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Fuzzy VIKOR with an application to water resources planning

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ABSTRACT

The fuzzy VIKOR method has been developed to solve fuzzy multicriteria problem with conflicting and noncommensurable (different units) criteria. This method solves problem in a fuzzy environment where both criteria and weights could be fuzzy sets. The triangular fuzzy numbers are used to handle imprecise numerical quantities. Fuzzy VIKOR is based on the aggregating fuzzy merit that represents distance of an alternative to the ideal solution. The fuzzy operations and procedures for ranking fuzzy numbers are used in developing the fuzzy VIKOR algorithm. VIKOR (VIsekriterijumska optimizacija i KOmpromisno Resenje) focuses on ranking and selecting from a set of alternatives in the presence of conflicting criteria, and on proposing compromise solution (one or more). It is extended with a trade-offs analysis. A numerical example illustrates an application to water resources planning, utilizing the presented methodology to study the development of a reservoir system for the storage of surface flows of the Mlava River and its tributaries for regional water supply. A comparative analysis of results by fuzzy VIKOR and few different approaches is presented.

1. Introduction

There are situations when the evaluation of alternatives must handle the imprecision of established criteria, and the development of a fuzzy multicriteria decision model is necessary to deal with either "qualitative" (unquantifiable or linguistic) or incomplete information (Vanegas & Labib, 2001; Zadeh et al., 1987). Imprecision in multicriteria decision making (MCDM) can be modeled using fuzzy set theory to define criteria and the importance of criteria. According to Bellman and Zadeh "much of the decision-making in the real world takes place in an environment in which the goals, the constraints, and consequences of possible actions are not known precisely" (Bellman & Zadeh, 1970). Ribeiro provides an overview of the concepts and theories of decision making in a fuzzy environment (Ribeiro, 1996). Von Altrock explains the elements of fuzzy logic system design, presenting case studies of real-world applications, of which the most visible applications are in the realms of consumer products, intelligent control, and industrial systems (Von Altrock, 1995). Less visible, but of growing importance, are applications relating to decision support systems (Zimmermann, 1991, 1987). Although fuzzy set theory has been and still remains somewhat controversial, its successes are too clear to be denied. However, Ribeiro warns that "too much fuzzification does not imply better modeling of reality, it can be counterproductive". Fuzzy ranking methods have been developed that can be used to compare fuzzy numbers (Chen & Hwang, 1992), but this is still an interesting research area.

There are two approaches to MCDM in a fuzzy environment, "conventional" and "fuzzy" (Perny & Roubens, 1998). The conventional approach is based on a nonfuzzy decision model, whereas the fuzziness dissolution (defuzzification) is performed at an early stage (Chen & Hwang, 1992; Wu, Tzeng, & Chen, 2009). The fuzzy approach is based on processing fuzzy data for decision making, then dissolving the fuzziness at a later stage (Opricovic, 2007). In both cases, defuzzification is necessary since MCDM results must provide a crisp conclusion. Defuzzification is selection of a specific crisp element based on the output fuzzy set, and it also includes converting fuzzy numbers into crisp scores. There are several defuzzification methods, although the operation defuzzification cannot be defined uniquely (Chen & Cheng, 2005; Detyniecki & Yager, 2000; Lee & Li, 1988; Opricovic & Tzeng, 2003; Yager & Filev, 1994).

The multicriteria decision making (MCDM) procedure consists of generating alternatives, establishing criteria, evaluation of alternatives, assessment of criteria weights, and application of a ranking method (Vincke, 1992). The alternatives are evaluates according to different criteria depending on the objectives of the problem. The evaluation of alternatives should be performed according to each criterion from the set of established criteria. A comparative analysis of MCDM methods is presented in several publications (Escobar & Moreno-Jimenez, 2002; Opricovic & Tzeng, 2007; Triantaphyllou, 2000).

The VIKOR method has been developed as an MCDM method to solve a discrete multicritea problem with noncommensurable and conflicting criteria (Opricovic, 1998). It focuses on ranking and selecting from a set of alternatives, and determines compromise

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solutions for a problem with conflicting criteria, which can help the decision makers to reach a final decision. The compromise solution is a feasible solution which is the closest to the ideal (Opricovic & Tzeng, 2004). VIKOR is based on old ideas of compromise programming (Duckstein & Opricovic, 1980; Yu, 1973). An extension of VIKOR to determine fuzzy compromise solution for multicriteria is presented in (Opricovic, 2007).

The fuzzy VIKOR method is developed as a fuzzy MCDM method to solve a discrete fuzzy multicritea problem with noncommensurable and conflicting criteria. It is presented in Section 2. The background for this method, including aggregation, normalization, DM's preference assessment, and operations on fuzzy numbers are discussed, as a study of rationality that in someway justifies the fuzzy VIKOR method and shows the position of its background in the literature on MCDM. This new method provides a contribution to the practice of MCDM. In Section 3, a numerical example illustrates an application of fuzzy VIKOR to water resources planning, aiming to numerical justification. Comparisons of the results by different methods are presented in Section 4.

2. The fuzzy VIKOR method

The fuzzy VIKOR method has been developed to determine the compromise solution of the fuzzy multicriteria problem

$$\underset{j}{mco}\left\{(\tilde{f}_{ij}(A_j), j = 1,\ldots,J), i = 1,\ldots,n\right\}$$

where: J is the number of feasible alternatives; $A_j = \{x_1, x_2, \ldots\}$ is the jth alternative obtained (generated) with certain values of system variables x; f_{ij} is the value of the ith criterion function for the alternative A_j; n is the number of criteria; *mco* denotes the operator of a multicriteria decision making procedure for selecting the best (compromise) alternative in multicriteria sense. Alternatives can be generated and their feasibility can be tested by mathematical models (determining variables x), physical models, and/or by experiments on the existing system or other similar systems. Constraints are seen as high-priority objectives, which must be satisfied in the alternatives generating process. In this paper we assume the alternatives are evaluated by the triangular fuzzy numbers $\tilde{f}_{ij} = (l_{ij}, m_{ij}, r_{ij}),\ i = 1,\ldots,n,\ j = 1,\ldots,J$. The set of criteria representing benefits (good effects) is denoted by I^b, and a set I^c for costs. Here $|I^b \cup I^c| = n$, where $|\cdot|$ denotes a cardinal number.

