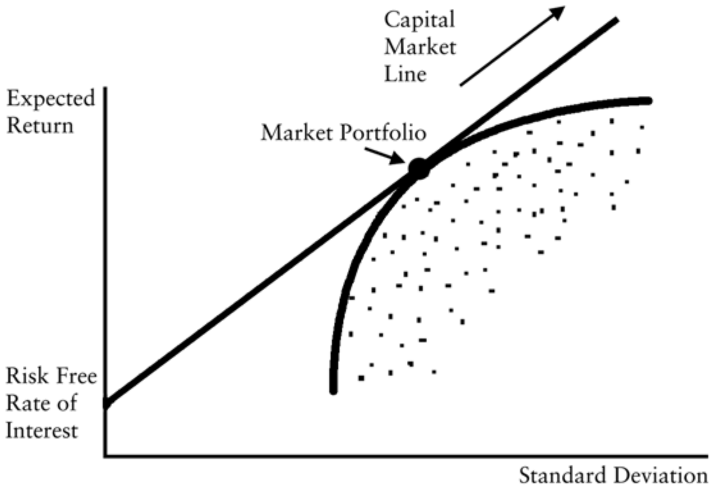
A portfolio comprised of 50% of the risky portfolio and 50% of the risk-free asset has an expected return of (0.50)4% + (0.50)2% = 3% and a portfolio standard deviation calculated as follows:

$$\begin{aligned}
&\text{Portfolio standard deviation}\\
&= \sqrt{0.50^2(0.03) + 0.50^2(0.00) + 2(0.00)0.50(0.50)}\\
&= \sqrt{0.0075} = 0.0866
\end{aligned}$$

If we look at all possible combinations of portfolios along the efficient frontier and the risk-free asset, we see that the best portfolios are no longer those along the entire length of the efficient frontier; rather, the best portfolios are now the combinations of the risk-free asset and one—and only one—portfolio of risky assets on the frontier. The portfolios comprised of the risk-free asset and this one risky portfolio are shown in Exhibit 10.8. These portfolios differ from one another by the proportion invested in the risk-free asset; as less is invested in the risk-free asset, both the portfolio's expected return and standard deviation increase.

[5] William F. Sharpe, "A Simplified Model of Portfolio Analysis," *Management Science* (January 1963), pp. 277–293.

EXHIBIT 10.8 **The Expected Return and Risk for All Possible Portfolios of Assets, Including a Risk-Free Asset**



Sharpe demonstrates that this one and only one portfolio of risky assets is the ***market portfolio***—a portfolio that consists of all assets, with the weights of these assets being the ratio of their market value to the total market value of all assets.

If investors are all risk averse—they only take on risk if there is adequate compensation—and if they are free to invest in the risky assets as well as the risk-free asset, the best deals lie along the line that is tangent to the efficient frontier. This line is referred to as the ***capital market line (CML)***. If the portfolios along the capital market line are the best deals and are available to all investors, it follows that the returns of these risky assets will be priced to compensate investors for the risk they bear *relative to that of the market portfolio*. Since the portfolios along the capital market line are the best deals, they are as diversified as they can get—no other combination of risky assets or risk-free asset provides a better expected return for the level of risk or provides a lower risk for the level of expected return.

The CML specifies the returns an investor can expect for a given level of risk. The CAPM uses this relationship between expected return and risk to describe how assets are priced.

The CAPM specifies that the return on any asset is a function of the return on a risk-free asset plus a risk premium. The return on the risk-free asset is compensation for the time value of money. The ***risk premium*** is the compensation for bearing risk. Putting these components of return together, the CAPM says:

Expected return on an asset
= Expected return on a risk-free asset + Risk premium

The market portfolio therefore represents the most well-diversified portfolio—the portfolio that consists of all the assets in a market. The only risk in a portfolio comprising all assets is nondiversifiable risk. As far as diversification goes, the market portfolio is the best you can do, because you have included everything in it.

Thus, if we assume that investors hold well-diversified portfolios (approximating the market portfolio), the only risk they have is nondiversifiable risk. If assets are priced to compensate for the risk of assets *and* if the only risk in your portfolio is nondiversifiable risk, then it follows that compensation for risk applies to only nondiversifiable risk. Let's refer to this nondiversifiable risk as ***market risk***.