The ranking algorithm VIKOR has the following steps:

(i) Determine the ideal $\tilde{f}_i^* = (l_i^*, m_i^*, r_i^*)$ and the nadir $\tilde{f}_i^\circ = (l_i^\circ, m_i^\circ, r_i^\circ)$ values of all criterion functions, $i = 1, 2, \ldots, n$.

$$\tilde{f}_i^* = \underset{j}{MAX}\tilde{f}_{ij}, \quad \tilde{f}_i^\circ = \underset{j}{MIN}\tilde{f}_{ij}, \quad \text{for} \quad i \in I^b;$$

$$\tilde{f}_i^* = \underset{j}{MIN}\tilde{f}_{ij}, \quad \tilde{f}_i^\circ = \underset{j}{MAX}\tilde{f}_{ij}, \quad \text{for} \quad i \in I^c.$$

(ii) Compute normalized fuzzy difference $\tilde{d}_{ij},\ j = 1,\ldots,J,\ i = 1,\ldots,n$:

$$\begin{aligned} \tilde{d}_{ij} &= (\tilde{f}_i^* \ominus \tilde{f}_{ij})/(r_i^* - l_i^\circ) \quad \text{for} \quad i \in I^b; \\ \tilde{d}_{ij} &= (\tilde{f}_{ij} \ominus \tilde{f}_i^*)/(r_i^\circ - l_i^*) \quad \text{for} \quad i \in I^c \end{aligned} \tag{1}$$

(iii) Compute $\tilde{S}_j = (S_j^l, S_j^m, S_j^r)$ and $\tilde{R}_j = (R_j^l, R_j^m, R_j^r),\ j = 1, 2, \ldots, J$, by the relations

$$\tilde{S}_j = \sum_{i=1}^{n}{}_{\oplus}\left(\tilde{w}_i \otimes \tilde{d}_{ij}\right) \tag{2}$$

$$\tilde{R}_j = \underset{i}{MAX}\left(\tilde{w}_i \otimes \tilde{d}_{ij}\right) \tag{3}$$

where $\tilde{S}$ is a fuzzy weighted sum, $\tilde{R}$ is a fuzzy operator MAX (see Appendix B), $\tilde{w}_i$ are the weights of criteria, expressing the DM's preference as the relative importance of the criteria.

(iv) Compute the values $\tilde{Q}_j = (Q_j^l, Q_j^m, Q_j^r),\ j = 1, 2, \ldots, J$, by the relation

$$\tilde{Q}_j = v\left(\tilde{S}_j \ominus \tilde{S}^*\right)/(S^{\circ r} - S^{*l}) \oplus (1 - v)\left(\tilde{R}_j \ominus \tilde{R}^*\right)/(R^{\circ r} - R^{*l}) \tag{4}$$

where: $\tilde{S}^* = \underset{j}{MIN}\,\tilde{S}_j$, $S^{\circ r} = \max_j S_j^r$, $\tilde{R}^* = \underset{j}{MIN}\,\tilde{R}_j$, $R^{\circ r} = \max_j R_j^r$, and v is introduced as a weight for the strategy of "the majority of criteria" (or "the maximum group utility"), whereas $1 - v$ is the weight of the individual regret. These strategies could be compromised by $v = 0.5$, and here v is modified as $v = (n + 1)/2n$ (from $v + 0.5(n - 1)/n = 1$) since the criterion (1 of n) related to R is included in S, too. The best values of S and R are denoted by $\tilde{S}^*$ and $\tilde{R}^*$, respectively.

(v) "Core" ranking

Rank the alternatives by sorting the core values $Q_j^m,\ j = 1, 2, \ldots, J$, in decreasing order. The obtained ordering is denoted by $\{A\}_{Q^m}$.

(vi) Fuzzy ranking

The jth ranking position in $\{A\}_{Q^m}$ of an alternative $A^{(j)}, j = 1,\ldots,J$, is confirmed if $\underset{k \in J^j}{MIN}\,\tilde{Q}^{(k)} = \tilde{Q}^{(j)}$, where $J^j = \{j, j + 1,\ldots,J\}$ and $\tilde{Q}^{(k)}$ is the fuzzy merit for the alternative $A^{(k)}$ at the kth position in $\{A\}_{Q^m}$ (see Appendix A). Confirmed ordering represents "exact" fuzzy ranking $\{A\}_{\tilde{Q}}$, although the set $\{A\}_{\tilde{Q}}$ could not be complete ordering (it may be partially ranking).

(vii) Defuzzification of $\tilde{S}_j,\ \tilde{R}_j,\ \tilde{Q}_j,\ j = 1, 2, \ldots, J$, by the relations

$$Crisp\left(\tilde{N}\right) = (2m + l + r)/4 \tag{5}$$

Here the defuzzification method "2nd weighted mean" is applied to convert a fuzzy number into crisp score (see Appendix A).

(viii) Rank the alternatives, sorting by the crisp values S, R and Q in decreasing order. The results are three ranking lists $\{A\}_S$, $\{A\}_R$, $\{A\}_Q$.

(ix) Propose as a compromise solution the alternative $(A^{(1)})$ which is the best ranked by the measure Q (in $\{A\}_Q$) if the following two conditions are satisfied:

C1. "Acceptable Advantage": $Adv \geqslant DQ$

where: $Adv = [Q(A^{(2)}) - Q(A^{(1)})]/[Q(A^{(J)}) - Q(A^{(1)})]$ is the advantage rate of the alternative $A^{(1)}$ ranked first, $A^{(2)}$ is the alternative with second position in $\{A\}_Q$, and the threshold $DQ = 1/(J - 1)$.

C2. "Acceptable Stability in decision making":

The alternative $A^{(1)}$ must also be the best ranked by S or/and R.

If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of:

- Alternatives $A^{(1)}$ and $A^{(2)}$ if only the condition C2 is not satisfied, or
- Alternatives $A^{(1)}, A^{(2)}, \ldots, A^{(M)}$ if the condition C1 is not satisfied; $A^{(M)}$ is determined by the relation $Q(A^{(M)}) - Q(A^{(1)}) < DQ$ for maximum M (the positions of these alternatives are "in closeness").

(x) Determine crisp trade-offs, $tr_{ik} = (D_i w_k)/(D_k w_i),\ k \neq i$, $k = 1,\ldots,n$, where tr_{ik} is the number of units of the ith criterion evaluated the same as one unit of the kth criterion; $D_i = r_i^* - l_i^\circ$ for $i \in I^b$, $D_i = r_i^\circ - l_i^*$ for $i \in I^c$, and $w = Crisp(\tilde{w})$ obtained by defuzzification used in step (vii). The index i is given by the VIKOR user. The VIKOR method introduces these trade-offs as a result of normalization used in Eq. (1) for operations in (2) and (3).