Because the market portfolio is made up of all assets, each asset possesses some degree of market risk. Since market risk is systematic across assets, it is often referred to as ***systematic risk***, and diversifiable risk is referred to as ***unsystematic risk***. Further, the risk that is not associated with the market as a whole is often referred to as ***company-specific risk*** when referring to stocks, since it is risk that is specific to the company's own situation—such as the risk of lawsuits and labor strikes—and is not part of the risk that pervades all securities.

The measure of an asset's return sensitivity to the market's return, its market risk, is referred to as that asset's ***beta, ß***.

The expected return on an individual asset is the sum of the expected return on the risk-free asset and the premium for bearing market risk. Let r_i represent the expected return on asset i, r_f represent the expected return on the risk-free asset, r_m represent the expected return on the market, and β_i represent the degree of market risk for asset i. Then:

$$r_i = r_f + (r_m - r_f)\beta_i \tag{10-12}$$

The term $(r_m - r_f)$, is the ***market risk premium***—if you owned all the assets in the market portfolio, you would expect to be compensated $(r_m - r_f)$ for bearing the risk of these assets. β is measure of market risk, which serves to fine-tune the risk premium for the individual asset. For example, if the market risk premium were 2% and the β for an individual asset were 1.5, you would expect to receive a risk premium of 3% since you are taking on 50% more risk than the market.

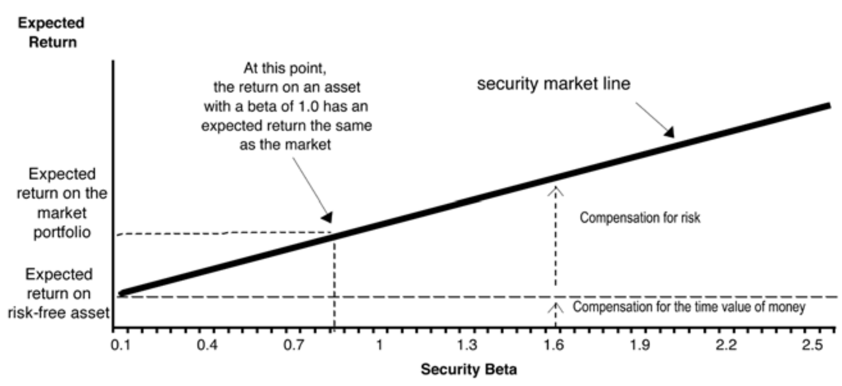
For each asset there is a beta. If we represent the expected return on each asset and its beta as a point on a graph and connect all the points, the result is the ***security market line (SML)***, as shown in Exhibit 10.9.[6] As you can see in the figure:

1. The greater the β, the greater the expected return.
2. If there were no market risk (beta = 0.0) on an asset, its expected return would be the expected return on the risk-free asset.
3. If the asset's risk is similar to the risk of the market as a whole (beta = 1.0), that asset's expected return is the return on the market portfolio.

For an individual asset, beta is a measure of sensitivity of its returns to changes in return on the market portfolio. If beta is *one*, we expect that for a given change of 1% in the market portfolio return, the asset's return is expected to change by 1%. If beta is *less than one*, then for a 1% change in the expected market return, the asset's return is expected to change by less than 1%. If the beta is *greater than one*, then for a 1% change in the expected market return, the asset's return is expected to change by more than 1%.

We typically estimate the beta for a common stock by looking at the historical relation between its return and the return on the market as a whole. The betas of some U.S. companies' common stocks are listed in Exhibit 10.10.

EXHIBIT 10.9 **Security Market Line that Describes the Relation Between Expected Asset Returns and Beta**



[6] You'll notice that in the discussion of the asset pricing models we refer to the risk and expected return for *assets*, which can be any asset, not exclusively stocks. The Security Market Line describes a relation between expected return and market risk that is applicable to any asset, even though the name implies securities.

EXHIBIT 10.10 **Security Betas of U.S. Companies' Common Stock**

Company	Industry	Security Beta
Amazon.com	Retailer (Internet)	1.85
Bank of America	Financial services	1.30
Baxter International, Inc.	Pharmaceuticals	0.80
Carnival Corporation	Recreation	1.50
Coca-Cola	Beverages	0.80
Cooper Tire and Rubber	Tire and rubber	0.95
Dole Foods	Food processing	0.75
FedEx Corporation	Air transport	0.80
Hershey Foods	Food processing	0.60
Home Depot	Retail (building supply)	1.40
Limited brands	Retail (clothing)	1.20
Mattel Inc.	Recreation	0.75
N.Y. Times	Newspaper	1.00
Papa John's International	Restaurant	0.75
Toys R Us	Retail (special lines)	1.15
Unisys	Computer and office equipment	1.25

Source: Value Line Investment Survey, September 20, 2002

The betas of some firms' stocks are close to 1.0, indicating that the returns on these stocks tend to move along with the market. There are several with betas less than 1.0, indicating that the return on this security does not move along with the market: If the market were to go up 10%, we would expect the return to go down only about 8% for FedEx, for example. Then there are some stocks whose beta is much higher than 1.0. For example, Amazon.com has a beta of 1.85. This means that if the market is expected to go up 1%, we expect Amazon's return to go up 1.85%; if the market is expected to go down 1%, we expect Amazon's return to go down 1.85%.

If we know part of the risk of a particular asset is common to all assets, and we have a large enough representation of all in the assets in our portfolio, then we don't need to be concerned with the diversifiable risk. We are concerned about the market risk of each asset in the portfolio and how it contributes to the market risk of the entire portfolio.

We can get a good idea of the portfolio's market risk by using a beta that represents the composition of the assets in the portfolio. To determine the portfolio's beta, we need to know the weighted average of the betas of the assets that make up the portfolio, where each weight is the proportion invested in each asset. Let β_p indicate the beta of the portfolio, w_i indicate the proportion invested in each the asset i, and β_i indicate the beta for asset i. If there are S assets in the portfolio, then:

$$\beta_p = w_1\beta_1 + w_2\beta_2 + w_3\beta_3 + \ldots + w_S\beta_S \tag{10-13}$$

or more compactly,

$$\beta_p = \sum_{i=1}^{S} w_i\beta_i \tag{10-14}$$

Suppose we have three securities in our portfolio, with the amount invested in each and their security beta as follows:

Security	Security Beta	Amount Invested
AAA	1.00	\$10,000
BBB	1.50	\$20,000
CCC	0.75	\$20,000

The portfolio's beta is:

$$\beta_p = \frac{\$10{,}000}{\$50{,}000}1.00 + \frac{\$20{,}000}{\$50{,}000}1.50 + \frac{\$20{,}000}{\$50{,}000}0.75 = 1.10$$

If the expected risk-free rate of interest is 4% and the expected return on the market is 7%, the $\beta_p = 1.1$ means:

Expected return on portfolio of AAA, BBB, and CCC
$= 4\% + 1.10(7\% - 4\%) = 7.3\%$

Limitations of the CAPM

As we have seen, the CAPM allows us to focus on the risk that is important in asset pricing—market risk. However, there are some drawbacks to applying the CAPM.

1. A beta is an estimate. For stocks, the beta is typically estimated using historical returns. But the estimate for beta depends on the method and period in which is it is measured. For assets other than stocks, beta estimation is more difficult.
2. The CAPM includes some unrealistic assumptions. For example, it assumes that all investors can borrow and lend at the same rate.

3. The CAPM is really not testable. The market portfolio is theoretical and not really observable, so we cannot test the relation between the expected return on an asset and the expected return of the market to see if the relation specified in the CAPM holds.
4. In studies of the CAPM applied to common stocks, the CAPM does not explain the differences in returns for securities that differ over time, differ on the basis of dividend yield, and differ on the basis of the market value of equity (the so called "size effect").

Though it lacks realism and is difficult to apply, the CAPM makes some sense regarding the role of diversification and the type of risk we need to consider in investment decisions.

The Arbitrage Pricing Model

An alternative to CAPM in relating risk and return is the arbitrage pricing model, which was developed by Stephen Ross. The ***arbitrage pricing model (APM)*** is an asset pricing model that is based on the idea that identical assets in different markets should be priced identically.[7]

While the CAPM is based on a market portfolio of assets, the APM doesn't mention a market portfolio at all. Instead, the APM states that an asset's returns should compensate the investor for the risk of the asset where the risk is due to a number of economic influences or company factors. Therefore, the expected return on the asset i, r_i, is:

$$r_i = r_f + \delta_1 \text{First factor} + \delta_2 \text{Second factor} + \delta_3 \text{Third factor} + \ldots \quad (10\text{-}15)$$

where each of the δ's reflect the asset's return sensitivity to the corresponding economic factor. The APM looks much like the CAPM, but the CAPM has one factor—the market portfolio. There are many factors in the APM.