(xi) The decision maker may give a new value of tr_{ik}, $k \neq i$, $k = 1,\ldots,n$ if he or she does not agree with computed values in step (x). The new values of weights are computed $w_k = |(D_k w_i - tr_{ik})/D_i|$, $k \neq i$, $k = 1,\ldots,n$; w_i is the previous value from the step (x). Then, VIKOR performs a new ranking from step (iii) using $\tilde{w}_k = (w_k, w_k, w_k),\ k = 1,\ldots,n$. The trade-offs determined in step (x) could help the decision maker to assess new values, although that task is very difficult.

(xii) The VIKOR algorithm ends if the new values are not given in step (xi).

The ranking algorithm VIKOR uses fuzzy operations presented in Appendix B.

This method focuses on ranking and selecting from a set of alternatives in the presence of conflicting criteria, and on proposing compromise solution (one or more). It is assumed that compromising is acceptable for conflict resolution, the decision maker (DM) is willing to approve solution that is the closest to the ideal, and the alternatives are evaluated (fuzzy or crisp) according to all established criteria.

The VIKOR method is an effective tool in multicriteria decision making. The obtained compromise solution could be accepted by the decision makers because it provides a maximum group utility of the "majority" (represented by min S, Eq. (2)), and a minimum individual regret of the "opponent" (represented by min R, Eq. (3)). The VIKOR algorithm can be performed without interactive participation of DM, but the DM is in charge of approving the final solution and his/her preference must be included. The compromise solutions could be the base for negotiation, involving the decision makers' preference by criteria weights. The VIKOR method may be incorporated within a DSS for MCDM (Liu & Stewart, 2004).

The fundamental issues of the VIKOR method are discussed in the previous articles, studying aggregation and normalization (Opricovic & Tzeng, 2004), DM's preference assessment (Opricovic & Tzeng, 2004; Opricovic, 2009), and operations on fuzzy numbers (Opricovic, 2007), that in someway justifies the VIKOR method and its rationality, and shows the position of its background in the literature on MCDM. Several papers presented application of the VIKOR method (Chang & Hsu, 2009; Ou Yang, Shieh, & Tzeng, 2009; Sanayei, Farid Mousavi, & Yazdankhah, 2010; Tong, Chen, & Wang, 2007). In Section 3, a numerical example illustrates an application of fuzzy VIKOR method aiming to numerical justification.

3. Fuzzy VIKOR application to water resources planning

Previous studies of the Mlava water resources system, in Serbia, have selected potential dam sites for reservoirs to provide water. In addition, comprehensive analysis was required to resolve conflicting technical, social and environmental features. Even if the topographic surveys confirm that the required reservoir capacity is available, a hydrological solution may conflict with environmental, social, and cultural features.

The VIKOR method was applied to evaluate alternative systems on the Mlava River. The alternatives were generated by varying two system parameters, dam site and dam height. The following six alternatives were selected for multicriteria optimization.

A1. The alternative A1 is the reservoir Vukan with normal level of 215 m.a.s.l. and useful storage of 86×10^6 m^3 could provide 4.08 m^3/s (average) for planed regional water supply. The dam site is 1.5 km downstream of the monastery Gornjak, and the implementation would require the removal of the monastery. There will be a loss of agricultural land of 120 ha. A section (1.5 km) of the regional road and parts of local roads will be flooded by the reservoir.

A2. Reservoir Vukan with normal level of 205 m.a.s.l. and useful storage of 40×10^6 m^3 would have less social and environmental impacts on local areas and could provide 2.87 m^3/s for water supply. It requires the removal of the monastery Gornjak. The loss of agricultural land is less than alternative A1.

A3. Reservoir Vitman I with normal level of 215 m.a.s.l. could provide 2.97 m^3/s. The dam site is 3 km upstream of the monastery Gornjak, but there will be a loss of agricultural land (120 ha).

A4. Reservoir Gradac with normal level of 275 m.a.s.l. could provide 2.73 m^3/s. The dam site is in the gorge Ribarska, upstream of the Gornjak gorge. There will be an impact on agricultural area in the region of Zagubica (a loss of 300 ha). The area of several households in two villages will be flooded and they have to be removed.

A5. System of three reservoirs, Vitman II (205) and Gradac (251) on Mlava, and Dubocica (255) on the tributary, could provide 2.5 m^3/s. All three dam sites are upstream of the monastery Gornjak. The loss of agricultural is relatively small since normal levels are lower.

A6. System similar to the alternative A5, Vitman III (203), Gradac (251) and Dubocica (255), which could provide 2.74 m^3/s. The Vitman III dam site is shortly downstream of Vitman II.

The designed reservoir systems are evaluated according to the following criteria:

f1. Investment costs (in 10^6 US$) including dam construction, expropriation of the area occupied by the reservoir, construction of new buildings for the households which have to move, and building new roads that will substitute flooded sections.

f2. Water supply discharge – yield (m^3/s) is the average annual value of discharge from the reservoir system available for regional water supply. The required reservoir capacity has been determined by the "sequent peak" algorithm for required total water demands. Water supply discharge has been determined by simulation of reservoir system with required capacity using historical hydrological series. Beside this discharge each reservoir has to realize downstream a biological minimum flow.

f3. Social impact (%) on urban and agricultural area expressing local regret as percentage of the regret in the alternative with maximum social impact.

f4. Impact on the monastery Gornjak is graded by the experts. The worst grade has the alternative that required the removal of monastery. The construction of a dam could have impact on ambient beauty of the Gornjak gorge.

The multicriteria task is to minimize the criterion functions f1, f3, and f4, and to maximize function f2. The four criterion functions are expressed in different units and they are noncommensurable. The values of criterion functions are obtained by a comprehensive study of this reservoir system on Mlava river system, and the results are presented in Table 1.

The criteria weights $\tilde{\mathscr{W}}_i = (1, 1, 1),\ i = 1, 2, 3, 4$ express equal importance (no preference), and $v = 0.625$ (see step (iv) in Section 2).

The results obtained by the fuzzy VIKOR algorithm are presented in Tables 2 and 3. Preliminary ranking ("core") of alternatives by the values Q^m is A3, A6, A5, A2, A4, A1. "Exact" fuzzy ranking by fuzzy VIKOR is not complete ordering in this example, since the position of A3 is confirmed (see step (vi) in Section 2, and Fig. 1).

Table 1
Performance matrix.

Criteria			Alternatives					
Name	extr.		A1	A2	A3	A4	A5	A6
$\tilde{f}_1$ Investment costs (10^6 $)		*l*	38.00	20.00	24.58	44.54	33.33	33.86
	Min	*m*	40.01	21.06	25.87	46.89	33.33	33.86
		r	48.00	24.00	29.75	56.27	43.33	42.32
$\tilde{f}_2$ Water supply (m^3/s)		*l*	3.26	2.57	2.82	2.46	2.25	2.47
	Max	*m*	4.08	2.87	2.97	2.73	2.50	2.74
		r	4.08	2.87	2.97	2.73	2.62	2.85
$\tilde{f}_3$ Social impact (%)		*l*	43	6	38	60	6	6
	Min	*m*	47	6	42	62	6	6
		r	48	6	50	68	6	6
$\tilde{f}_4$ Impact on monastery (grade)	*Min*	*l m r*	10	10	1	0	2	3

Table 2
Results by fuzzy VIKOR.