What if an asset's price is out of line with what is expected? That's where arbitrage comes in. Any time an asset's price is out of line with how market participants feel it should be priced—based on the basic economic influences—investors will enter the market and buy or sell the asset until its price is in line with what they think it should be.

The APM provides theoretical support for an asset pricing model where there is more than one risk factor. Consequently, models of this type are referred to as ***multifactor risk models***. There are three types of

[7] Stephen A. Ross, "The Arbitrage Theory of Capital Asset Pricing," *Journal of Economic Theory*, December 1976.

multifactor risk models: statistical factor models, macroeconomic factor models, and fundamental factor models.[8]

In a ***statistical factor model*** a statistical technique called factor analysis is used to derive risk factors that best explain observed asset returns. Let's suppose that there are six "factors" identified by the model that are statistically found to best explain common stock returns. These "factors" are statistical artifacts. The objective in a statistical factor model then becomes to determine the economic meaning of each of these statistically derived factors. Because of the problem of interpretation, it is difficult to use the factors from a statistical factor model. Instead, practitioners prefer the two other models described below, which allow them to prespecify meaningful factors, and thus produce a more intuitive model.

In a ***macroeconomic factor model***, observable macroeconomic variables are used to try to explain observed asset returns. An example of a proprietary macroeconomic factor model is the Burmeister, Ibbotson, Roll, and Ross model.[9] In this model, there are five macroeconomic factors that have been found that do a good job of explaining common stock returns. They are unanticipated changes in the following macroeconomic variables: investor confidence (confidence risk); interest rates (time horizon risk); inflation (inflation risk); real business activity (business cycle risk); and market index (market timing risk).

The most common model used by practitioners is the ***fundamental factor model***. It uses company and industry attributes and market data to determine the factors that best explain observed asset returns.[10] The most often used fundamental factor model for explaining common stock returns is the one developed by the firm of Barra. In the Barra model the risk factors, referred to as risk indexes, are indexes of stock price volatility, stock price momentum, market capitalization (size) of the firm, earnings growth, earnings yield, book-to-value ratio, earnings variability, exposure to foreign currencies, dividend yield, and leverage. In addition, the Barra model indicates that the industry that a firm is in is another factor that explains the return on common stock. In the Barra model there are 55 industry groups.

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[8] Gregory Connor, "The Three Types of Factor Models: A Comparison of Their Explanatory Power," *Financial Analysts Journal* (May–June 1995), pp. 42–57.

[9] Edwin Burmeister, Roger Ibbotson, Richard Roll, and Stephen A. Ross, "Using Macroeconomic Factors to Control Portfolio Risk," unpublished paper. The information used in this chapter regarding the BIRR model is obtained from various pages of the BIRR website (*www.birr.com*).

[10] For a further discussion of fundamental factor models, see Frank J. Fabozzi, Frank J. Jones, and Raman Vardharaj, "Multi-Factor Equity Risk Models," Chapter 13 in Frank J. Fabozzi and Harry M. Markowitz, *The Theory and Practice of Investment Management* (New York: John Wiley & Sons, 2002).

Financial Decision Making and Asset Pricing

Portfolio theory and asset pricing models lay the groundwork for financial decisions. While portfolio theory and asset pricing theory are complex and rely on many assumptions, they do get us thinking about what is important:

- Return and risk must both be considered.
- Because investors must be compensated for risk, a greater return is expected for bearing greater risk.
- Investors hold portfolios of assets, therefore the relevant risk in the valuation of assets is the portfolio's risk.

If a corporation is considering investing in a new product, there are two levels of thinking to work through in evaluating its risk and returns:

- If a firm takes on the product, it is adding it to its portfolio of assets and needs to consider the effect of this product on the firm's overall risk.
- Because a firm is owned by the investors, who themselves may own portfolios of assets, the relevant risk to consider is how the change in the firm's risk affects the owners' portfolio risk.

Therefore, when we evaluate the new product's future cash flows, the discount rate that we apply to value these future cash flows must reflect how that product affects the owners' portfolio risk.

SUMMARY

- Financial decision makers must consider both expected return and risk from investments.
- To evaluate an investment, the financial manager needs to consider the different types of risk, including cash flow risk, reinvestment rate risk, interest rate risk, purchasing power risk, and currency risk.
- Cash flow risk comprises sales risk, operating risk, and financial risk. Sales risk is the degree of uncertainty regarding the number of units of a good or service the firm will be able to sell and the price of these units. Operating risk is the uncertainty arising from the mix of variable and fixed operating costs. Financial risk is the uncertainty arising from the firm's financing decisions.
- Interest rate risk is the uncertainty associated with the change in the value of an asset that is caused by changes in the discount rate used to

translate future values into present ones. With a bond, for example, interest rate risk is the sensitivity of the bond's price to change in the yield on the bond.

- Purchasing power risk is the uncertainty associated with the change in the value of the currency. The greater the unanticipated inflation, the greater the purchasing power risk.
- Currency risk is the uncertainty arising from the change in exchange rates between different currencies. If future cash flows are denominated in a currency other than the domestic currency, the value of those cash flows is dependent, in part, on the exchange rate between the domestic and the foreign currency.
- When a firm invests in assets whose cash flows are not perfectly correlated with the firm's other assets, the firm's risk may be reduced. This is diversification.
- The risk that cannot be diversified away, the asset's market risk, is what investors demand compensation for in the form of higher expected returns.
- The Capital Asset Pricing Model and the Arbitrage Pricing Model are descriptions of the relation between risk and expected return. The CAPM specifies the expected return on an asset in terms of the expected return on a risk-free asset plus a premium for market risk.
- The Arbitrage Pricing Model specifies the expected return on an asset in terms of the expected return on the risk-free asset plus premiums for several risk factors. These models are called multifactor risk models.

QUESTIONS

1. The Global Company is considering investing in a project in another country. This project will generate cash flows—in the other country's currency—each year for ten years, at which time the project will be terminated. What types of risk does Global need to consider in its investment decision?
2. Consider the two firms Tweedle Dee and Tweedle Dum. Both firms operate in the same industry, but Tweedle Dum has a greater portion of fixed operating costs relative to variable costs than does Tweedle Dee. Which firm has greater operating risk? Which firm has a higher degree of operating leverage?
3. Abel, an astute investor, buys bonds and always holds them to maturity. He claims that because he holds these bonds to maturity, there is no risk. Is he correct? Explain.
4. If you invest in corporate bonds, what types of risk do you assume?

5. Consider the following investments and their expected returns and standard deviations of expected returns:

Investment	Expected Return	Standard Deviation
1	10%	10%
2	11%	10%
3	9%	9%
4	11%	9%

If you a risk averse investor, which investment would he or she prefer from each of the following pairs?
a. 1 and 2
b. 2 and 4
c. 3 and 4
d. 1 and 4

6. The covariance of returns on Asset A and Asset B are negative. What does this tell us about the correlation coefficient for their returns? If we form a portfolio comprised of Asset A and Asset B, what is the relation between the portfolio's risk and that of Asset A and Asset B considered separately?

7. Consider the following common stocks and their return characteristics:

Stock	Expected Return	Standard Deviation	Security Beta
1	10%	5%	1.00
2	8%	5%	1.20
3	10%	6%	0.80

Which stock would a risk averse investor prefer between:
a. 1 and 2?
b. 1 and 3?
c. 2 and 3?

8. a. What are the major features of the capital asset pricing model and the arbitrage pricing model that distinguish them from one another?
b. What are the three types of arbitrage pricing models?

9. The Gearing Company has provided you with the following information regarding their operating and financing costs:

Price per unit	=	$50
Variable cost per unit	=	$30
Fixed operating cost	=	$100,000
Fixed financing cost	=	$50,000

a. Calculate its degree of operating leverage at 10,000 units sold.
b. Calculate its degree of financial leverage at 10,000 units sold.
c. Calculate its degree of total leverage at 10,000 units sold.
d. If there is a 1% increase in units sold, what do you expect to be the change in operating cash flows?
e. If there is a 3% decrease in units sold, what do you expect to be the change in cash flows to owners?