		A1	A2	A3	A4	A5	A6
$\tilde{S}$	S^l	1.535	1.103	0.791	1.727	0.807	0.796
	S^m	2.184	1.661	1.420	2.353	1.402	1.385
	S^r	2.897	1.935	1.767	2.885	1.843	1.795
	Crisp S	2.200	1.590	1.349	2.330	1.363	1.340
$\tilde{R}$	R^l	1.0	1.0	0.516	0.871	0.350	0.300
	R^m	1.0	1.0	0.607	0.903	0.863	0.732
	R^r	1.0	1.0	0.710	1.0	1.0	0.880
	Crisp R	1.0	1.0	0.610	0.919	0.769	0.661
$\tilde{Q}$	Q^l	0.087	−0.041	−0.393	0.075	−0.478	−0.508
	Q^m	0.448	0.293	0.010	0.446	0.142	0.067
	Q^r	1.0	0.715	0.509	0.996	0.687	0.609
	Crisp Q	0.495	0.315	0.034	0.491	0.124	0.059

Crisp values (by defuzzification) of $\tilde{S}_j$, $\tilde{R}_j$, $\tilde{Q}_j$, $j = 1, 2, \ldots, J$, are presented in Table 2. Ranking by crisp values are $\{A\}_S$ = A6, A3, A5, A2, A1, A4, $\{A\}_R$ = A3, A6, A5, A4, A1, A2, $\{A\}_Q$ = A3, A6, A5, A2, A4, A1. The compromise solution for final decision is the set {A3, A6, A5} (see step (ix) in Section 2).

A 3. Vitman I (215) (advantage 5.4%).
A 6. Vitman III (203), Gradac (251), Dubocica (255).
A 5. Vitman II (205), Gradac (251), Dubocica (255).

Table 3
Ranking by fuzzy VIKOR.

		Ordering					
		1	2	3	4	5	6
"Core" ranking $\{A\}_{Q^m}$		A3	A6	A5	A2	A4	A1
"Exact" fuzzy ranking $\{A\}_{\tilde{Q}}$			A6	A5	A2	A4	A1
Defuzzification	*Q*	A3	A6	A5	A2	A4	A1
	S	A6	A3	A5	A2	A1	A4
	R	A3	A6	A5	A4	A1	A2

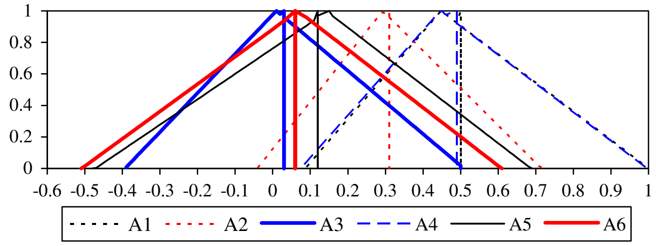

Fig. 1. Fuzzy merit $\tilde{Q}(A_j)$ and crisp Q_j, $j = 1, 2, \ldots, J$.

The trade-offs values determined by VIKOR (the step x) are presented in Table 4, showing how many $10^6$$ are evaluated as one unit of *k*th criterion. The determined tradeoffs are: 19.82 × $10^6$$/(m^3/s), 0.58 × $10^6$$/% and 3.63 $10^6$$/mark-unit, for example, 1 m^3/s of water supply discharge worth as 19.82 × $10^6$$ of investment costs, and removing monastery Gornjak worth as 25.41 × $10^6$$. This values seem too high in economic sense, although assessing trade-offs between economic and qualitative criteria is a very difficult task. The trade-offs determined by VIKOR are the result of normalizing noncommensurable criteria. The new trade-offs are given in Table 4. New weights are determined and ranking by VIKOR has been repeated.

The results obtained by the fuzzy VIKOR algorithm with given tradeoffs in Table 4 are presented in Tables 5 and 6. New weights in Table 4 are determined by the procedure presented in step (xi) in Section 2. For example, $w_5 = |(D_5 w_2 tr_{25})/D_2| = ((4.08 - 2.25) * 1 * 15)/(56.27 - 20.0) = 0.757$, w_2 is the previous value from the step (x).

Preliminary ranking ("core") of alternatives by the values Q^m is A3, A2, A6, A5, A1, A4. "Exact" fuzzy ranking by fuzzy VIKOR is not complete ordering in this example, although the positions of five alternatives are confirmed (see step (vi) in Section 2). The position of A2 is not confirmed since $Q^l(A_2) = -0.171$ is greater then $Q^l(A_5) = -0.290$ (Fig. 2).

Table 4
Trade-offs by VIKOR.

	$\vec{1}$	$\vec{2}$	$\vec{3}$	$\vec{4}$
tr_{1k}, $k = 1, \ldots, n$ ($10^6\$/\vec{k}$)	1.0	19.82	0.58	3.63
New given trade-offs tr_{1k}, $k = 1, \ldots, n$	1	15	0.2	2
New weights	1.0	0.757	0.342	0.551

Table 5
Results by fuzzy VIKOR with given trade-offs.

		A1	A2	A3	A4	A5	A6
$\tilde{S}$	S^l	0.802	0.602	0.368	1.083	0.632	0.607
	S^m	1.300	1.052	0.845	1.579	1.102	1.073
	S^r	1.894	1.286	1.088	2.012	1.510	1.447
	Crisp S	1.324	0.998	0.786	1.563	1.087	1.050
$\tilde{R}$	R^l	0.551	0.551	0.176	0.566	0.265	0.272
	R^m	0.551	0.551	0.459	0.712	0.653	0.554
	R^r	0.772	0.624	0.521	1.000	0.757	0.666
	Crisp R	0.607	0.570	0.404	0.748	0.582	0.512
$\tilde{Q}$	Q^l	−0.095	−0.171	−0.431	0.019	−0.290	−0.296
	Q^m	0.215	0.121	0.0	0.394	0.186	0.130
	Q^r	0.851	0.553	0.431	1.000	0.699	0.633
	Crisp Q	0.297	0.156	0.0	0.452	0.195	0.149

Table 6
Ranking by fuzzy VIKOR with given trade-offs.

		Ordering					
		1	2	3	4	5	6
“Core” ranking	$\{A\}_{Q^m}$	A3	A2	A6	A5	A1	A4
“Exact” fuzzy ranking	$\{A\}_{\tilde{Q}}$	A3		A6	A5	A1	A4
Defuzzification	Q	A3	A6	A2	A5	A1	A4
	S	A3	A2	A6	A5	A1	A4
	R	A3	A6	A2	A5	A1	A4

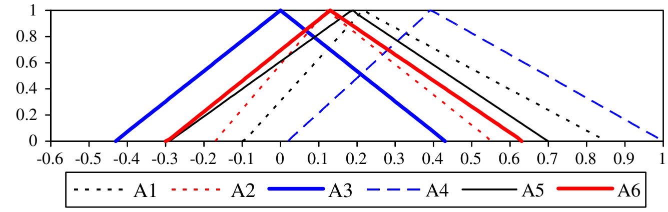

Fig. 2. New fuzzy merit $\tilde{Q}(A_j),\ j = 1, 2, \ldots, J$ for given trade-offs.

Crisp values (by defuzzification) of $\tilde{S}_j,\ \tilde{R}_j,\ \tilde{Q}_j,\ j = 1, 2, \ldots, J$, are presented in Table 5. Ranking by crisp values are is $\{A\}_Q$ = A3, A6, A2, A5, A1, A4. The compromise solution for final decision is A3 Vitman I (215) with the advantage rate of 33%.

The alternative A3 is a single reservoir system (Vitman I (215)). Second ranked is the alternative A6, a three-reservoir system (Vitman III (203), Gradac (251), Dubocica (255)). The decision makers for the Mlava project may adopt alternative A6, which could be developed in three phases, building one reservoir in each phase.

4. Some comparisons

Comparisons of the results by different methods are made in this section. These comparisons could challenge the readers to compare fuzzy VIKOR with particular methods.

4.1. Non-fuzzy model

Non-fuzzy MCDM methods are compared many times (Triantaphyllou, 2000; Opricovic & Tzeng, 2007). Original (non-fuzzy) VIKOR uses crisp input data. The input data for VIKOR are core (m) values from Table 1. The result from (Opricovic, 2009) is the ranking list: A3, A6, A5, A2, A1, A4, and the set of compromise solutions consists of

A 3. Vitman I (215) (advantage 19%).
A 6. Vitman III (203), Gradac (251), Dubocica (255).

Preliminary ranking (“core”) of alternatives by the values Q^m is A3, A6, A5, A2, A4, A1 (Table 3), the set of compromise solutions is {A3, A6}. Crisp values (by defuzzification) of $\tilde{S}_j,\ \tilde{R}_j,\ \tilde{Q}_j,\ j = 1, 2, \ldots, J$, are presented in Table 2. Ranking by defuzzified values of $\tilde{Q}_j$ is $\{A\}_Q$ = A3, A6, A5, A2, A4, A1, and the set of compromise solutions is

A 3. Vitman I (215) (advantage 5.4%).
A 6. Vitman III (203), Gradac (251), Dubocica (255) .
A 5. Vitman II (205), Gradac (251), Dubocica (255).

The results are similar because of small spread (support) of fuzzy numbers $r - l$ in Table 1.

4.2. Predefuzzification

Two approaches to fuzzy multicriteria decision making are presented, “fuzzy” and “conventional”. The fuzzy VIKOR method is a fuzzy approach. The conventional approach is based on a nonfuzzy decision model, here VIKOR method, whereas the fuzziness dissolution (defuzzification) is performed at an early stage (predefuzzification). The predefuzzification approach is used in (Wu et al., 2009), utilizing the COA (center of area) method to find “the Best Nonfuzzy Performance value (BNP)”. The ranking of the alternatives then proceeds based on the value of the derived BNP for each of the alternatives. Three MCDM analytical tools of SAW, TOPSIS, and VIKOR were adopted to rank alternatives. The same approach is used in (Wu, Chen, & Chen, 2010), adopting VIKOR to rank alternatives.

The fuzzy numbers from Table 1 are defuzified by the procedure used in step (vii), Section 2, and discussed in Appendix A. The defuzzified performance matrix is presented in Table 7.

The results in Table 7 are used as input data for non-fuzzy VIKOR. The non-fuzzy VIKOR is published in Opricovic and Tzeng (2007). The ranking result is A3, A6, A5, A2, A1, A4 and the compromise solution is A3 (advantage 21.9%).

The non-fuzzy VIKOR results (Opricovic, 2009) with core (m) values from Table 1 as input data are: ranking list: A3, A6, A5, A2, A1, A4, and the set of compromise solutions consists of A3 (advantage 19%) and A6.

Table 7
Defuzified performance matrix.

	A1	A2	A3	A4	A5	A6
f_1	41.50	21.53	26.52	48.65	35.83	35.97
f_2	3.87	2.79	2.93	2.66	2.47	2.70
f_3	46.25	6	43	63	6	6
f_4	10	10	1	0	2	3

These two results are very close, since the threshold for advantage in this example is 20% (see step (ix) in Section 2). Main reason is small spread (support) of fuzzy numbers $r - l$ in Table 1.

4.3. Ranking fuzzy numbers

Ranking fuzzy numbers using α-weighted valuations is considered in (Detyniecki & Yager, 2000). Fuzzy numbers $\widetilde{N}_1 = (1,7,9)$, $\widetilde{N}_2 = (4,6,12)$, and $\widetilde{N}_1 = (1,7,9)$, $\widetilde{N}_3 = (2,4,10)$ are compared. The result is $\widetilde{N}_1 < \widetilde{N}_2$ for "pro-support" ($0 \leqslant q < 2$) and $\widetilde{N}_1 > \widetilde{N}_2$ for $q > 2$ (importance of high α–levels). Analogous result is for $\widetilde{N}_1$ and $\widetilde{N}_3$.

Applying fuzzy VIKOR the result is as follows: "Core" ranking $\widetilde{N}_1$, $\widetilde{N}_2$, $\widetilde{N}_3$; "Exact" ranking $\widetilde{N}_2 > \widetilde{N}_3$; Ranking by crisp values (defuzzification) $\widetilde{N}_2(Q = 0.026)$, $\widetilde{N}_1(Q = 079)$, $\widetilde{N}_3(Q = 0.132)$. "Exact" ranking provides consistent result. Ordering $\widetilde{N}_1$ and $\widetilde{N}_2$ is an inconsistent result.

Crisp values by the center-of-gravity ($k = 1$, Appendix A) are $N_1 = 5.667$, $N_2 = 7.333$, $N_3 = 5.333$, and by the CFCS method (Opricovic & Tzeng, 2003) $N_1 = 6.29$, $N_2 = 6.71$, $N_3 = 4.88$.