10. Suppose that the contribution margin is $55 per unit. If fixed costs (operating and financing combined) are $10 million, what is the break-even number of units produced and sold?
11. The Jonhaux Company produces a product that has a contribution per unit of $40. Fixed operating costs are $140,000. The Jonhaux Company currently has $10 million of bonds outstanding with a coupon rate of 5%.
    a. What is the current break-even number of units for Jonhaux considering all fixed costs?
    b. The board of Jonhaux is considering a proposal to issue $1 million additional bonds, with a a coupon rate of 6%. How would this proposed financing affect the break-even point?
    c. If 20,000 units are produced and sold, what is the degree of operating leverage, the degree of financial leverage, and the degree of total leverage under the current and proposed financial structures?
12. Consider two bonds, MM and NN:

    - Bond MM has face value of $1,000, matures in five years, and pays 6% interest semiannually.
    - Bond NN has a face value of $1,000, matures in five years, and pays 2% interest semiannually.

    a. If the yield-to-maturity on these bonds changes from 4% to 6%, which bond's value changes the most?
    b. Which bond has the greatest interest rate risk? Why?
    c. Which bond has the greatest reinvestment rate risk? Why?
13. Suppose you want to earn a rate of 8% after inflation. If you expect inflation to be 4% during the next year, what nominal rate of return would you require on your investment?
14. Your firm is considering investing in a new product. Marketing research has determined that the sales of the new product depend, in large part, on the whether or not competitors jump in to mimic the product. Their assessment of sales, and the likelihood of mimicking is as follows:

Competitor's Reaction	Probability	Sales
Mimic	80%	$1,000,000
Do not mimic	20%	$10,000,000

a. What are the expected sales from this new product?
b. What is the standard deviation of possible sales of this new product?

15. Suppose you are offered two investments with the following expected cash flows:

Economic Scenario	Probability of Economic Scenario	Possible Outcome for Investment 1	Possible Outcome for Investment 2
Boom	20%	$1,000	$1,200
Normal	50%	750	750
Bust	30%	250	117

a. Calculate the expected value of each investment.
b. Calculate the standard deviation for each investment's possible outcomes.
c. Which investment is riskier? Explain.

16. Consider two bonds, HI and LI. The HI bond has a 10% coupon rate and the LI bond has a 5% coupon rate. Both bonds pay interest annually and are priced to yield 10%. Suppose the following interest scenarios are possible at the point in time when both bonds have five years remaining to maturity:

Possible Interest Rate	Probability of Interest Rate
5%	10%
10	50
15	40

a. Calculate the expected value for each bond.
b. Calculate the standard deviation of possible values for each bond.
c. Which bond is riskier? Why?

17. Consider a portfolio comprised of Security A and Security B, with an equal investment in each. Security A's returns have an expected return of 3% and a standard deviation of 4%. Security B's returns have an expected return of 5% and standard deviation of 6%. Complete the following table:

Correlation Coefficient of Returns on Securities A and B	Portfolio Return	Covariance between Returns on Securities A and B	Portfolio Variance	Portfolio Standard Deviation
1.00	______	____________	_______	_________
0.50	______	____________	_______	_________
0.00	______	____________	_______	_________
–0.50	______	____________	_______	_________
–1.00	______	____________	_______	_________

18. Consider a portfolio comprised of Asset P and Asset Q. The expected return on Asset P is 10% and the standard deviation is 6%. The expected return on Asset Q is 12% and the standard deviation is 8%. The correlation between the returns on these two assets is 0.500. Complete the following table.

Proportion of Portfolio Invested in Asset P	Proportion of Portfolio Invested in Asset Q	Portfolio Return	Covariance Between Returns on Assets P and Q	Portfolio Variance	Portfolio Standard Deviation
100%	0%	______	____________	_______	_______
0%	100%	______	____________	_______	_______
50%	50%	______	____________	_______	_______
25%	75%	______	____________	_______	_______
75%	25%	______	____________	_______	_______

19. If the expected return on a risk-free asset is 5% and the market premium is 4%, what is the expected security return if the security's beta is: a. 0.00? b. 0.50? c. 1.00? d. 1.25? e. 2.00?
20. Suppose the expected risk-free rate is 5% and the expected return on the market is 12%. Further suppose that you have a portfolio comprised of the four securities, with equal investments in each:

Security	Security Beta
AA	1.00
BB	1.25
CC	1.50
DD	1.00

a. What is the expected return for each security in your portfolio?
b. What is the portfolio's beta?
c. What is the expected return on your portfolio?