An interesting example is ordering fuzzy numbers $\widetilde{N}_1 = (5,6,13)$, $\widetilde{N}_2 = (3,7,13)$, $\widetilde{N}_3 = (5,8,9)$, $\widetilde{N}_4 = (2,9,10)$, (Detyniecki & Yager, 2000). Applying fuzzy VIKOR there is no ordering by "exact' fuzzy ranking; and ranking by crisp values (defuzzification) $\widetilde{N}_1 = \widetilde{N}_2 = \widetilde{N}_3 = \widetilde{N}_4$.

Crisp values by the center-of-gravity are $N_1 = 8$, $N_2 = 7.667$, $N_3 = 7.333$, $N_4 = 7$, and by the CFCS method (Opricovic & Tzeng, 2003) $N_1 = 7.128$, $N_2 = 7.366$, $N_3 = 7.574$. $N_4 = 7.872$.

These are examples of inconsistent ranking. The explanation of an inconsistent result is that low α-levels are compensated with the high α-levels (Detyniecki & Yager, 2000).

4.4. NFWA and fuzzy VIKOR

The NFWA method (new fuzzy-weighted average) is applied and an example is presented in Vanegas and Labib (2001). Fuzzy result by NFWA is $\widetilde{D}(A_1) = (0.15, 0.32, 0.58)$, $\widetilde{D}(A_2) = (0.37, 0.61, 0.85)$, $\widetilde{D}(A_3) = (0.15, 0.38, 0.61)$, and ranking result (crisp) is $A_2(D = 0.61)$, $A_3(0.38)$, $A_1(0.35)$.

The same problem is solved by the fuzzy VIKOR algorithm and the result is as follows: $\widetilde{Q}(A_1) = (-0.54, 0.28, 0.98)$, $\widetilde{Q}(A_2) = (-0.75, 0.0, 0.75)$, $\widetilde{Q}(A_3) = (-0.64, 0.15, 0.93)$, "Exact" fuzzy ranking A_2, A_3, A_1 (complete ranking); and ranking by crisp values (defuzzification) $A_2(Q = 0.0)$, $A_3(0.15)$, $A_1(0.25)$. Ranking results by these two methods are very close, and ordering is the same.

4.5. Fuzzy AHP and fuzzy VIKOR

An example of fuzzy multicriteria problem is presented in (Gu & Zhu, 2006). Comparison results between the proposed algorithm and other algorithms are presented. There is a conclusion "It is apparent that the proposed improving fuzzy AHP algorithm based on fuzzy eigenvector of fuzzy attribute evaluation space is more efficient than others. It has good objectivity and resolution." Fuzzy result by the improved fuzzy AHP algorithm is $\widetilde{W}(A_1) = (0.3375, 0.8195, 1)$, $\widetilde{W}(A_2) = (0.3164, 0.6740, 1)$, $\widetilde{W}(A_3) = (0.3770, 0.8941, 1)$, $\widetilde{W}(A_4) = (0.3387, 0.7491, 1)$, and ranking result (crisp) is $A_3(W = 0.791)$, $A_1(0.744)$, $A_4(0.709)$, $A_2(0.666)$.

The same problem is solved by the fuzzy VIKOR algorithm and the result is as follows. $\widetilde{Q}(A_1) = (-0.455, 0.051, 0.669)$, $\widetilde{Q}(A_2) = (-0.396, 0.193, 1.0)$, $\widetilde{Q}(A_3) = (-0.491, 0.0, 0.5)$, $\widetilde{Q}(A_4) = (-0.474, 0.110, 0.784)$. "Exact" fuzzy ranking is A_3, A_4, A_2; and ranking by crisp values (defuzzification) $A_3(Q = 0.002)$, $A_1(0.079)$, $A_4(0.132)$, $A_2(0.248)$. Ranking results by these two methods are very close, and ordering is the same.

4.6. "Distance" method and fuzzy VIKOR

The procedure FMCGDSS (fuzzy multi-criteria group decision support system) based on metric distance method is presented and applied in (Chen & Cheng, 2005). The ranking results are the following: A_3(fuzzy mean = 3.662, fuzzy spread = 0.881), A_2(3.577, 0.97), A_1(3.477, 0.952).

The same problem is solved by the fuzzy VIKOR algorithm and the result is as follows: $\widetilde{Q}(A_1) = (-0.86, 0.088, 0.946)$, $\widetilde{Q}(A_2) = (-0.863, 0.054, 0.954)$, $\widetilde{Q}(A_3) = (-0.871, 0.0, 0.877)$, "Exact" fuzzy ranking is A_3, A_1; and ranking by crisp values (defuzzification) $A_3(Q = 0.002)$, $A_2(0.05)$, $A_1(0.065)$. Ordering by these two methods is the same.

5. Conclusions

The fuzzy VIKOR method focuses on ranking and selecting from a set of alternatives in a fuzzy environment. Imprecision in multicriteria decision making is modeled using fuzzy set theory to define criteria and the importance of criteria (weights). The triangular fuzzy numbers are used to handle imprecise numerical quantities. The VIKOR method is based on the aggregating fuzzy merit $\widetilde{Q}$ that represents distance of an alternative to the ideal solution. The fuzzy operations and procedures for ranking fuzzy numbers are used in developing VIKOR algorithm.

A numerical example illustrates an application of the fuzzy VIKOR method to water resources planning, aiming to numerical justification. It is an intention to illustrate the conceptual and operational validation of the application of this method in real world problem. The fuzzy VIKOR method background and comparisons of the results by different methods are presented in order to show the position of this new method in the literature on fuzzy MCDM.

Researchers are challenged to provide a guide for choosing the method that is both theoretically well founded and practically operational to solve actual problems.

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Appendix A

A.1. Ranking fuzzy numbers and defuzzification

MCDM in a fuzzy environment requires the comparison of fuzzy numbers. The problem of comparing fuzzy numbers has been studied and appears to be an important and difficult problem. A fuzzy number is characterized by its shape, spread, height, and relative location on the x-axis. A good ranking method would be one that takes into account all these factors. Since a fuzzy number represents many possible real numbers that have different membership values, one will face a difficult problem of comparing two different fuzzy numbers. Over 20 ranking methods for fuzzy numbers have been proposed, but none of these existing methods is perfect (Chen & Hwang, 1992; Detyniecki & Yager, 2000; Gu & Zhu, 2006; Lai & Hwang, 1994; Lee & Li, 1988; Yager, 1981). In general, two approaches are used: (1) comparison of fuzzy numbers and (2) converting fuzzy number into crisp score (deffuzzification).

The fuzzy VIKOR algorithm in step (vi) uses a ranking procedure with consistent results providing complete ranking only if fuzzy numbers have separated membership functions (no cross-overlapping). If there is cross-overlapping, this procedure does not provide complete ordering. Fuzzy operator *MIN* is used for ranking and to confirm ranking (domination). An alternative A_j is better ranked than A_k if $\widetilde{Q}_j = MIN\left(\widetilde{Q}_j, \widetilde{Q}_k\right)$, or $Q_j^l \leqslant Q_k^l,\ Q_j^m < Q_k^m,\ Q_j^r \leqslant Q_k^r$.

The fuzzy VIKOR algorithm uses a defuzzification procedure in step (vii) to convert fuzzy numbers into real (crisp) numbers. Then, in step (viii), the ranking of fuzzy numbers is performed through the comparison of the corresponding real numbers.

The kth weighted mean method has been developed to be used as defuzzification procedure in this paper. It uses membership function to the power of k as a weighted factor. The crisp value $Crisp(\widetilde{N})$ for the triangular fuzzy number $\widetilde{N} = (l, m, r)$ is determined by the following formula

$$Crisp(\widetilde{N}) = \int_l^r x\mu^k(x)dx \bigg/ \int_l^r \mu^k(x)dx$$

Integrating the integrals the following formula is obtained:

$$Crisp(\widetilde{N}) = (km + l + r)/(k + 2)$$

or

$$C = m + (s_r - s_l)/(k + 2)$$

and

$$\mu(C) = \begin{cases} \frac{k+1}{k+2} + \frac{s_r}{(k+2)s_l}, & C \leqslant m \\ \frac{k+1}{k+2} + \frac{s_l}{(k+2)s_r}, & C \geqslant m \end{cases}$$

where $C = Crisp\left(\widetilde{N}\right),\ s_l = m - l$ and $s_r = r - m$ are left and right support (spread), respectively.

The parameter (power) k has the impact on defuzzification result as follows:

$k = 1 : C = (m + l + r)/3$ or $C = m + (s_r - s_l)/3$ and $\mu(C) \geqslant 2/3$

Increasing k ($k = 2,3,\ldots$): C moves toward m (core) and membership $\mu(C)$ increases; for example, for $k = 4$: $C = (m + (s_r - s_l)/6)$; and, $\lim_{k\to\infty} C(k) = m$, $\lim_{k\to\infty} \mu(C,k) = 1$. The *Centroid* (*Center-of-gravity*) method, which provides a crisp value based on the center-of-gravity of the fuzzy set could be considered as a special case for $k = 1$. Within multicriteria decision making, $k \geqslant 4$ could be preferred by a "risk aversion" decision maker (increasing membership $\mu(C)$), this is "pro-core" defuzzification. A "gambler" decision maker could have different preference (Yu, 1990). A general suggestion could be to use one of the values {2, 3, 4} for power k, and the value of power k should be the same for defuzzifying all fuzzy numbers within a study.

The fuzzy VIKOR algorithm in step (vii) uses the 2nd weighted mean as a practical defuzzification tool for converting a fuzzy number into crisp number. A weighted factor include the membership function $\mu(x)$ that denotes the degree of truth that the fuzzy value is equal to x within the real interval $[l, r]$. The greater the value of $\mu(x)$, the higher the confidence in the value of x. The wider the support of the membership function, the higher the fuzziness (impreciseness, uncertainty).

Appendix B

B.1. Operations on triangular fuzzy numbers

To express an imprecise value, as "about m" ("approximately m"), the triangular fuzzy number (TFN) $\widetilde{N} = (l, m, r)$ is used, associated with the membership triangular function defined as follows:

$$\mu_{\widetilde{N}}(x) = \begin{cases} (x - l)/(m - l), & x \leqslant m \\ (r - x)/(r - m), & x \geqslant m \\ 0, & x \notin [l, r] \end{cases}$$

The membership function $\mu(x)$ denotes the degree of truth that the fuzzy value is equal to x within the real interval $[l, r]$. The fuzzy number $\widetilde{N}$ has the core m with $\mu(m) = 1$ and the support $[l, r]$.

The fuzzy VIKOR method has been developed applying mathematical operations on TFNs defined as follows

$$\begin{aligned}
&\text{Summation}: && \sum_{i=1}^{n} {}_{\oplus}\widetilde{N}_i = \left(\sum_{i=1}^{n} l_i, \sum_{i=1}^{n} m_i, \sum_{i=1}^{n} r_i\right) \\
&\text{Scalar summation}: && \widetilde{N} \oplus K = (l + K, m + K, r + K) \\
&\text{Subtraction}: && \widetilde{N}_1 \ominus \widetilde{N}_2 = (l_1 - r_2, m_1 - m_2, r_1 - l_2) \\
&\text{Scalar subtraction}: && \widetilde{N} - K = (l - K, m - K, r - K) \\
&\text{Scalar multiplication}: && K \times \widetilde{N} = (K \times l, K \times m, K \times r), \\
& && \text{for} \quad K \geqslant 0 \\
&\text{Multiplication}: && \widetilde{N}_1 \otimes \widetilde{N}_2 = (l_1 \times l_2, m_1 \times m_2, r_1 \times r_2), \\
& && \text{for} \quad l_1 \geqslant 0 (positive \widetilde{N}_1) \\
&\text{Scalar division}: && \widetilde{N}/K = (l/K, m/K, r/K), \\
& && \text{for} \quad K > 0 \\
&\text{Operator } MAX: && \underset{i}{MAX}\, \widetilde{N}_i = (\max_i l_i, \max_i m_i, \max_i r_i) \\
&\text{Operator } MIN: && \underset{i}{MIN}\, \widetilde{N}_i = (\min_i l_i, \min_i m_i, \min_i r_i)
\end{aligned}$$

The result of summation or subtraction on TFNs is TFN. The result of fuzzy multiplication is considered as an approximation of TFN, especially when applying α-cut (Giachetti & Young, 1997a). The result of $MAX\left(\widetilde{N}_1, \widetilde{N}_2\right)$ (or MIN) is not a TFN only if $\widetilde{N}_1$ and $\widetilde{N}_2$ are overlapping in two cases: 1. $l_1 < l_2$, $m_1 \neq m_2$, $r_1 > r_2$; and 2. $l_1 < l_2$, $m_1 > m_2$, $r_1 < r_2$; in these cases TFN is used as an approximation. Definitions and characteristics of the above operations are discussed in several articles (Chiu & Wang, 2002; Giachetti & Young, 1997b; Klir & Yuan, 1995).

References

Bellman, R. E., & Zadeh, L. A. (1970). Decision-making in a fuzzy environment. *Management Science, 17*, 141–164.

Chang, C.-L., & Hsu, C.-H. (2009). Multi-criteria analysis via the VIKOR method for prioritizing land-use restraint strategies in the Tseng–Wen reservoir watershed. *Journal of Environmental Management, 90*(11), 3226–3230.

Chen, L. S., & Cheng, C. H. (2005). Selecting IS personnel use fuzzy GDSS based on metric distance method. *European Journal of Operational Research, 160*(3), 803–820.

Chen, S. J., & Hwang, C. L. (1992). *Fuzzy multiple attribute decision making: Methods and applications*. Berlin: Springer-Verlag.

Chiu, C. H., & Wang, W. J. (2002). A simple computation of MIN and MAX operations for fuzzy numbers. *Fuzzy Sets and Systems, 126*, 273–276.

Detyniecki, M., & Yager, R. (2000). Ranking fuzzy numbers using α-weighted valuations. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, 8*(5), 573–591.

Duckstein, L., & Opricovic, S. (1980). Multiobjective optimization in river basin development. *Water Resources Research, 16*(1), 14–20.

Escobar, M. T., & Moreno-Jimenez, J. M. (2002). A linkage between the analytic hierarchy process and the compromise programming models. *Omega, 30*, 359–365.

Giachetti, R. E., & Young, R. E. (1997a). Analysis of the error in the standard approximation used for multiplication of triangular and trapezoidal fuzzy numbers and the development of a new approximation. *Fuzzy Sets and Systems, 91*, 1–13.

Giachetti, R. E., & Young, R. E. (1997b). A parametric representation of fuzzy numbers and their arithmetic operators. *Fuzzy Sets and Systems, 91*, 185–202.

Gu, X., & Zhu, Q. (2006). Fuzzy multi-attribute decision-making method based on eigenvector of fuzzy attribute evaluation space. *Decision Support Systems, 41*, 400–410.

Klir, G. J., & Yuan, B. (1995). *Fuzzy sets and fuzzy logic: Theory and applications*. Englewood Cliffs: Prentice-Hall.

Lai, Y. J., & Hwang, C. L. (1994). *Fuzzy multiple objective decision making: Methods and applications*. Berlin: Springer-Verlag.

Lee, E. S., & Li, R. L. (1988). Comparison of fuzzy numbers based on the probability measure of fuzzy events. *Computers and Mathematics with Applications, 15*, 887–896.

Liu, D., & Stewart, T. J. (2004). Integrated object-oriented framework for MCDM and DSS modeling. *Decision Support Systems, 38*, 421–434.

Opricovic, S. (1998). *Multicriteria optimization of civil engineering systems (in Serbian, Visekriterijumska optimizacija sistema u gradjevinarstvu)*. Belgrade: Faculty of Civil Engineering.

Opricovic, S. (2007). A fuzzy compromise solution for multicriteria problems. *International Journal of Uncertainty, Fuzziness and Knowledge-based Systems, 15*(3), 363–380.

Opricovic, S. (2009). A compromise solution in water resources planning. *Water Resources Management, 23*(8), 1549–1561.

Opricovic, S., & Tzeng, G. H. (2003). Defuzzification within a multicriteria decision model. *International Journal of Uncertainty, Fuzziness And Knowledge-Based Systems, 11*(5), 635–652.

Opricovic, S., & Tzeng, G. H. (2004). The compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS. *European Journal of Operational Research, 156*(2), 445–455.

Opricovic, S., & Tzeng, G. H. (2007). Extended VIKOR method in comparison with outranking methods. *European Journal of Operational Research, 178*(2), 514–529.

Ou Yang, Y. P., Shieh, H. M., & Tzeng, G. H. (2009). A VIKOR technique with applications based on DEMATEL and ANP. *Communications in Computer and Information Science, 35*, 780–788.

Perny, P., & Roubens, M. (1998). Fuzzy preference modeling. In R. Slowinski (Ed.), *Fuzzy Sets in Decision Analysis, Operations Research and Statistics* (pp. 3–30). Boston: Kluwer Academic Publishers.

Ribeiro, R. A. (1996). Fuzzy multiple attribute decision making: A review and new preference elicitation techniques. *Fuzzy Sets and Systems, 78*, 155–181.

Sanayei, A., Farid Mousavi, S., & Yazdankhah, A. (2010). Group decision making process for supplier selection with VIKOR under fuzzy environment. *Expert Systems with Applications, 37*(1), 24–30.

Tong, L., Chen, C. C., & Wang, C. H. (2007). Optimization of multi-response processes using the VIKOR method. *International Journal of Advanced Manufacturing Technology, 31*(11–12), 1049–1057.

Triantaphyllou, E. (2000). *Multi-criteria decision making methods: A comparative study*. Dordrecht: Kluwer Academic Publishers.

Vanegas, L. V., & Labib, A. W. (2001). Application of new fuzzy-weighted average (NFWA) method to engineering design evaluation. *International Journal of Production Research, 39*(6), 1147–1162.

Vincke, P. (1992). *Multicriteria decision-aid*. New York: John Wiley & Sons.

Von Altrock, C. (1995). *Fuzzy logic & neuro-fuzzy applications explained*. Englewood Cliffs: Prentice Hall.

Wu, H. Y., Chen, J. K., & Chen, I. S. (2010). Innovation capital indicator assessment of Taiwanese Universities: A hybrid fuzzy model application. *Expert Systems with Applications, 37*, 1635–1642.

Wu, H. Y., Tzeng, G. H., & Chen, Y. H. (2009). A fuzzy MCDM approach for evaluating banking performance based on balanced scorecard. *Expert Systems with Applications, 36*, 10135–10147.

Yager, R. R. (1981). A procedure for ordering fuzzy subsets of the unit interval. *Information Sciences, 24*, 143–161.

Yager, R. R., & Filev, D. P. (1994). *Essentials of fuzzy modeling and control*. New York: John Wiley & Sons, Inc.

Yu, P. L. (1973). A class of solutions for group decision problems. *Management Science, 19*(8), 936–946.

Yu, P. L. (1990). *Forming winning strategies: An integrated theory of habitual domains*. Heidelberg: Springer-Verlag.

Zadeh, L. A. (1987). The concept of a linguistic variable and its application to approximate reasoning. In R. R. Yager et al. (Eds.), *Fuzzy sets and applications: Selected papers by L.A. Zadeh* (pp. 219–269). New York: John Wiley & Sons.

Zimmermann, H. J. (1987). *Fuzzy sets, decision making, and expert systems*. Boston: Kluwer Academic Publishers.

Zimmermann, H. J. (1991). *Fuzzy set theory and its applications*. Boston: Kluwer Academic Publishers.